Derivasjon av trigonometriske funksjoner

(kap. 10.8-10.9)

Derivasjonsregler:

$$(\sin x)' = \cos x \qquad (\tan x)' = \frac{1}{\cos^2 x}$$

$$(\cos x)' = -\sin x \qquad = 1 + \tan^2 x$$

Eks: $(2\sin x - 1)' = 2 \cdot (\sin x)' - 0$

$$= \underline{2\cos x}$$

$$(\cos^2 x)' = ((\cos x)^2)'$$

$$= 2 \cdot \cos x \cdot (\cos x)'$$

$$= 2 \cdot \cos x \cdot (-\sin x)$$

$$= \underline{-2 \sin x \cdot \cos x}$$

$$(\tan x)' = \left(\frac{\sin x}{\cos x}\right)' = \left(\frac{u}{v}\right)'$$

$$= \frac{u' \cdot v - u \cdot v'}{v^2} = \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \underline{\underline{\frac{1}{\cos^2 x}}}$$

$$= \underline{\underline{1 + \tan^2 x}}$$

Eks:

$$f(x) = 2 \cdot \sin(\pi x - \pi/2) + 3\cos(2x)$$

$$f'(x) = 2\cos(\pi x - \pi/2) \cdot \pi + 3 \cdot (-\sin(2x)) \cdot 2$$

$$= \underline{2\pi \cdot \cos(\pi x - \pi/2) - 6\sin(2x)}$$

Eks:

$$f(x) = \frac{\cos x + 1}{\sin x} = \frac{u}{v}$$

$$\boxed{\begin{array}{ll} u = \cos x + 1 & v = \sin x \\ u' = -\sin x & v' = \cos x \end{array}}$$

$$f'(x) = \frac{u' \cdot v - u \cdot v'}{v^2}$$

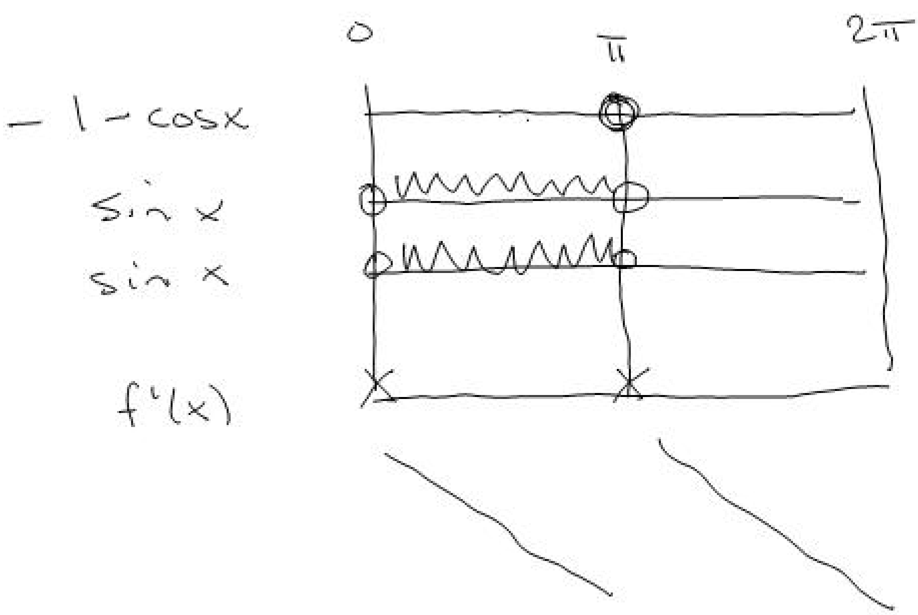
$$= \frac{-\sin x \cdot \sin x - (\cos x + 1) \cdot \cos x}{\sin^2 x}$$

$$= \frac{-\sin^2 x - \cos^2 x - \cos x}{\sin^2 x}$$

$$-(\sin^2 x + \cos^2 x)$$

$$= \underline{\underline{\frac{-1 - \cos x}{\sin^2 x}}}$$

$$\underline{D_f = [0, 2\pi)}$$



$\underline{\sin x = 0:}$

$x = 0 + n \cdot 2\pi = \underline{0}$

eller

$x = \pi + n \cdot 2\pi = \underline{\pi}$

$\underline{-1 - \cos x = 0:}$

$-1 = \cos x$

$x = \cos^{-1}(-1) + n \cdot 2\pi$

$= \pi + n \cdot 2\pi = \pi$

eller

$\left(\begin{array}{l} x = 2\pi - \pi + n \cdot 2\pi \\ = \pi + n \cdot 2\pi = \pi \end{array} \right)$

