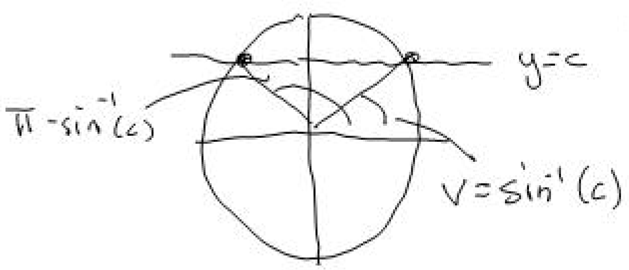
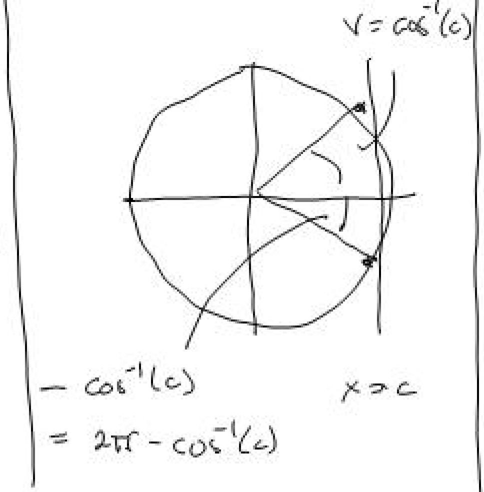
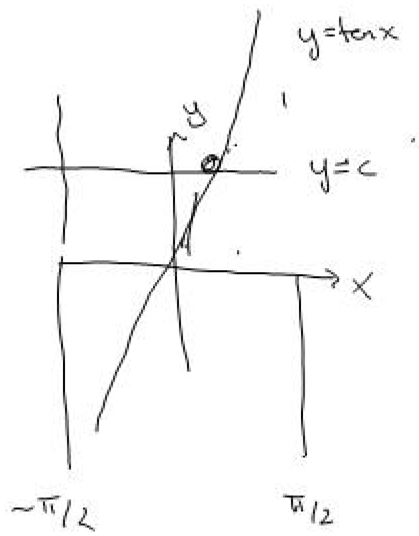
Trigonometriske likninger (kap. 10.1, 10.2, 10.7) + kap. 6

① Grunnleggende trigonometriske likninger:

$\sin v = c$	$\cos v = c$	$\tan v = c$
$v = \sin^{-1}(c) + n\cdot 2\pi$ eller $v = \pi - \sin^{-1}(c) + n\cdot 2\pi$	$v = \cos^{-1}(c) + n\cdot 2\pi$ eller $v = -\cos^{-1}(c) + n\cdot 2\pi$	$v = \tan^{-1}(c) + n\cdot\pi$







Eks: $\cos v = 1/2$

$$\begin{cases} v = \cos^{-1}(1/2) + n\,2\pi = \underline{\frac{\pi}{3} + n\cdot 2\pi} \\ \text{eller} \\ v = -\cos^{-1}(1/2) + n\cdot 2\pi = \underline{-\frac{\pi}{3} + n\cdot 2\pi} \end{cases}$$

$\tan v = -1$ $\qquad v = \tan^{-1}(-1) + n\cdot\pi = \underline{-\frac{\pi}{4} + n\cdot\pi}$

② Likninger med kun én trigonometrisk funksjon.

Eks:

$$2\sin x - 1 = 0$$

$$\frac{\cancel{2}\sin x}{\cancel{2}} = \frac{1}{2}$$

$$\sin x = 1/2$$

$$x = \sin^{-1}(1/2) + n\cdot 2\pi = \underline{\frac{\pi}{6} + n\cdot 2\pi}$$

eller

$$x = \pi - \sin^{-1}(1/2) + n\cdot 2\pi = \underline{\frac{5\pi}{6} + n\cdot 2\pi}$$

Eks:

$$\cos(2x - \pi) = 0 \qquad \boxed{u = 2x - \pi}$$

$$\cos(u) = 0$$

(1) $u = \cos^{-1}(0) + n\cdot 2\pi = \frac{\pi}{2} + n\cdot 2\pi$

eller

(2) $u = -\cos^{-1}(0) + n\cdot 2\pi = -\frac{\pi}{2} + n\cdot 2\pi$

(1):

$$2x - \pi = \frac{\pi}{2} + n\cdot 2\pi$$

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{\pi}{2} + \pi + n\cdot 2\pi = \frac{\frac{3\pi}{2} + n\cdot 2\pi}{2}$$

$$x = \underline{\frac{3\pi}{4} + n\cdot\pi}$$

(2): $2x - \pi = -\frac{\pi}{2} + n \cdot 2\pi$

$$\frac{\cancel{2}x}{\cancel{2}} = -\frac{\pi}{2} + \pi + n \cdot 2\pi = \frac{\frac{\pi}{2} + n \cdot 2\pi}{2}$$

$$\underline{x = \frac{\pi}{4} + n \cdot \pi}$$

Løsning: $\underline{x = \frac{3}{4}\pi + n \cdot \pi}$ eller $\underline{x = \frac{\pi}{4} + n \cdot \pi}$

