Repetisjon : Trigonometriske likninger
(kap. 10.1–10.2, kap. 6)

* Grunnleggende likninger :

$$\begin{cases} \sin x = c \\ \cos x = c \\ \tan x = c \end{cases}$$

* Likninger med én trigonometrisk funksjoner

Eks. $\sin x - 1 = 4$

$\cos(\pi x - \pi) = 1$

* Likninger med flere trigonometriske funksjoner

Løsningsmetoder:

* faktorisering ($\sin x \cdot \cos x = 0$)

* bruk at $\tan x = \frac{\sin x}{\cos x}$

($\sin x - \cos x = 0$)

* bruk at $\sin^2 x + \cos^2 x = 1$

($\cos^2 x = \sin x + 3$)

Viktig: – Addisjon av harmoniske svingninger
– Løsning av likningen

$$\boxed{a \cdot \sin x + b \cdot \cos x = c}$$

(a, b, c er gitte tall).

Addisjon av harmoniske svingninger:

Eks: $\sin x + \sqrt{3} \cdot \cos x = A \cdot \sin(x - \varphi)$

Formel:
$$f(x) = A_1 \cdot \sin(\omega(x - \varphi_1)) + C_1$$
$$g(x) = A_2 \sin(\omega(x - \varphi_2)) + C_2$$

$$f(x) + g(x) = A \cdot \sin(\omega(x - \varphi)) + C$$

$$\boxed{\begin{aligned} A &= \sqrt{A_1^2 + A_2^2 - 2A_1A_2 \cdot \cos(\omega(\varphi_1 - \varphi_2))} \\ \varphi &= \frac{1}{\omega} \arctan\left(\frac{A_1 \sin(\omega\varphi_1) + A_2 \sin(\omega\varphi_2)}{A_1 \cos(\omega\varphi_1) + A_2 \cos(\omega\varphi_2)}\right) \\ C &= C_1 + C_2 \end{aligned}}$$

$\cos x = \sin(x + \pi/2)$

$f(x) = \sin(x)$ $\quad A_1 = 1,\ \omega = 1,\ \varphi_1 = 0,\ C_1 = 0$

$g(x) = \sqrt{3} \cos(x)$ $\quad A_2 = \sqrt{3},\ \omega = 1,\ \varphi_2 = -\pi/2,\ C_2 = 0$

$$f(x) + g(x) = \sin x + \sqrt{3} \cos x = A \cdot \sin(x - \varphi) + C$$
$$= 2 \cdot \sin(x - \quad)$$

$c = c_1 + c_2 = 0 + 0 = 0$

$$A = \sqrt{1^2 + (\sqrt{3})^2 - 2 \cdot 1 \cdot \sqrt{3} \cdot \cos(\pi/2)}$$
$$= \sqrt{4} = 2$$

$$\varphi = \frac{1}{4} \cdot \arctan\left(\frac{1 \cdot \sin 0 + \sqrt{3} \cdot \overset{= -1}{\sin(-\pi/2)}}{1 \cdot \cos 0 + \sqrt{3} \cdot \cos(-\pi/2)} \right)$$
$$= \arctan\left(\frac{-\sqrt{3}}{1} \right) = \arctan(-\sqrt{3}) = \underline{-\pi/3}$$

Ecs: $\sin x + \sqrt{3} \cos x = \underline{2 \cdot \sin(x + \pi/3)}$

Formler: $a \sin x + b \cos x = A \cdot \sin(x - \varphi)$

$$\boxed{\begin{aligned} A &= \sqrt{a^2 + b^2} \\ \varphi &= \arctan(-b/a) \end{aligned}}$$

$$\sin x + \cos x = 1$$

① Regner ut $\sin x + \cos x$:

$$\sin x + \cos x = A \cdot \sin(x - \varphi)$$

a=1, b=1: $A = \sqrt{1^2 + 1^2} = \sqrt{2}$

$$\varphi = \arctan(-1/1) = \arctan(-1) = -\pi/4$$

$$\sin x + \cos x = \sqrt{2} \cdot \sin(x + \pi/4)$$

②
$$\sin x + \cos x = 1$$

$$\frac{\cancel{\sqrt{2}} \cdot \sin(x + \pi/4)}{\cancel{\sqrt{2}}} = \frac{1}{\sqrt{2}}$$

$$\sin(x + \pi/4) = 1/\sqrt{2}$$

$$x + \pi/4 = \sin^{-1}(1/\sqrt{2}) + n \cdot 2\pi = \pi/4 + n \cdot 2\pi$$

eller

$$x + \pi/4 = \pi - \pi/4 + n \cdot 2\pi = \frac{3\pi}{4} + n \cdot 2\pi$$

$$x = n \cdot 2\pi$$

eller

$$x = \frac{\pi}{2} + n \cdot 2\pi$$

Alternativer: $\sin x + \cos x = 1$

(a)

$$\cos x = 1 - \sin x$$
$$(\cos x)^2 = (1-\sin x)^2$$
$$\cos^2 x = 1 - 2\sin x + \sin^2 x$$
$$\cancel{1} - \sin^2 x = \cancel{1} - 2\sin x + \sin^2 x$$
$$0 = 2\sin^2 x - 2\sin x$$
$$0 = 2\sin x(\sin x - 1)$$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\sin x = 0$ eller $\sin x = 1$