Utregning av $(\sin x)'$:

$$f(x) = \sin x$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h \to 0} \frac{\sin x \cdot \cos h + \cos x \cdot \sin h - \sin x}{h}$$

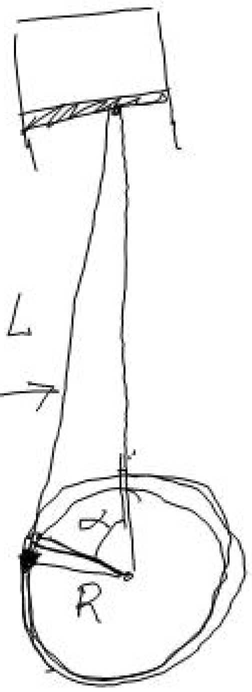
$$= \lim_{h \to 0} \frac{\sin x \cdot (\cos h - 1) + \cos x \cdot \sin h}{h}$$

$$= \sin x \cdot \boxed{\lim_{h \to 0} \frac{\cos h - 1}{h}} + \cos x \cdot \boxed{\lim_{h \to 0} \frac{\sin h}{h}}$$

$$= \sin x \cdot 0 + \cos x \cdot 1 = \underline{\underline{\cos x}}$$

Eks:

$$v(\alpha) = \omega R \cdot \left(\sin\alpha + \frac{R}{2L}\cdot\sin(2\alpha)\right)$$



v : hastigheten til stempelet

α : vinkelen

L : lengden av stangen

R : radius i sirkelen

ω : vinkelhastighet

Eks:

n omdreiinger per min.

$$\frac{2\pi R \cdot n}{60} = \text{gjennomsnittshastighet i m/s.}$$

$$\parallel$$

$$\omega R$$

$$\omega = \frac{2\pi n}{60} \quad \left(\frac{1}{s}\right) \quad \text{vinkelhastighet}$$

$\omega R = 7\ m/s$	$R = 1\ m$
$R/L = 0.24$	$\omega = 7\ rad/s$
$R/2L = 0.12$	$L \approx 4\ m$

$$v(\alpha) = \omega R \cdot \left(\sin\alpha + \frac{R}{2L}\sin(2\alpha)\right)$$

$$= 7 \cdot (\sin\alpha + 0.12 \cdot \sin(2\alpha))$$

α : rad.
v : m/s

~~MAKE~~

$$v(\alpha) = 7 \cdot \sin\alpha + 0.84 \cdot \sin(2\alpha) \qquad \alpha \in [-\pi\ \pi]$$

$$v'(\alpha) = 7 \cdot \cos\alpha + 0.84 \cdot \cos(2\alpha) \cdot 2$$

$$= 7\cos\alpha + 1.68 \cdot \cos(2\alpha)$$

$$= 7\cos\alpha + 1.68\,(\cos^2\alpha - \sin^2\alpha)$$

$$= 7 \cdot \cos\alpha + 1.68\cos^2\alpha - 1.68\sin^2\alpha$$

$$\boxed{\sin^2\alpha = 1-\cos^2\alpha}$$

$\underline{v'(\alpha) = 0:}$

$$7\cos\alpha + 1.68\cos^2\alpha - 1.68 \cdot \sin^2\alpha = 0$$

$$7\cos\alpha + 1.68\cos^2\alpha - 1.68(1-\cos^2\alpha) = 0$$

$$3.36\cos^2\alpha + 7\cos\alpha - 1.68 = 0$$

$\underline{\cos\alpha \cong 0.217}$ eller ~~$\cos\alpha \cong -2.30$~~

$$\alpha \simeq 1.35 + n \cdot 2\pi \simeq 1.35 \qquad (\simeq 77.5^\circ)$$

eller

$$\alpha \simeq -1.35 + n \cdot 2\pi \simeq -1.35 \qquad (\simeq -77.5^\circ)$$

$\rightarrow$ Fortegnsskjema $\Rightarrow$ $v \cong 7.2$ m/s ved $\alpha \cong \pm 1.35$ rad. er høyeste hastighet.