Derivasjon:

$$\boxed{\begin{aligned}(\sin x)' &= \cos x \\ (\cos x)' &= -\sin x\end{aligned}}$$

③ Trigonometriske likninger med flere trigonometriske funksjoner

Triks 1: Bruk at $\tan x = \frac{\sin x}{\cos x}$.

Typisk bruk: $a \cdot \sin x + b \cdot \cos x = 0$

$a \sin^2 x + b \cdot \sin x \cdot \cos x + c \cdot \cos^2 x = 0$

(a, b, c er tall)

Triks 2: Faktorisering $u \cdot v = 0$

$u = 0$ eller $v = 0$

Typisk bruk: $\sin x \cdot \cos x = 0$

Triks 3: Bruk at $\sin^2 x + \cos^2 x = 1$.

$\sin^2 x = 1 - \cos^2 x$ $\quad \sin x = \pm\sqrt{1 - \cos^2 x}$

$\cos^2 x = 1 - \sin^2 x$ $\quad \cos x = \pm\sqrt{1 - \sin^2 x}$

$\boxed{\sin^2 x = 1 - \cos^2 x}$

<u>Eks:</u>

$$\sin^2 x + 1 = \cos x$$
$$(1 - \cos^2 x) + 1 = \cos x$$
$$2 - \cos^2 x = \cos x$$

$$-\cos^2 x - \cos x + 2 = 0$$

$$\cos x = \frac{1 \pm \sqrt{1 - 4 \cdot (-1) \cdot 2}}{2 \cdot (-1)} = \frac{1 \pm 3}{-2}$$

$\cos x = -2$ eller $\cos x = 1$

ingen løsning

$x = \cos^{-1}(1) + n \cdot 2\pi$

eller

$x = -\cos^{-1}(1) + n \cdot 2\pi$

$x = 0 + n \cdot 2\pi = \underline{n \cdot 2\pi}$

eller

$x = 0 + n \cdot 2\pi = \underline{n \cdot 2\pi}$

<u>Løsn:</u> $x = \underline{\underline{n \cdot 2\pi}}$

Eks:

$$\sin x + 3\cos x = 1 \qquad \boxed{\sin^2 x + \cos^2 x = 1}$$

$$\sin x = 1 - 3\cos x$$

$$\sin^2 x = (1 - 3\cos x)^2 = 1 - 6\cos x + 9\cos^2 x$$

$$\sin^2 x = 1 - 6\cos x + 9\cos^2 x \qquad \boxed{\sin^2 x = 1 - \cos^2 x}$$

$$1 - \cos^2 x = 1 - 6\cos x + 9\cos^2 x$$

$$-10\cos^2 x + 6\cos x = 0$$

$$\cos x(-10\cos x + 6) = 0$$

(1) $\underline{\cos x = 0}$ eller $-10\cos x + 6 = 0$

(2) $\underline{\cos x = 0.6}$

(1) $\cos x = 0$

$x = \underline{\pi/2 + n \cdot 2\pi}$

eller

$\cancel{x = -\pi/2 + n \cdot 2\pi}$

(2) $\cos x = 0.6$

$x \approx \cancel{0.927 + n \cdot 2\pi}$

eller

$x \approx \underline{-0.927 + n \cdot 2\pi}$

Sett prøve: $\sin x + 3\cos x = 1$

$x = \pi/2$:	VS $= 1 + 3 \cdot 0 = 1$	HS $= 1$	ok.
$x = -\pi/2$:	VS $= -1 + 3 \cdot 0 = -1$	HS $= 1$	ikke ok.
$x \approx 0.927$:	VS $= 0.8 + 3 \cdot 0.6 = 2.6$	HS $= 1$	— " —
$x \approx -0.927$:	VS $= -0.8 + 3 \cdot 0.6 = 1$	HS $= 1$	ok.

$$\boxed{\begin{aligned} &x = \underline{\pi/2 + n \cdot 2\pi} \\ &x \approx \underline{-0.927 + n \cdot 2\pi} \end{aligned}}$$

Likningen $\boxed{a \cdot \sin x + b \cdot \cos x = c}$

(a, b, c er tall)

① Hvis $c = 0$ kan vi bruke $\frac{\sin x}{\cos x} = \tan x$.

