

Kopi av Matematikk Føreløp (Skolelærere).

- Særlig kap 8.2 i kopien (likninger og ulikheter) står bedre enn i sinus.

Eksempel V07, oppg. 3

Les: $f(x) = 4 \sin^2 x - 1, x \in [0, 2\pi)$

a) Skjæringspunkt med x -aksen
= nullpunkt: $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$$f(x) = 0$$

$$4 \sin^2 x - 1 = 0$$

$$\frac{4 \sin^2 x}{4} = \frac{1}{4}$$

$$\sin^2 x = 1/4$$

$$\sin x = \pm \sqrt{1/4} = \pm 1/2$$

$$\underline{\sin x = 1/2}$$

eller

$$\underline{\sin x = -1/2}$$

$$x = \sin^{-1}(1/2) + n \cdot 2\pi \\ = \pi/6 + n \cdot 2\pi = \underline{\pi/6}$$

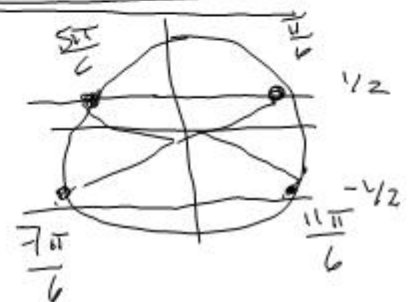
eller

$$x = \pi - \pi/6 + n \cdot 2\pi \\ = \underline{\underline{5\pi/6}}$$

$$x = \sin^{-1}(-1/2) + n \cdot 2\pi \\ = -\pi/6 + n \cdot 2\pi = \underline{\underline{11\pi/6}}$$

eller

$$x = \pi - (-\pi/6) + n \cdot 2\pi \\ = \underline{\underline{7\pi/6}}$$

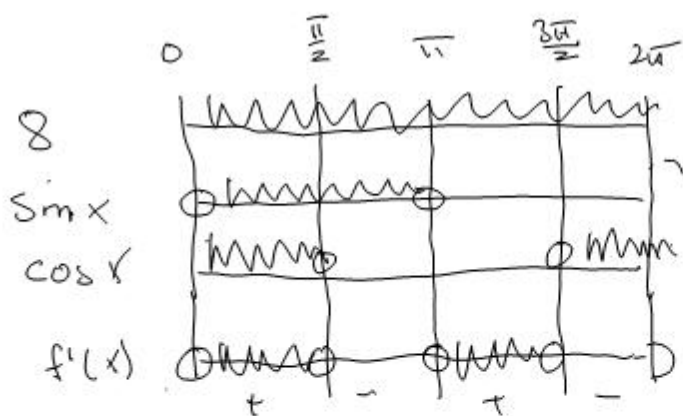


b) $f'(x) = (4\sin^2 x - 1)' = 4 \cdot (\sin^2 x)' = 0$ u med u = \sin x

$$= 4 \cdot \underbrace{2\sin x}_{2u} \cdot \underbrace{\cos x}_{u'} = \underline{\underline{8\sin x \cdot \cos x}}$$

c) Fortegnsskjema for $f'(x)$:

$$f'(x) = 8 \sin x \cdot \cos x$$



$$\underline{\sin x = 0}$$

$x = 0$
eller
 $x = \pi$

$$\underline{\cos x = 0:}$$

$x = \frac{\pi}{2}$
eller
 $x = 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$



$f'(x) \geq 0$: $x \in [0, \frac{\pi}{2}]$
eller
 $x \in [\pi, \frac{3\pi}{2}]$

$$f(x) = 4\sin^2 x - 1$$

d) Lokale topp-plot:

