17/03/09: $\begin{cases} \text{Logaritmer} \\ \text{Eksponensial funksjoner} \end{cases}$

Derivasjonsregler:

(1) $(e^x)' = e^x$

(2) $(\ln x)' = \frac{1}{x}$

Eks:

$$f(x) = \frac{e^x}{e^x+1}$$

$$f'(x) = \frac{(e^x)' \cdot (e^x+1) - e^x \cdot (e^x+1)'}{(e^x+1)^2}$$

$$= \frac{e^x \cdot (e^x+1) - e^x \cdot e^x}{(e^x+1)^2}$$

$$= \frac{\overset{e^{2x}}{\cancel{(e^x)^2}} + e^x - \overset{e^{2x}}{\cancel{(e^x)^2}}}{(e^x+1)^2} = \frac{e^x}{(e^x+1)^2}$$

Eks:

$$f(x) = \ln(x^2+1)$$

$$f'(x) = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$$

Omforming og derivasjon av $\begin{cases} a^x \\ \log_a(x) \end{cases}$

Omforming av eksponentialfunksjoner

Eks: $f(x) = 4 \cdot 2^x$

$= 4 \cdot (e^{\ln 2})^x$

$= 4 \cdot e^{\ln 2 \cdot x}$

$\approx \underline{\underline{4 \cdot e^{0.69x}}}$

Skriver om til eksponentialfunksjon med grunntall e

$2 = e^{\ln 2}$

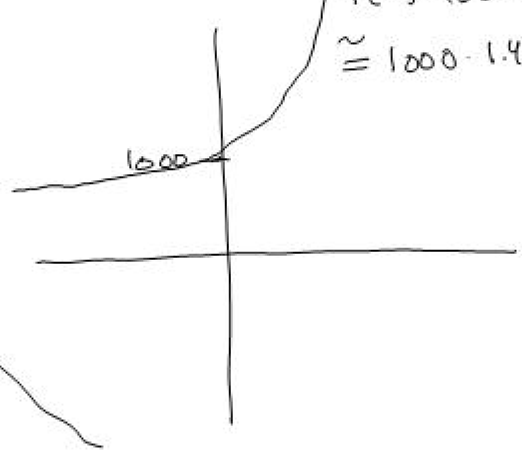
Eks: $f(x) = C \cdot e^{kx}$

$\begin{cases} C = 1000 \\ k = 0.4 \end{cases}$

$= 1000 \cdot e^{0.4x}$

$= 1000 \, (e^{0.4})^x$

$= 1000 \cdot 1.49^x$



$f(x) = 1000 \cdot e^{0.4x}$
$\approx 1000 \cdot 1.49^x$

Eks: $f(x) = a^x = e^{\ln a \cdot x}$

$C = 1000$

$k = 0.4 \rightsquigarrow a = e^{0.4}$

≈ 1.49

$\rightsquigarrow r \approx 49\%$

$f'(x) = (a^x)'$

$= (e^{\ln a \cdot x})'$

$= e^{\ln a \cdot x} \cdot \ln a$

$(e^u)' = e^u \cdot u'$

$= a^x \cdot \ln a$

$\lim_{h \to 0} \frac{a^h - 1}{h}$

Formel: $\boxed{(a^x)' = a^x \cdot \ln a}$

Eks: $(2^x)' = 2^x \cdot \ln 2 \approx \underline{0.69 \cdot 2^x}$

$(1.07^x)' = \underline{1.07^x \cdot \ln 1.07}$

Omforming av logaritmer

Formel: $\boxed{\log_a(x) = \frac{\ln x}{\ln a}}$

Eks: $\log_3(x) = \frac{\ln(x)}{\ln(3)} \simeq \frac{\ln(x)}{1.09}$

$\log_3(x) = \frac{1}{\ln(3)} \cdot \ln(x) \simeq \frac{1}{1.09} \cdot \ln(x)$

Eks: $(\log_3(x))' = \left(\frac{1}{\ln(3)} \cdot \ln(x)\right)'$

$= \frac{1}{\ln(3)} \cdot (\ln x)' = \frac{1}{\ln 3} \cdot \frac{1}{x} \simeq \underline{\underline{0.9 \cdot \frac{1}{x}}}$