② Hvis $c \neq 0$: Metode 1: Kvadrere likningen (etter å ha ordnet den) og bruke $\sin^2 x + \cos^2 x = 1$

Husk: sett prøve!

Metode 2:

En harmonisk svingning med vinkelhastighet ω:

$$f(x) = A_1 \cdot \sin(\omega(x - \varphi_1)) + c_1$$
$$g(x) = A_2 \cdot \sin(\omega(x - \varphi_2)) + c_2$$

$f(x) + g(x)$ er en ny harmonisk svingning med vinkelhastighet ω.

$\underbrace{\sin x + 3\cos x}$ $= 1$

$\sin x$: $\begin{cases} A = 1 & \omega = 1 \\ c = 0 & \varphi = 0 \end{cases}$

$3\cos x = 3\sin(x + \pi/2)$: $\begin{cases} A = 3 & \omega = 1 \\ c = 0 & \varphi = -\pi/2 \end{cases}$

$$\sin x + 3\cos x = A \cdot \underline{\sin(x-\alpha)} + \cancel{C}$$

$$\sin(x-\alpha) = \sin x \cdot \cos\alpha - \cos x \sin\alpha$$

$$\sin x + 3\cos x = A \cdot (\sin x \cdot \cos\alpha - \cos x \cdot \sin\alpha)$$
$$= (A \cdot \cos\alpha) \sin x - (A \cdot \sin\alpha) \cos x$$

$$\left.\begin{array}{l} A \cdot \cos\alpha = 1 \\ -A \cdot \sin\alpha = 3 \end{array}\right\} \quad \frac{-\cancel{A} \sin\alpha}{\cancel{A} \cdot \cos\alpha} = \frac{3}{1}$$

$$-\tan\alpha = 3$$
$$\tan\alpha = -3$$
$$\alpha = \arctan(-3)$$
$$\alpha \simeq \underline{-1.249}$$

$$A = \frac{1}{\cos\alpha}$$
$$\cong \frac{1}{\cos(-1.249)}$$
$$A \simeq \underline{3.16}$$

$$\sin x + 3\cos x = 3.16 \cdot \sin(x + 1.249)$$

Hvordan legger sammen to bølgefunksjoner (harmoniske svingninger) med samme ω.

$$f(x) = A_1 \cdot \sin(\omega(x-\varphi_1)) + c_1$$
$$g(x) = A_2 \cdot \sin(\omega(x-\varphi_2)) + c_2$$

$$h(x) = f(x) + g(x) = \underline{A} \cdot \sin(\omega(x-\underline{\varphi})) + \underline{c}$$

$$\boxed{\begin{aligned} c &= c_1 + c_2 \\ A &= \sqrt{A_1^2 + A_2^2 - 2A_1A_2\cos(\omega(\varphi_1-\varphi_2))} \\ \varphi &= \frac{1}{\omega}\arctan\left(\frac{A_1\sin(\omega\varphi_1) + A_2\sin(\omega\varphi_2)}{A_1\cdot\cos(\omega\varphi_1) + A_2\cdot\cos(\omega\varphi_2)}\right) \end{aligned}}$$

Bruk addisjonsformel for sin til å finne φ.

Eks:

$$\sin x + 3\cos x = 1$$
$$\frac{3.16 \cdot \sin(x + 1.249)}{3.16} = \frac{1}{3.16}$$

$$\sin(x + 1.249) = 0.316$$

① $x + 1.249 = \sin^{-1}(0.316) + n \cdot 2\pi = 0{,}322 + n \cdot 2\pi$

$$x = 0.322 - 1.249 + n \cdot 2\pi$$
$$x = \underline{-0.927 + n \cdot 2\pi}$$

② $x + 1.249 = \pi - 0.322 + n \cdot 2\pi$

$$x = \pi - 0.322 - 1.249 + n \cdot 2\pi = \underline{1.571 + n \cdot 2\pi}$$