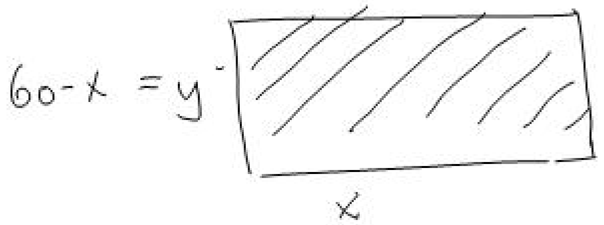
Optimering (kap. 9.3)

Eks: Skal sette opp rektangulært gjerde



Maksimalt 120 m med gjerde.

Ønsker arealet størst mulig

$$x + y + x + y = 120$$

$$2x + 2y = 120 \quad \rightarrow \quad \frac{2y}{2} = \frac{120 - 2x}{2}$$

$$y = 60 - x$$

$$A = x \cdot y$$

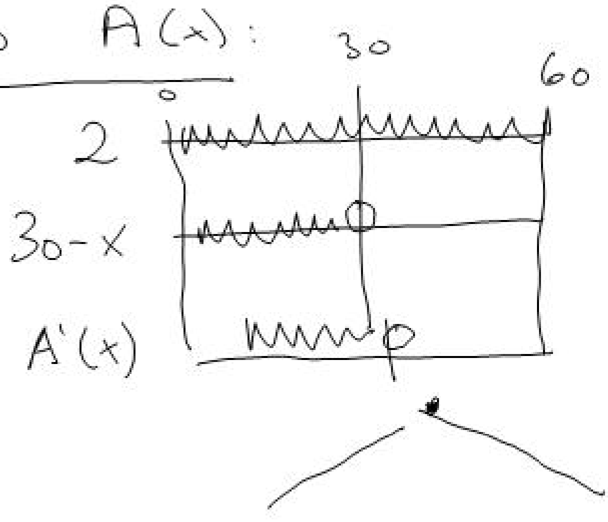
$$A(x) = x \cdot (60 - x)$$

$$\boxed{A(x) = 60x - x^2 \quad , x \in (0, 60)}$$

Finner største verdi av $A(x)$:

$$A'(x) = 60 - 2x$$

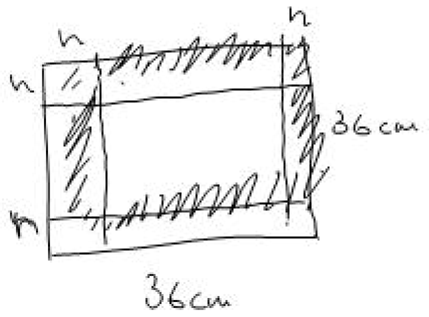
$$= 2(30 - x)$$



Størst verdi for A:

$x = 30 \qquad A = A(30) = \underline{\underline{900}}$

Eks: Et kvadratisk pappstykke med side = 36cm. skal brettes en boks med størst mulig volum. Hva blir volumet?



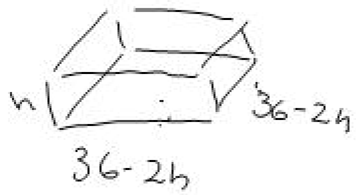
Konklusjon:

