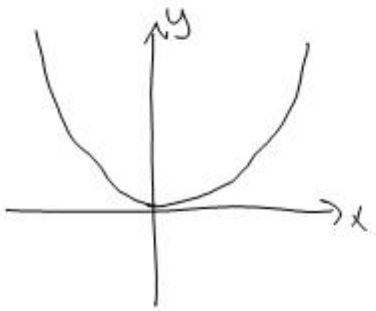
Implisitt derivasjon (Notat, LF)

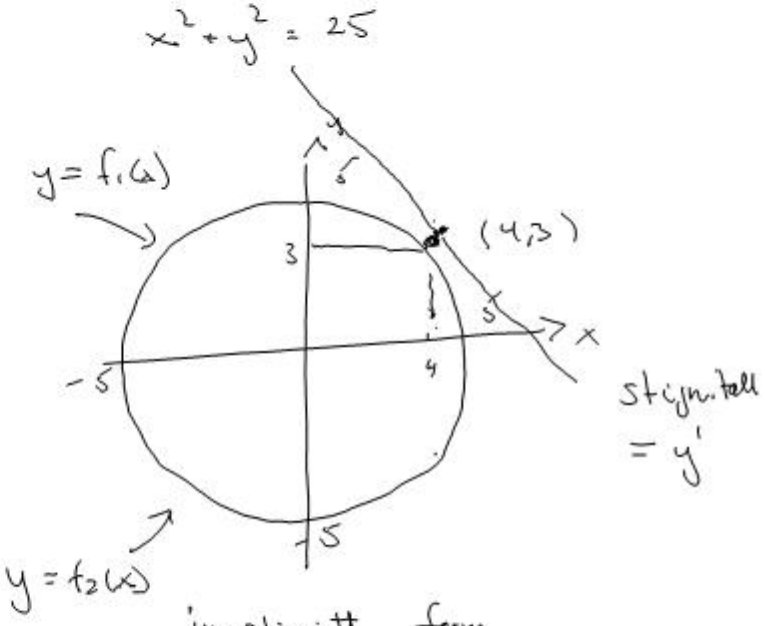
$y = x^2$



eksplisitt form

$y = f(x)$

$y' = f'(x)$



implisitt form

$F(x,y) = 0$

$y' = ?$ implisitt derivasjon.

Metode 1:

$x^2 + y^2 = 25$

$y^2 = 25 - x^2$

$y = \pm\sqrt{25 - x^2}$

$f_1(x) = \sqrt{25 - x^2}$

$f_2(x) = -\sqrt{25 - x^2}$

y' i (4,3): $f_1'(4)$

$$f_1'(x) = \left(\sqrt{25 - x^2}\right)' = \frac{1}{2\sqrt{25 - x^2}} \cdot (-2x)$$

$$f_1'(x) = \frac{-2x}{2\sqrt{25-x^2}} = -\frac{x}{\sqrt{25-x^2}}$$

$$f_1'(4) = -\frac{4}{\sqrt{25-16}} = -\frac{4}{3}$$

Metode 2: Implisitt derivasjon

$$x^2 + y^2 = 25$$

$$(x^2+y^2)' = (25)'$$

$$2x + 2y \cdot y' = 0$$

$$\frac{2y \cdot y'}{2y} = -\frac{2x}{2y}$$

$$y' = -\frac{x}{y}$$

$$\frac{d}{dx}(y^2) = \frac{dy^2}{dy} \cdot \frac{dy}{dx}$$

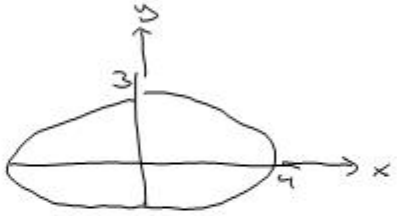
$$\frac{dy^2}{dx} = \frac{dy^2}{dy} \cdot \frac{dy}{dx}$$

$$= 2y \cdot y'$$

I punktet $(4,3)$: $y' = -\frac{4}{3}$

I punktet $(4,-3)$: $y' = -\frac{4}{(-3)} = 4/3$

Ex5: $9x^2 + 16y^2 = 144$



Implicit derivation:

$$(9x^2 + 16y^2)' = (144)'$$

$$18x + 32y \cdot y' = 0$$

$$\frac{32y \cdot y'}{32y} = \frac{-18x}{32y}$$

$$y' = -\frac{9x}{16y}$$

$(xy)' = (x)' \cdot y + x \cdot (y)'$

$= 1 \cdot y + x \cdot 1 \cdot y'$

$= y + x \cdot y'$

Ex6: $y^3 + xy = 2x$

$$(y^3 + xy)' = (2x)'$$

$$3y^2 \cdot y' + y + xy' = 2$$

$$3y^2 \cdot y' + xy' = 2 - y$$

$$\frac{(3y^2 + x)y'}{3y^2 + x} = \frac{2 - y}{3y^2 + x}$$

$$y' = \frac{2 - y}{3y^2 + x}$$