

Harmoniske svingninger

(Kap. 10.3-10.6)

$$f(x) = A \cdot \sin(\omega(x-\varphi)) + C$$

Standard-form

$A > 0$: amplitude

C : likevektslinjen

$\omega = k > 0$: vinkelhastighet

φ : faseforskyvning

$$y = C$$

$$\omega = \frac{2\pi}{T} \quad T = \frac{2\pi}{\omega}$$

Ex:

$$f(x) = 7 + 10 \sin\left(\frac{2\pi}{365}x - \frac{2\pi}{365} \cdot 100\right)$$

normal temperatur : °C etter
x dager

$$f(x) = 7 + 10 \cdot \sin\left(\frac{2\pi}{365}x - \frac{200\pi}{365}\right) \\ = A \cdot \sin(\omega(x-\varphi)) + C$$

$$\left. \begin{array}{l} A = \underline{10} \\ C = \underline{7} \end{array} \right\} \begin{array}{l} \text{maks} = 7 + 10 = \underline{17} \\ \text{min} = 7 - 10 = \underline{-3} \end{array}$$

$$\textcircled{\omega}x - \omega\varphi = \textcircled{\frac{2\pi}{365}}x - \frac{200\pi}{365}$$

$$\omega = \frac{2\pi}{365} \implies T = \frac{2\pi}{\omega} = \underline{\underline{365}}$$

$$\frac{2\pi}{365} \cdot \varphi = \frac{200\pi}{365} \implies \varphi = \underline{\underline{100}}$$

Leser av fra grafen:

$$\left. \begin{array}{l} \text{maks: } y = 17 \\ \text{min: } y = -3 \end{array} \right\} \begin{array}{l} C = \frac{17 + (-3)}{2} = 7 \\ A = 17 - 7 = 10 \\ \quad 7 - (-3) = 10 \end{array}$$

fase forskyvning: $\phi = 100$

Vinkelhastighet: periode $T \approx 555 - 191 = 364$
(riktig verdi $T = 365$)

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{365}$$

Funktionsgraph als $f(x) = 7 + 10 \cdot \sin\left(\frac{2\pi}{365}x - \frac{200\pi}{365}\right)$

$$f(x) = 7 + 10 \cdot \sin\left(\frac{2\pi}{365}x - \frac{200\pi}{365}\right)$$

(a) Nullpunkt für f:

$$f(x) = 0$$

$$7 + 10 \cdot \sin\left(\overbrace{\frac{2\pi}{365}x - \frac{200\pi}{365}}^u\right) = 0$$

$$7 + 10 \cdot \sin(u) = 0$$

$$\frac{10 \cdot \sin(u)}{10} = \frac{-7}{10}$$

$$\sin(u) = -0.7$$

$$u = \arcsin(-0.7) + n \cdot 2\pi \quad \text{oder} \quad u = \pi - \arcsin(-0.7) + n \cdot 2\pi$$

$$u = -0.775 + n \cdot 2\pi \quad \text{oder} \quad u = 3.917 + n \cdot 2\pi$$

$$\frac{2\pi}{365}(x-100) = -0.775 + n \cdot 2\pi \quad \text{oder} \quad \frac{2\pi}{365}(x-100) = 3.917 + n \cdot 2\pi$$

$$x-100 = \frac{365}{2\pi}(-0.775 + n \cdot 2\pi) \quad \text{oder} \quad x-100 = \frac{365}{2\pi}(3.917 + n \cdot 2\pi)$$

$$x = -\frac{365 \cdot 0.775}{2\pi} + 100 + n \cdot 365 \quad \text{oder} \quad x = \frac{365 \cdot 3.917}{2\pi} + 100 + n \cdot 365$$
$$= \underline{\underline{55 + n \cdot 365}} \quad \quad \quad = \underline{\underline{327.5 + n \cdot 365}}$$

b) Lokale topp-punkt:

$$y = C + A = 7 + 10 = 17$$

$$7 + 10 \cdot \sin\left(\frac{2\pi}{365}(x-100)\right) = 17$$

$$7 + 10 \sin u = 17$$

$$\frac{10 \sin u}{10} = \frac{17-7}{10}$$

$$\sin(u) = 1$$

$$u = \arcsin(1) + n \cdot 2\pi$$

$$= \pi/2 + n \cdot 2\pi$$

$$\frac{2\pi}{365}(x-100) = \pi/2 + n \cdot 2\pi$$

$$x-100 = \frac{365}{2\pi} \left(\pi/2 + n \cdot 2\pi \right)$$

$$x-100 = \frac{365}{4} + n \cdot 365$$

$$x = \frac{365}{4} + 100 + n \cdot 365$$

$$= \underline{\underline{191.25 + n \cdot 365}}$$

Lokale bunnepunkt:

$$y = C - A = 7 - 10 = -3$$

$$7 + 10 \cdot \sin\left(\frac{2\pi}{365}(x-100)\right) = -3$$

$$\underline{\underline{\dots \frac{365}{4} + 100 + n \cdot 365}}$$

$$\begin{aligned}
 x &= -\frac{365}{4} + 100 + n \cdot 365 \\
 &= \underline{\underline{8.75 + n \cdot 365}}
 \end{aligned}$$

Exo: $f(x) = \underline{2 \cos x - 1} = 2 \sin(x + \pi/2) - 1$

(a) Nullpunkt: $f(x) = 0$
 $2 \cos x - 1 = 0$
 $\frac{2 \cos x}{2} = \frac{1}{2}$
 $\cos x = 1/2$

$$x = \arccos(1/2) + n \cdot 2\pi = \underline{\underline{\pi/3 + n \cdot 2\pi}}$$

oder

$$x = \arccos(1/2) + n \cdot 2\pi = \underline{\underline{-\pi/3 + n \cdot 2\pi}}$$

(b) Lokale tpp-punkt:

$$y = 1$$

$$f(x) = 2 \sin(x + \pi/2) - 1$$

$$\left. \begin{array}{l}
 A = 2 \quad c = -1 \\
 \omega = 1 \quad \varphi = -\pi/2
 \end{array} \right\} \begin{array}{l}
 \text{max: } y = 1 \\
 \text{min: } y = -3
 \end{array}$$

$$2 \cos x - 1 = 1$$

$$\cos x = 1$$

$$x = 0 + n \cdot 2\pi = \underline{\underline{n \cdot 2\pi}}$$

e) lokale Nullstellen:

$$y = -3$$

$$2\cos x - 1 = -3$$

$$\cos x = \frac{-3+1}{2} = -1$$

$$x = \arccos(-1) + n \cdot 2\pi$$

$$= \underline{\underline{\pi + n \cdot 2\pi}}$$