Størst volum

$h = 6$ cm

$V = 24^2 \cdot 6 \text{ cm}^3$

$= \underline{\underline{3456 \text{ cm}^3}}$



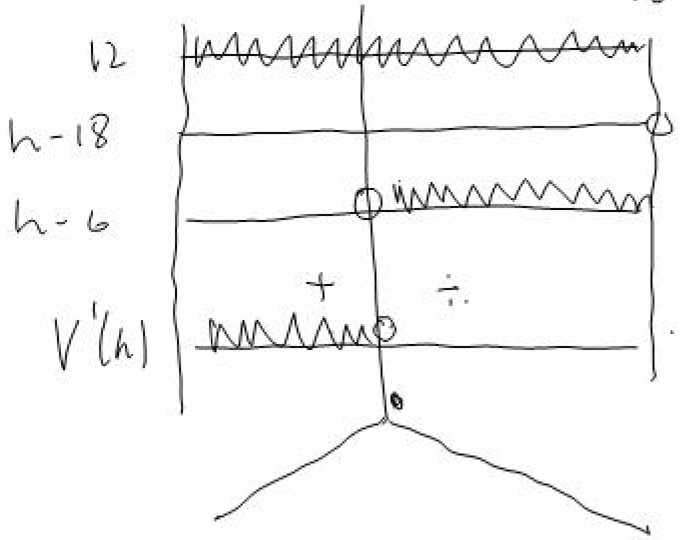
$$V = (36-2h)^2 \cdot h$$

$$= (36^2 - 144h + 4h^2) \cdot h$$

$$\boxed{V(h) = 4h^3 - 144h^2 + 36^2 \cdot h, \quad h \in (0, 18)}$$

$$V'(h) = 12h^2 - 288h + 36^2$$

$$= 12 \cdot (h-18)(h-6)$$



$12h^2 - 288h + 36^2 = 0$

~~$h = 288 \pm \sqrt{288^2 - \ldots}$~~

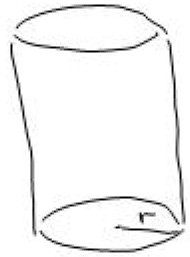
$$h = \frac{288 \pm \sqrt{288^2 - 4 \cdot 12 \cdot 36^2}}{2 \cdot 12}$$

$$= \frac{288 \pm 144}{24}$$

$$x_1 = \frac{432}{24} = 18$$

$$x_2 = \frac{144}{24} = 6$$

Eks:



$h = \frac{350}{\pi r^2}$

$V = \cdot 0{,}35\ l$

$= 350\ cm^3$

Hvordan skal vi velge r og h for at overflaten blir minst mulig.

$$\boxed{\begin{aligned} V &= \pi r^2 \cdot h \\ O &= 2\pi r^2 + h \cdot 2\pi r \end{aligned}}$$

$$\pi r^2 \cdot h = 350$$

$$\boxed{h = \frac{350}{\pi r^2}}$$

$\frac{350}{\pi r^2} = 0$

skjer aldri

$$O(r) = 2\pi r^2 + \frac{350}{\pi r^2} \cdot 2\pi r$$

$$= 2\pi r^2 + \frac{350 \cdot 2\cancel{\pi}\cancel{r}}{\cancel{\pi} r \cancel{2}}$$

$$= 2\pi r^2 + \frac{700}{r}$$

$$= \frac{2\pi r^2 \cdot r + 700}{r}$$

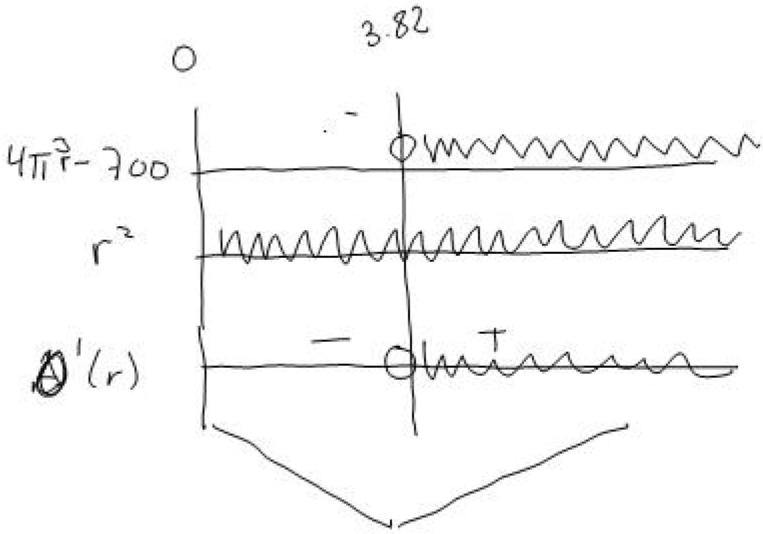
$$O(r) = \frac{2\pi r^3 + 700}{r}, \quad r \in (0, \infty)$$

$$O'(r) = 2\pi \cdot 2r + 700 \cdot \left(-\frac{1}{r^2}\right)$$

$$= 4\pi r - \frac{700}{r^2} = \frac{4\pi r \cdot r^2}{r^2} - \frac{700}{r^2}$$

$$= \frac{4\pi r^3 - 700}{r^2}$$

$$O'(r) = \frac{4\pi r^3 - 700}{r^2} \qquad O(r) = 2\pi r^2 + \frac{700}{r}$$



$$4\pi r^3 - 700 = 0$$

$$\frac{4\pi r^3}{4\pi} = \frac{700}{4\pi}$$

$$r^3 = \frac{700}{4\pi} \approx$$

$$r = \sqrt[3]{\frac{700}{4\pi}}$$

$$\approx \underline{3.82}$$

<u>Minst overflate:</u>

$$r = \sqrt[3]{\frac{700}{4\pi}} \approx 3.82 \text{ cm}$$

$$O \approx \underline{288 \text{ cm}^2}$$

Eks: Oppg. 4, Eks. Jul 2006/07

$$f(x) = \frac{6x^2+3x}{3x+2}, \quad x \neq -2/3$$

a) Finn nullpunkt til f ved regning

$$f(x) = 0$$

$$\frac{6x^2+3x}{3x+2} = 0 \quad | \cdot (3x+2)$$

$$6x^2+3x = 0$$

$$3x(2x+1) = 0$$

$3x = 0$ eller $2x+1=0$

$x = 0$ $\qquad$ $x = -1/2$

$$\underline{\underline{x=0}} \text{ eller } \underline{\underline{x=-1/2}}$$

b) $f'(x) = \left(\frac{6x^2+3x}{3x+2}\right)'$

$\boxed{\begin{array}{ll} u = 6x^2+3x & v = 3x+2 \\ u' = 12x+3 & v' = 3 \end{array}}$