Formel: $\boxed{(\log_a(x))' = \frac{1}{\ln a} \cdot \frac{1}{x}}$

Derivasjons regler:

(1) $(e^x)' = e^x$

(2) $(\ln x)' = \frac{1}{x}$

(3) $(a^x)' = \ln a \cdot a^x$

(4) $(\log_a x)' = \frac{1}{\ln a} \cdot \frac{1}{x}$

Eks: Funksjons drøfting av

$$f(x) = \ln(x^2+1) \quad . \quad D_f = (-\infty, \infty)$$

(a) Nullpkt: Skjæring med x-aksen $\Leftrightarrow$ $y=0$

$$\ln(x^2+1) = 0 \quad | \text{ bruker } e^x$$

$$e^{\ln(x^2+1)} = e^0$$

$$x^2+1 = 1$$

$$x^2 = 0$$

$$\underline{x = 0}$$

Skjæringspkt med y-aksen $\Leftrightarrow$ $x=0$

$$y = f(0) = \ln(0^2+1) = \ln(1) = 0$$

$$\underline{y=0}$$

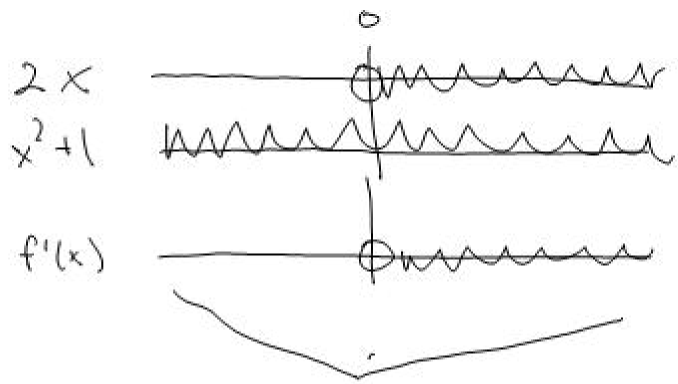
(b) Derivasjon:

$$f'(x) = (\underbrace{\ln(x^2+1)}_{\ln(u)})' = \frac{1}{u} \cdot u' = \frac{1}{x^2+1} \cdot 2x$$

$$= \underline{\underline{\frac{2x}{x^2+1}}}$$

(c) Lokale topp / bunnpkt

$$f'(x) = \frac{2x}{x^2+1}$$



(voksende for $x \geqslant 0$
avtagende for $x \leqslant 0$)

Lokalt bunnpkt: $x = 0 \quad y = f(0) = 0$

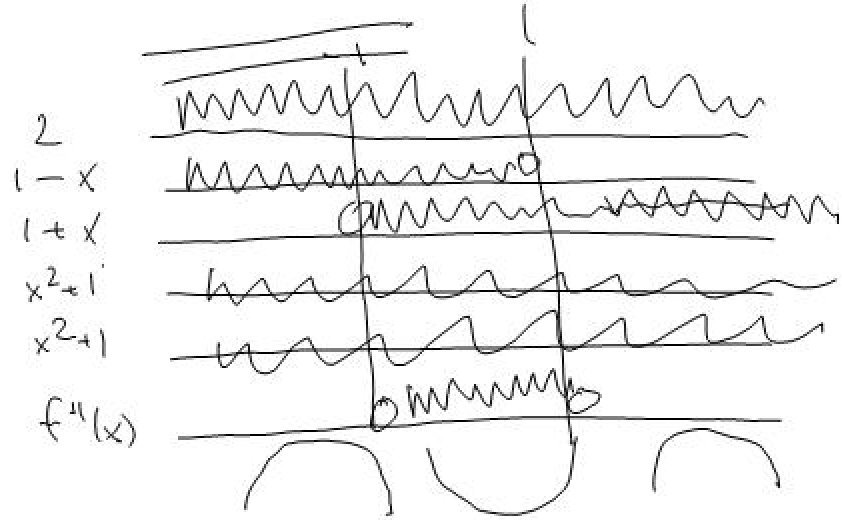
$\Rightarrow \underline{\underline{(0,0)}}$

