19/03/09: $\begin{cases} \text{Omvendte funksjoner} \\ \text{Inverse funksjoner} \end{cases}$ (kap. 11.9)

Eks: $f(x) = x^2 + 2x \;,\; \boxed{x \geq -1}$

$= (x+1)^2 - 1$

$y = 3 \longrightarrow x = ?$

$y = 4 \to x = ?$

$y = x^2 + 2x$

$3 = x^2 + 2x$

$0 = x^2 + 2x - 3$

$x = \dfrac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} = \dfrac{-2 \pm 4}{2}$

$\cancel{x = -3}$ eller $x = 1$

$x = 1$

Formel for x:

$y = x^2 + 2x$

$0 = x^2 + 2x - y$

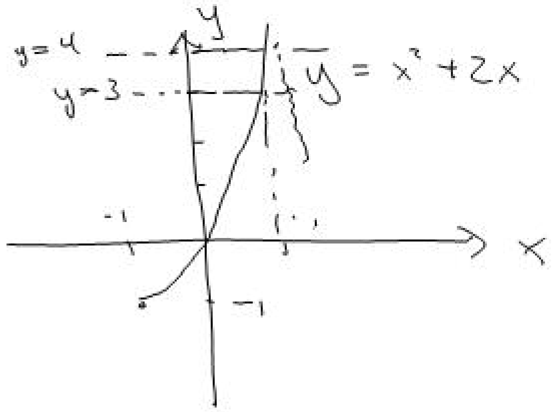
$x = \dfrac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot (-y)}}{2 \cdot 1}$

$= \dfrac{-2 \pm \sqrt{4 + 4y}}{2} = \dfrac{-2 \pm 2\sqrt{1+y}}{2}$

$= \sqrt{4} \cdot \sqrt{1+y}$

$x = -1 \pm \sqrt{1+y}$ $\qquad$ $x = -1 + \sqrt{1+y}$

$$f(x) = x^2 + 2x, \quad x \geq -1$$



$$x = -1 + \sqrt{1+y}$$

$$f^{-1}(y) = -1 + \sqrt{1+y}, \quad y \geq -1$$

den omvendte funksjonen til f

$$y=3: \quad x = f^{-1}(3) = -1 + \sqrt{1+3} = 1$$

$$y=4: \quad x = f^{-1}(4) = -1 + \sqrt{1+4} = \sqrt{5} - 1 \simeq 1.23$$

$$f(x) = x^2 + 2x, \quad D_f = [-1, \infty), \quad V_f = [-1, \infty)$$

$$f^{-1}(y) = -1 + \sqrt{1+y}, \quad D_{f^{-1}} = [-1, \infty), \quad V_{f^{-1}} = [-1, \infty)$$

Hvis $f(x)$ er en funksjon med defn. mengde D_f og verdimengde V_f:

* Hvis $f(x)$ er voksende på hele D_f eller avtagende på hele D_f, så fins en omvendt funksjon. f^{-1}

$$x = f^{-1}(y) \quad , \quad D_{f^{-1}} = V_f \quad (y\text{-verdier})$$
$$V_{f^{-1}} = D_f \quad (x\text{-verdiene})$$

Vanlige omvendte funksjoner:

① $f(x) = x^2$, $D_f = [0,\infty)$, $V_f = [0,\infty)$

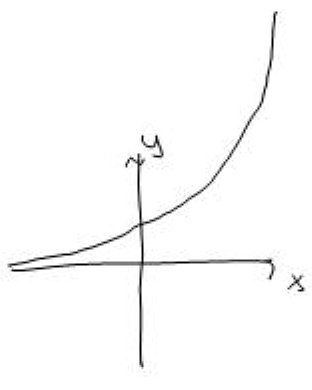
$f^{-1}(y) = \sqrt{y}$, $D_{f^{-1}} = [0,\infty)$, $V_{f^{-1}} = [0,\infty)$