Grafisk: Likning $\cos(2x - \pi) = 0$

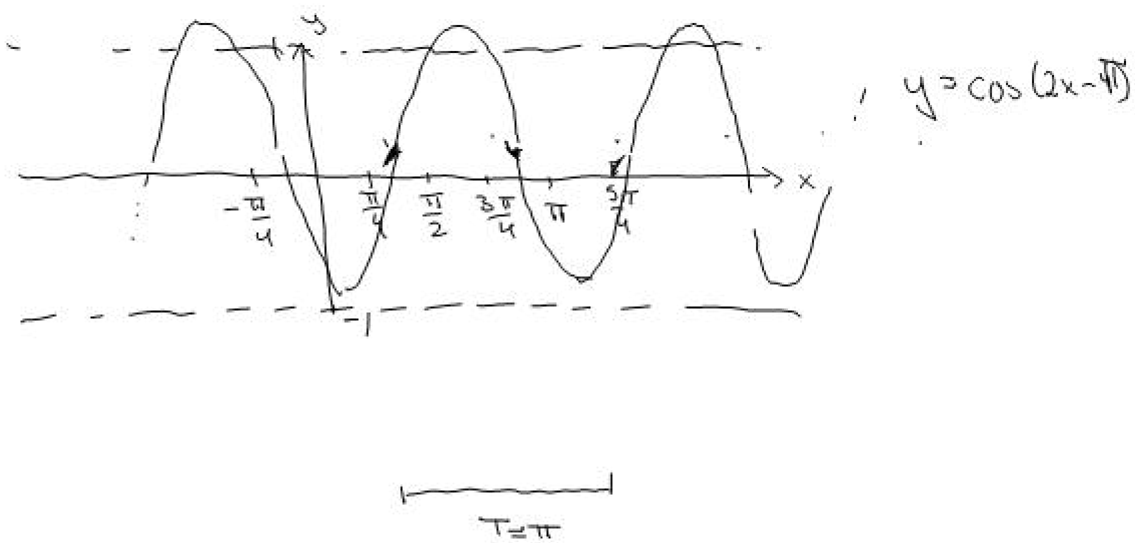
Løsning = nullpkt for $\underline{f(x) = \cos(2x - \pi)}$

$$\boxed{\begin{aligned} &\cos(u) \\ &= \sin(u + \pi/2) \end{aligned}}$$

$$\cos(2x - \pi) = \sin(2x - \pi + \pi/2) = \sin(2x - \pi/2)$$
$$= 1 \cdot \sin(2(x - \pi/4)) + 0$$

$C = 0 \qquad \omega = 2 \Rightarrow T = \frac{2\pi}{2} = \pi$

$A = 1 \qquad \varphi = \pi/4$



Eks:

$$\sin^2 x - 1 = 4\sin x$$

$$\boxed{\sin^2 x - 4\sin x - 1 = 0}$$

$$(\sin x)^2 - 4\cdot\sin x - 1 = 0$$

$$\boxed{u = \sin x}$$

$$u^2 - 4u - 1 = 0$$

$$u = \frac{4 \pm \sqrt{16 - 4\cdot(-1)}}{2} = \frac{4 \pm \sqrt{20}}{2}$$

$$= \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$$

$\sin x = 2+\sqrt{5}$ eller $\sin x = 2-\sqrt{5}$

$\simeq 4.23$ $\simeq -0.23$

ingen løsning

$$x = \sin^{-1}(2-\sqrt{5}) + n\cdot 2\pi$$

$$\cong \underline{-0.24 + n\cdot 2\pi}$$

eller

$$x = \pi - \sin^{-1}(2-\sqrt{5}) + n\cdot 2\pi$$

$$\cong \underline{3.38 + n\cdot 2\pi}$$

Eks:

$$2 \cdot \tan\left(\frac{x}{2} - \pi\right) = 1 \qquad \boxed{u = \frac{x}{2} - \pi}$$