$x = 0 + n\cdot 2\pi = n\cdot 2\pi$

eller

$\cancel{x = \pi + n\cdot 2\pi}$

$x = \frac{\pi}{2} + n\cdot 2\pi$

eller

$\cancel{x = \frac{\pi}{2} + n\cdot 2\pi}$

Sett prøve: $--- \rightarrow$ $x = n\cdot 2\pi$ eller $x = \frac{\pi}{2} + n\cdot 2\pi$

(b)

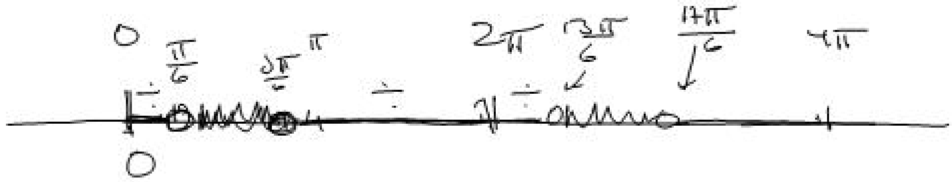
$$\sin x + \cos x = A\cdot \sin(x-a)$$
$$= A\cdot(\sin x \cdot \cos a - \cos x \cdot \sin a)$$
$$= (A\cdot\cos a)\cdot\sin x + (-A\sin a)\cdot\cos x$$

$$\left.\begin{aligned} A\cdot\cos a &= 1 \\ -A\cdot\sin a &= 1 \end{aligned}\right\} \Rightarrow \cdots \text{ finner } A \text{ og } a.$$

Trigonometriske ulikheter (kap. 10.7)

Eks: $2\sin x - 1 < 0$

$2\sin x - 1$



$2\sin x - 1 = 0$:

$$\frac{2\sin x}{2} = \frac{1}{2}$$

$$\sin x = 1/2$$

$$x = \pi/6 + n \cdot 2\pi \quad \text{eller} \quad x = \frac{5\pi}{6} + n \cdot 2\pi$$

Eks: $2\sin x - 1 < 0, \quad 0 \leq x < 2\pi$

Løsning: $0 \leq x \leq \pi/6$ eller $\frac{5\pi}{6} < x < 2\pi$

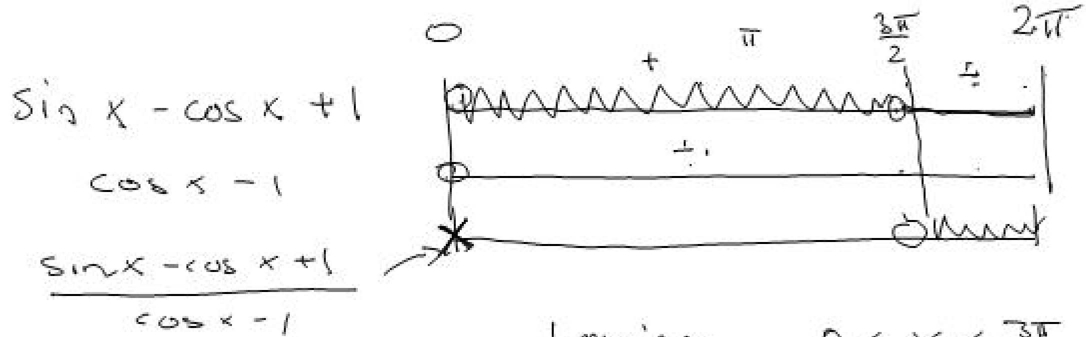
Eks: $\frac{\sin x}{\cos x - 1} < 1 \quad , \quad 0 \leq x < 2\pi$

$$\frac{\sin x}{\cos x - 1} - 1 < 0$$

$$\frac{\sin x}{\cos x - 1} - \frac{\cos x - 1}{\cos x - 1} < 0$$

$$\frac{\sin x - \cos x + 1}{\cos x - 1} < 0$$

$2\pi - \frac{\pi}{6} = -\frac{\pi}{6}$



Løsning: $\underline{\underline{0 < x < \frac{3\pi}{2}}}$

$\underline{\cos x - 1 = 0}$: $\cos x = 1$, $x = 0 + n \cdot 2\pi$ eller ~~$x = 0 + n \cdot 2\pi$~~ $\Rightarrow \underline{x = 0}$

$\underline{\sin x - \cos x + 1 = 0}$: $\sin x - \cos x = -1$

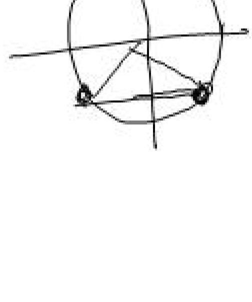
$A = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$\varphi = \arctan\left(\frac{+1}{1}\right) = \arctan(1) = \pi/4$

$A \cdot \sin(x - \varphi) = -1$

$\frac{\sqrt{2} \cdot \sin(x - \pi/4)}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$

$\sin(x - \pi/4) = -1/\sqrt{2}$



$x - \pi/4 = \pi + \pi/4 + n \cdot 2\pi$ eller $x - \pi/4 = \sin^{-1}(-1/\sqrt{2}) + n \cdot 2\pi$

$x = \pi + \pi/4 + \pi/4 + n \cdot 2\pi$ $\quad = -\pi/4 + n \cdot 2\pi$

$= \frac{3\pi}{2} + n \cdot 2\pi = \frac{3\pi}{2}$ $\quad x = 0 + n \cdot 2\pi = \underline{0}$