$y = x^2$ ← løser likningen for x

$x = \sqrt{y}$

② $f(x) = e^x$, $D_f = \overset{x}{(-\infty,\infty)}$, $V_f = \overset{y}{(0,\infty)}$

$f^{-1}(y) = \ln(y)$, $D_{f^{-1}} = \underset{y}{(0,\infty)}$, $V_{f^{-1}} = \underset{x}{(-\infty,\infty)}$



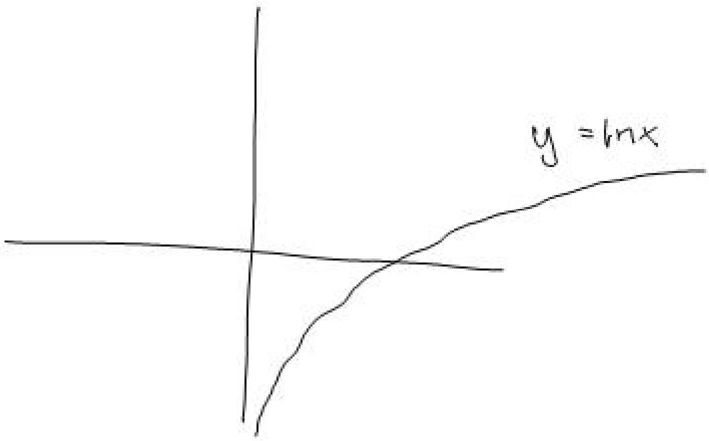
$y = e^x \qquad x = \ln(y)$

Asymptoter og logaritmer

$f(x) = \ln x$,

$x > 0$

$f'(x) = \frac{1}{x}$



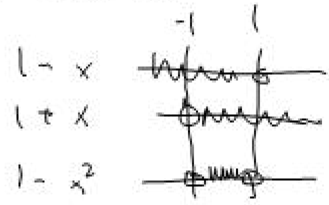
<u>Eks:</u> $f(x) = \ln(1 - x^2)$

$D_f = (-1, 1)$

$1 - x^2 > 0$

$\shortparallel$

$(1-x)(1+x)$



<u>Asymptoter:</u>

<u>Vertikale:</u> $x = a$ vertikal asymptote hvis $\lim_{x \to a} f(x) = \pm\infty$

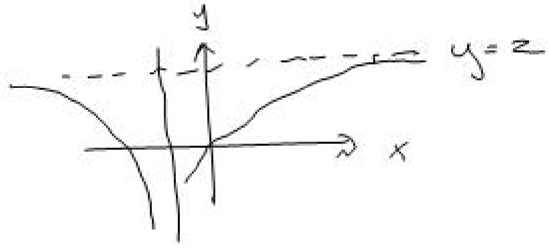
$\underline{x = 1}$ og $\underline{x = -1}$ er vertikale asymptoter for $f(x) = \ln(1 - x^2)$

$$\left(\lim_{x \to 1} \ln(1 - x^2) = -\infty \right)$$

Horisontale / skrå asymptoter:

$$f(x) = \ln(1 - x^2)$$

Horisontale:



$y = a$ horisontal asymptote
hvis

$$\lim_{x \to \infty} f(x) = a$$

eller

$$\lim_{x \to -\infty} f(x) = a$$

$$\lim_{x \to \infty} \ln(1 - x^2) = \text{ikke definert}$$

Konklusjon: $f(x) = \ln(1 - x^2)$ har ingen horisontalen eller skrå asymptoter.

Eks: $f(x) = \ln(1 + x^2)$

Horisontale:

$$\lim_{x \to \infty} \ln(1 + x^2) = \infty$$

(tilsvarende for $x \to -\infty$)

$x \to \infty$ betyr $1 + x^2 \to \infty$

$\ln(1 + x^2) \to \infty$

ingen horisontale asymptoter.

Skrå: $\lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{\ln(1 + x^2)}{x} = 0$

ingen skrå-asymptote (fordi $a = 0$)