$$x = \frac{\pi}{2}$$

$$y = f\left(\frac{\pi}{2}\right) = 3$$

$$x = \frac{3\pi}{2}$$

$$y = f\left(\frac{3\pi}{2}\right) = 3$$

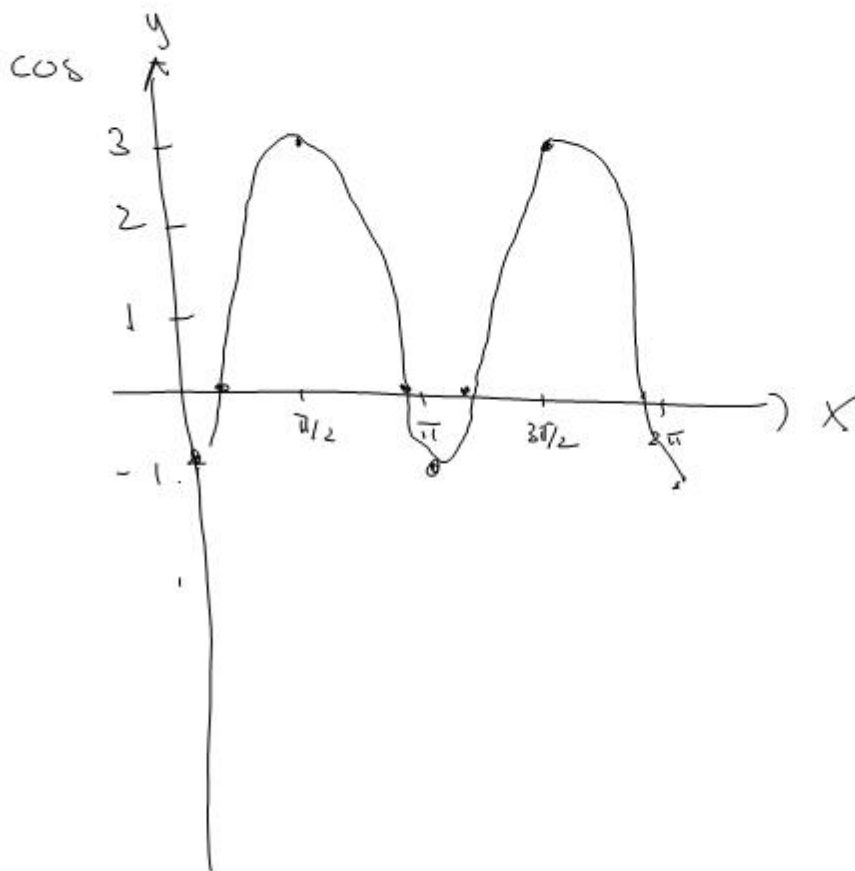
Lokale bunnplot:

$$x = 0$$

$$y = f(0) = -1$$

$$x = \pi$$

$$y = f(\pi) = -1$$



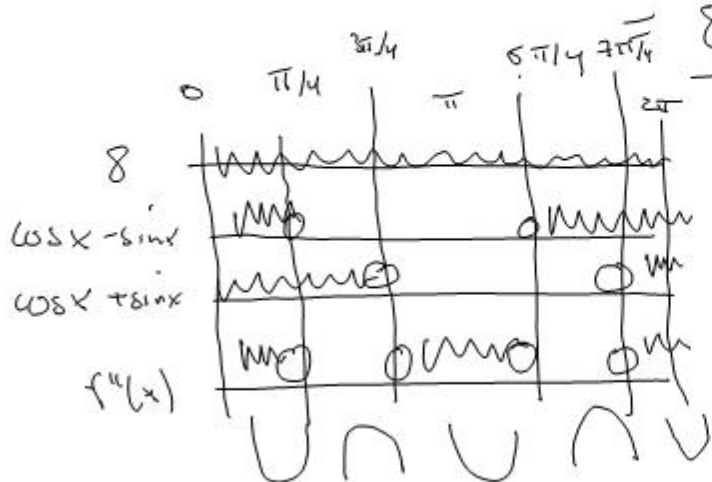
Verdehlt:

$$\begin{aligned}
 f''(x) &= (8 \sin x \cos x)' \\
 &= 8 \cdot (\sin x \cdot \cos x)' \\
 &= 8 \cdot (\cos x \cdot \cos x + \sin x \cdot (-\sin x)) \\
 &= \underline{8 \cdot (\cos^2 x - \sin^2 x)}
 \end{aligned}$$

$$(u \cdot v)' = u' \cdot v + u \cdot v'$$

$$\begin{aligned}
 u &= \sin x \\
 u' &= \cos x \\
 v &= \cos x \\
 v' &= -\sin x
 \end{aligned}$$

$$\underline{8 \cdot (\cos x - \sin x) \cdot (\cos x + \sin x)}$$



$$\cos x - \sin x = 0 \quad / \cos x$$

$$1 - \tan x = 0$$

$$\tan x = 1$$

$$x = \pi/4 + n \cdot \pi$$

$$= \pi/4, 5\pi/4$$

$$\cos x + \sin x = 0 \quad / \cos x$$

$$1 + \tan x = 0$$

$$\tan x = -1$$

$$x = -\frac{\pi}{4} + n \cdot \pi = \underline{\underline{\frac{3\pi}{4}, \frac{7\pi}{4}}}}$$

Konklusjon:

Vendepkt. i

$$x = \underline{\underline{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}}}$$