$$\frac{2 \cdot \tan(u)}{2} = \frac{1}{2}$$

$$\tan(u) = \frac{1}{2}$$

$$u = \tan^{-1}\left(\frac{1}{2}\right) + n \cdot \pi \cong 0.464 + n \cdot \pi$$

$$\frac{x}{2} - \pi \cong 0.464 + n \cdot \pi$$

$$\cancel{2} \cdot \frac{x}{\cancel{2}} \simeq (0.464 + \pi + n \cdot \pi) \cdot 2$$

$$x \simeq 0.928 + 2\pi + n \cdot 2\pi$$

$$\simeq \underline{\underline{0.928 + n \cdot 2\pi}}$$

$$\ldots, 0.93, 7.21, \ldots$$

$$\cong \underline{\underline{7.21 + n \cdot 2\pi}}$$

$$\ldots, 0.93, 7.21, 13.49, \ldots$$

③ Likninger med flere trigonometriske funksjoner

Eks:

$$\sin x - \cos x = 0$$

$$\sin x \cdot \cos x = 0$$

$$\sin x - 2\cos x = 1$$

Metode: Må redusere likningen slik at det inneholder kun én trig. funksjon.

Triks I: Bruk at $\frac{\sin x}{\cos x} = \tan x$:

Eks:

$$\frac{\sin x}{\cos x} - \frac{\cos x}{\cos x} = \frac{0}{\cos x} \quad | : \cos x$$

$$\tan x - 1 = 0$$

$$\tan x = 1$$

$$x = \tan^{-1}(1) + n \cdot \pi = \underline{\underline{\pi/4 + n \cdot \pi}}$$

Eks:

$$\frac{\sin x}{\cos x} - \frac{2\cos x}{\cos x} = \frac{1}{\cos x}$$

$$\tan x - 2 = \frac{1}{\cos x}$$

metoden hjelper ikke.

Rep: $a \cdot \sin x + b \cdot \cos x = 0 \quad | : \cos x$

$$\frac{a \cdot \sin x}{\cos x} + \frac{b \cancel{\cos x}}{\cancel{\cos x}} = \frac{0}{\cos x}$$

$$a \cdot \tan x + b = 0$$

⋮ kan løses

Eks: $$\frac{\sin^2 x}{\cos^2 x} + \frac{2 \cancel{\cos x} \cdot \sin x}{\cos^{\cancel{2}} x} + \frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x}} = \frac{0}{\cos^2 x} \quad | : \cos^2 x$$

$$\tan^2 x + 2 \tan x + 1 = 0$$

$$\tan x = \frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot 1}}{2} = -1$$

$$x = \tan^{-1}(-1) + n \cdot \pi = \underline{\underline{-\frac{\pi}{4} + n \cdot \pi}}$$

Triks 2: Faktorisering $\boxed{u \cdot v = 0}$ gir $u = 0$ eller $v = 0$

Eks: $\sin x \cdot \cos x = 0$

$\boxed{\sin x = 0}$ eller $\boxed{\cos x = 0}$

$0 + n \cdot 2\pi$ — $\underline{x = n \cdot 2\pi}$ eller

$\pi - 0 + n \cdot 2\pi$ — $x = \underline{\pi + n \cdot 2\pi}$

$x = \underline{\frac{\pi}{2} + n \cdot 2\pi}$ eller

$x = \underline{-\frac{\pi}{2} + n \cdot 2\pi}$

Triks 3: Bruk $\boxed{\sin^2 x + \cos^2 x = 1}$

$$\sin^2 x = 1 - \cos^2 x$$
$$\cos^2 x = 1 - \sin^2 x$$

Eks: $\sin x + 3\cos^2 x = 1$

$$\sin x + 3(1 - \sin^2 x) = 1$$
$$\sin x + 3 - 3\sin^2 x - 1 = 0$$
$$-3\sin^2 x + \sin x + 2 = 0$$
$$\sin x = \frac{-1 \pm \sqrt{1^2 - 4\cdot(-3)\cdot 2}}{2\cdot(-3)} = \frac{-1 \mp 5}{-6}$$

$\sin x = \underline{1}$ eller $\sin x = \underline{-2/3}$

$$x = \sin^{-1}(1) + n\cdot 2\pi = \underline{\pi/2 + n\cdot 2\pi}$$

(eller $x = \pi - \pi/2 + n\cdot 2\pi = \pi/2 + n\cdot 2\pi$)

$$x \simeq \sin^{-1}(-2/3) + n\cdot 2\pi \simeq \underline{-0.73 + n\cdot 2\pi}$$

eller

$$x \simeq \underline{3.87 + n\cdot 2\pi}$$