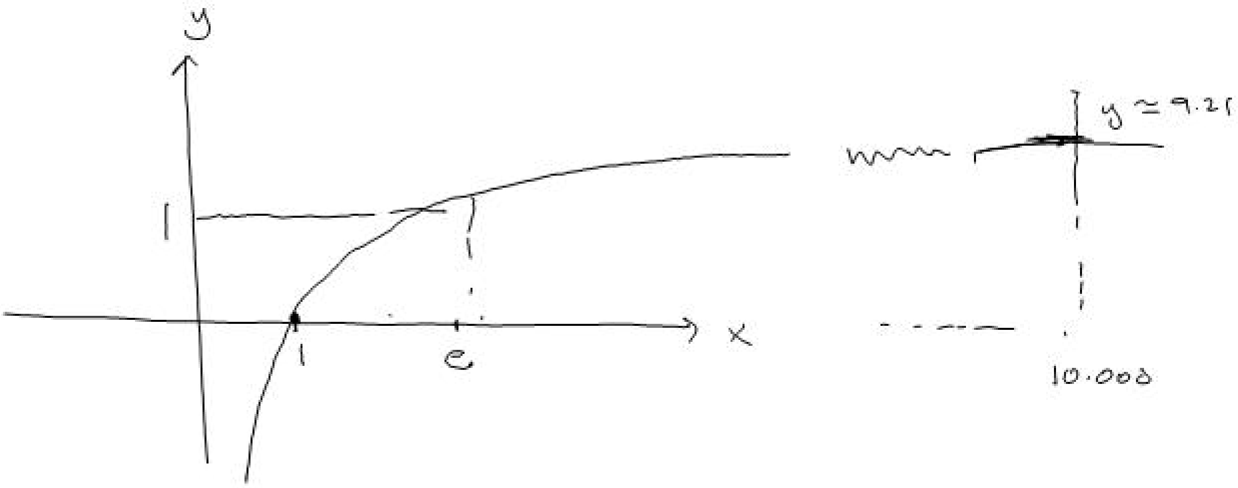
12/03/2009

Naturlig logaritme

$f(x) = \ln(x) \quad , \quad x > 0$



Eks:

$\ln(1) = 0$
$\ln(e) = 1$
$\ln(10.000) \simeq 9.21$

$f'(x) = \dfrac{1}{x}$

$\ln(e^x) = x$

$e^{\ln x} = x$

$(e^{\ln x})' = (x)'$

$(e^u)' = 1$

$e^u \cdot u' = 1$

$e^{\ln x} \cdot u' = 1$

$u' = \dfrac{1}{e^{\ln x}} = \dfrac{1}{x}$

$\boxed{\begin{array}{l} u = \ln x \\ u' = ? \end{array}}$

$\boxed{(e^x)' = e^x}$

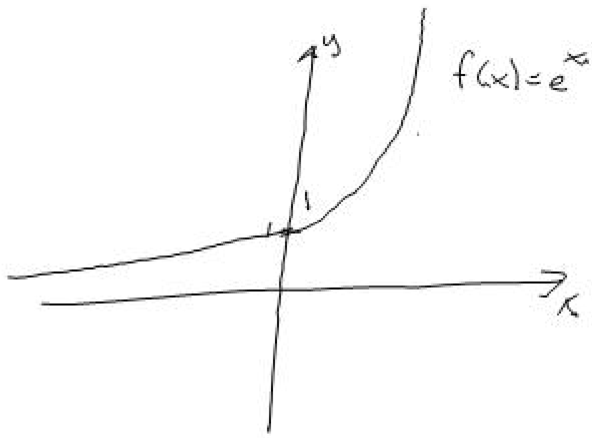
Oppsummering:

① $e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n \simeq 2.71828$ Euler-tallet

② Eksponensialfunksjonen med grunntall e

$$f(x) = e^x$$

$$f'(x) = e^x$$



③ Naturlig logaritme

$$f(x) = \ln(x) \quad , x > 0$$

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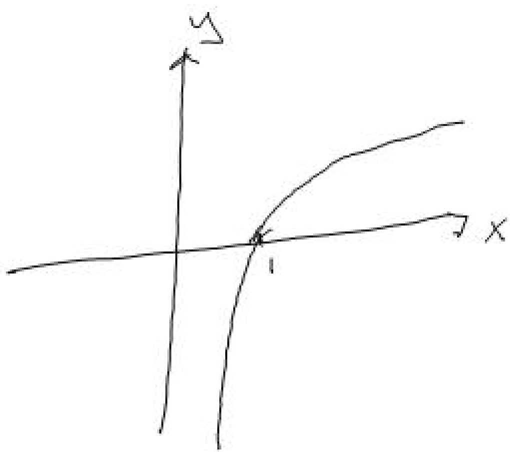

Løse likninger med e^x og $\ln x$

Eks: $e^x = 7$

$\ln(e^x) = \ln(7)$

$\underline{x = \ln(7)} \approx \underline{1.95}$

Eks: $2^x = 5$

$\ln(2^x) = \ln(5)$

$\frac{x \cdot \cancel{\ln(2)}}{\cancel{\ln(2)}} = \frac{\ln(5)}{\ln(2)}$

$x = \underline{\frac{\ln(5)}{\ln(2)}} \approx \underline{2.32}$

$$\boxed{\begin{aligned}
&(1)\quad \ln(a \cdot b) = \ln(a) + \ln(b) \\
&(2)\quad \ln(a/b) = \ln(a) - \ln(b) \\
&(3)\quad \ln(a^b) = b \cdot \ln(a) \\
&(4)\quad \log_a(b) = \frac{\ln(b)}{\ln(a)}
\end{aligned}}$$