(d) $$f''(x) = \left(\frac{2x}{x^2+1}\right)' = \frac{2\cdot(x^2+1) - 2x\cdot 2x}{(x^2+1)^2}$$

$$= \frac{2x^2+2-4x^2}{(x^2+1)^2} = \underline{\underline{\frac{2-2x^2}{(x^2+1)^2}}} = \frac{2(1-x^2)}{(x^2+1)^2}$$

$$= \frac{2(1-x)(1+x)}{(x^2+1)^2}$$

(e)



Vendepkt:

$x=1 \quad y = \ln 2$

$x=-1 \quad y = \ln 2$

$\underline{\underline{(1, \ln 2) \text{ og } (-1, \ln 2)}}$

Eks: $f(x) = x \cdot e^x$, $D_f = (-\infty, \infty)$

(a) Nullpkt: $f(x) = 0$

$x \cdot e^x = 0$

$\underline{\underline{x = 0}}$ eller $e^x = 0$

$(x = \ln(0))$

ingen løsning

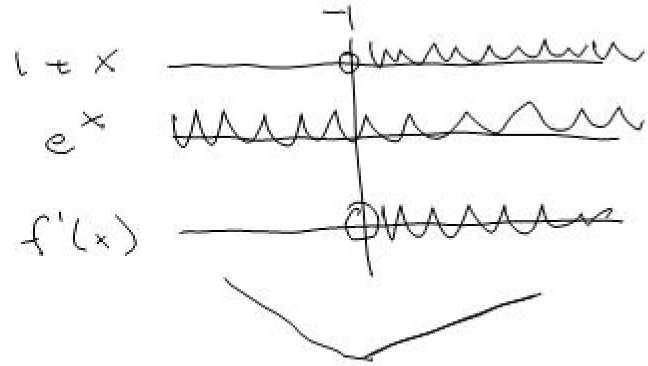
Nullpkt: $\underline{\underline{(0,0)}}$

(b) Derivasjon:

$f'(x) = (x \cdot e^x)' = 1 \cdot e^x + x \cdot e^x$

$= \underline{\underline{e^x + x \cdot e^x}} = \underline{\underline{(1+x)e^x}}$

(c) Lokale topp-/bunnpkt:



Lokalt bunnpkt:

$(-1, f(-1))$

$= (-1, -1 \cdot e^{-1})$

$= \underline{\underline{\left(-1, -\frac{1}{e}\right)}}$

d)

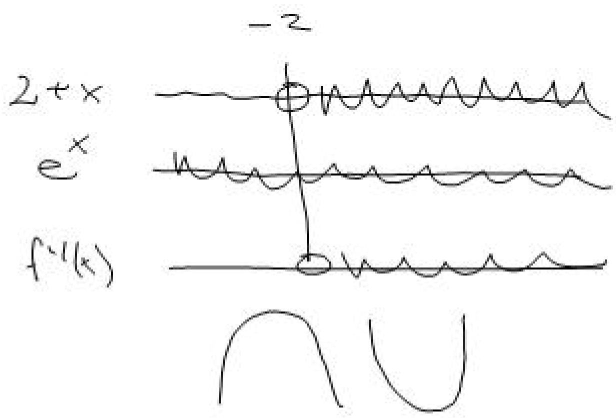
$$f''(x) = \big(\underbrace{e^x + x e^x}_{=(1+x)\cdot e^x}\big)'$$

$$= (1+x)' \cdot e^x + (1+x)\cdot (e^x)'$$

$$= 1 \cdot e^x + (1+x)\cdot e^x = e^x + (1+x)\cdot e^x$$

$$= \underline{\underline{(2+x)\cdot e^x}}$$

e)



Vendepkt:

$$(-2, f(-2))$$

$$= (-2, -2\cdot e^{-2})$$

$$= \underline{\underline{\left(-2, -\frac{2}{e^2}\right)}}$$