

Ubestemte integral

Regneregler:

$$\textcircled{1} \int (u \pm v) dx = \int u dx \pm \int v dx$$

$$\textcircled{2} \int c \cdot u dx = c \cdot \int u dx$$

$$\textcircled{3} \int x^n dx = \frac{1}{n+1} x^{n+1} + C$$

$$\textcircled{4} \int \sin x dx = -\cos x + C$$

$$\textcircled{5} \int \cos x dx = \sin x + C$$

$$\textcircled{6} \int (\tan^2 x + 1) dx = \tan x + C$$

$$\textcircled{7} \int e^x dx = e^x + C$$

$$\textcircled{8} \int \frac{1}{x} dx = \ln |x| + C$$

Kjerne-regel og substitusjon (15.5)

Ex:
$$\int \frac{1}{x+1} dx = \underline{\underline{\ln |x+1| + C}}$$

$$\ln |x+1|' = \frac{1}{x+1} \cdot 1 = \frac{1}{x+1}$$

Exo: $\int \frac{1}{1-x} dx = -\ln|1-x| + C$

$$\ln|1-x|' = \frac{1}{1-x} \cdot (-1)$$

$$\int \frac{1}{2x+1} dx = \frac{1}{2} \cdot \ln|2x+1| + C$$

$$\frac{1}{2} \ln|2x+1|' = \frac{1}{2} \left(\frac{1}{2x+1} \cdot 2 \right)$$

$$\int e^{2x} dx = \frac{1}{2} e^{2x} + C$$

$$(e^{2x})' = e^{2x} \cdot 2$$

$$\int \sin(2x) dx = -\frac{1}{2} \cos(2x) + C$$

Substitusjon:

$$\int \sin(2x) dx = \int \sin(u) \frac{du}{2}$$

$$= \int \frac{1}{2} \sin(u) du = \frac{1}{2} (-\cos u) + C$$

$$= -\frac{1}{2} \cos(2x) + C$$

$$\left. \begin{array}{l} u = 2x \\ \textcircled{du = u' \cdot dx} \end{array} \right|$$

$$\frac{du}{2} = \frac{2 dx}{2}$$

$$dx = \frac{du}{2}$$

Eks:

$$\begin{aligned}\int (2x+1)^3 dx &= \int u^3 \frac{du}{2} \\ &= \frac{1}{2} \int u^3 du = \frac{1}{2} \cdot \frac{1}{4} u^4 + C \\ &= \underline{\underline{\frac{1}{8} \cdot (2x+1)^4 + C}}\end{aligned}$$

$$\begin{cases} u = 2x+1 \\ u' = 2 \\ du = 2 dx \end{cases}$$

Eks:

$$\begin{aligned}\int \frac{x}{x^2+1} dx &= \\ &= \int \frac{\cancel{x}}{u} \frac{du}{2\cancel{x}} = \frac{1}{2} \int \frac{1}{u} du \\ &= \underline{\underline{\frac{1}{2} \ln |u| + C = \frac{1}{2} \ln(x^2+1) + C}}\end{aligned}$$

$$\begin{cases} u = x^2+1 \\ du = 2x \cdot dx \end{cases}$$

$$dx = \frac{du}{2x}$$

$$\int \frac{1}{x^2+1} dx = \int \frac{1}{u} \frac{du}{2x}$$

$$= \int \frac{1}{u \cdot 2x} \cdot du \quad \text{gør ikke}$$

$$\begin{cases} u = x^2+1 \\ du = 2x \cdot dx \end{cases}$$

Generelt:

$$u = \text{uttrykk i } x$$
$$\boxed{du = u' \cdot dx}$$

$$\left. \begin{aligned} u' &= \frac{du}{dx} \\ u' \cdot dx &= du \end{aligned} \right\}$$