$2^x = 5$

$\log_2(2^x) = \log_2(5)$

$x = \log_2(5)$

$= \frac{\ln(5)}{\ln(2)} \approx \underline{2.32}$

$= \frac{\log(5)}{\log(2)} \approx \underline{\underline{2.32}}$

Eb:

$$\frac{\cancel{1000}\cdot 1.07^{x}}{\cancel{1000}} = \frac{5000}{1000}$$

$$1.07^{x} = 5$$

$$\ln(1.07^{x}) = \ln(5)$$

$$x\cdot\ln(1.07) = \ln 5$$

$$x = \frac{\ln(5)}{\ln(1.07)} \approx \underline{\underline{23.8}}$$

$$1000\cdot 1.07^{x} = 5000$$

$$\ln(1000\cdot 1.07^{x}) = \ln(5000)$$

$$\ln(1000) + \ln(1.07^{x}) = \ln(5000)$$

$$\ln(1000) + x\cdot\ln(1.07) = \ln(5000)$$

$$\frac{x\cdot\cancel{\ln(1.07)}}{\cancel{\ln(1.07)}} = \frac{\ln(5000) - \ln(1000)}{\ln(1.07)}$$

$$x = \frac{\ln\left(\frac{5000}{1000}\right)}{\ln(1.07)} = \frac{\ln(5)}{\ln(1.07)} \approx \underline{\underline{23.8}}$$

Eb:

$$\frac{\cancel{1000}\cdot 3^{x}}{\cancel{1000}} = \frac{1500\cdot 2^{x}}{1000}$$

$$3^{x} = 1.5\cdot 2^{x} \longrightarrow \frac{3^{x}}{2^{x}} = 1.5 \Rightarrow \left(\frac{3}{2}\right)^{x} = 1.5$$

$$1.5^{x} = 1.5$$

$$\underline{\underline{x = 1}}$$

$$\ln(3^{x}) = \ln(1.5\cdot 2^{x})$$

$$x\cdot\ln(3) = \ln(1.5) + \ln(2^{x})$$

$$x\cdot\ln(3) = \ln(1.5) + x\cdot\ln(2)$$

$$x\cdot\ln(3) - x\cdot\ln(2) = \ln(1.5)$$

$$\frac{x\cdot\cancel{(\ln 3 - \ln 2)}}{\cancel{\ln 3 - \ln 2}} = \frac{\ln 1.5}{\ln 3 - \ln 2}$$

$$x = \frac{\ln 1.5}{\ln 3 - \ln 2} = \frac{\ln 1.5}{\ln\left(\frac{3}{2}\right)} = \underline{\underline{1}}$$

Eks:

$$e^x - 3 = e^{-x}$$

$$\ln(e^x - 3) = \ln(e^{-x})$$

$$\nearrow ? \quad = -x$$

ingen regneregler
for $\ln(a \pm b)$

må prøve en annen metode

$$e^x - 3 = e^{-x} \quad \Big| \cdot e^x \qquad\qquad e^{-x} = \frac{1}{e^x}$$

$$e^x \cdot e^x - 3e^x = 1$$

$$(e^x)^2 - 3e^x - 1 = 0 \qquad\qquad \underline{u = e^x}:$$

$$u^2 - 3u - 1 = 0$$

$$u = \frac{3 \pm \sqrt{9 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1} = \frac{3 \pm \sqrt{13}}{2}$$

$$e^x = \frac{3 + \sqrt{13}}{2} \quad \text{eller} \quad e^x = \frac{3 - \sqrt{13}}{2}$$

$$x = \ln\left(\frac{3 + \sqrt{13}}{2}\right) \approx \underline{\underline{1.19}}$$

ingen løsning
siden $\frac{3 - \sqrt{13}}{2} < 0$

<u>Eks:</u> $\ln x = 4$ | bruker e^x

$$e^{\ln x} = e^4$$

$$x = \underline{\underline{e^4}} \approx 54.6$$

<u>Eks:</u> $2\ln x - 1 = 3$

$$\frac{\cancel{2}\ln x}{\cancel{2}} = \frac{3+1}{2}$$

$\ln x = 2$ | bruker e^x på begge sider

$$e^{\ln x} = e^2$$

$$x = \underline{\underline{e^2}} \approx 7.4$$

<u>Eks:</u> (11.44b)

$$\ln(8x^2) - 2\ln(2x)$$

$$= \ln(8) + \ln(x^2) - 2\cdot(\ln(2) + \ln(x))$$

$$= \ln(8) + \ln(x^2) - 2\ln(2) - 2\ln(x)$$

$$= \ln(2^3) + 2\ln(x) - 2\ln(2) - 2\ln(x)$$

$$= 3\ln(2) + \cancel{2\ln(x)} - 2\ln(2) - \cancel{2\ln(x)}$$

$$= \underline{\underline{\ln(2)}}$$