$$= \frac{u' \cdot v - u \cdot v'}{v^2} = \frac{(12x+3)\cdot(3x+2) - (6x^2+3x)\cdot(3)}{(3x+2)^2}$$

$$= \frac{(36x^2+9x+24x+6) - (18x^2+9x)}{(3x+2)^2}$$

$$= \frac{18x^2 + 24x + 6}{(3x+2)^2} = \underline{\underline{\frac{6 \cdot (3x^2 + 4x + 1)}{(3x+2)^2}}}$$

c) $3x^2 + 4x + 1 = 0$

$$x = \frac{-4 \pm \sqrt{16 - 4 \cdot 3 \cdot 1}}{2 \cdot 3}$$

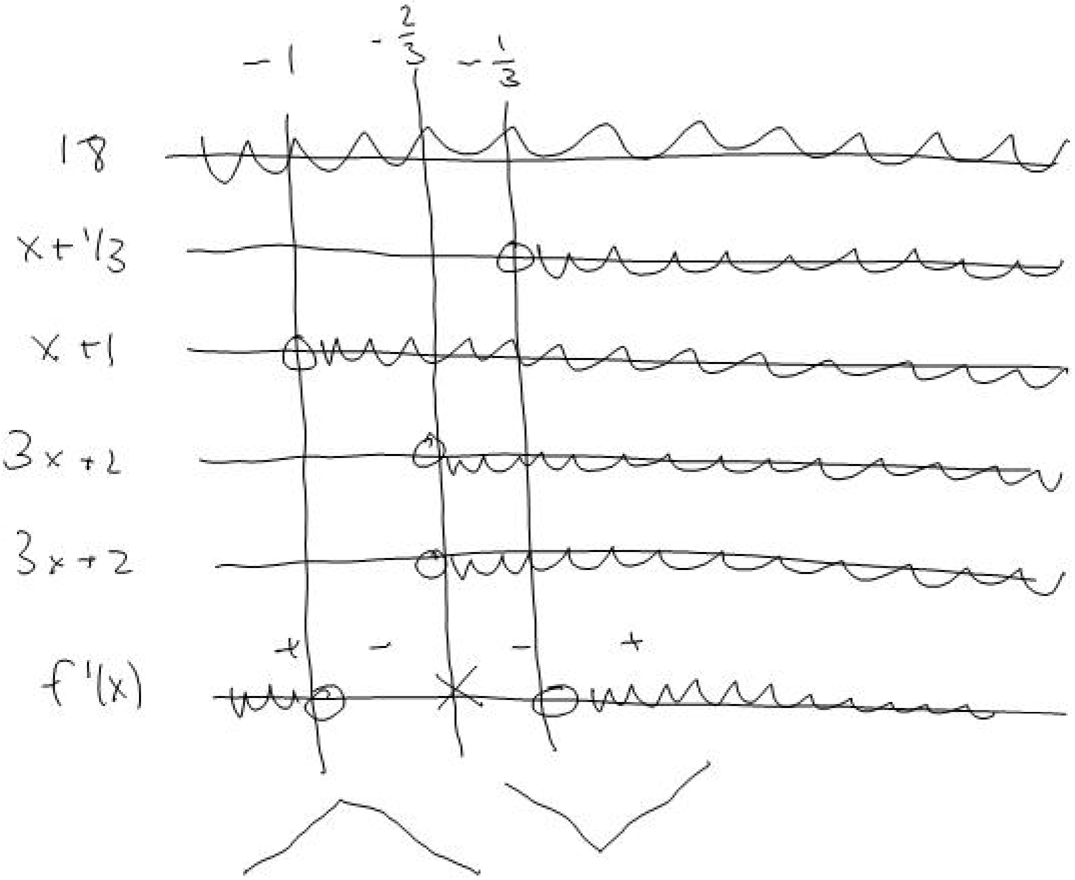
$$= \frac{-4 \pm 2}{6}$$

$\underline{x = -1/3}$ $\qquad$ $\underline{x = -1}$

$3x^2 + 4x + 1$

$= 3(x + 1/3) \cdot (x+1)$

$$f'(x) = \frac{6 \cdot 3 \cdot (x + 1/3) \cdot (x+1)}{(3x+2)^2}$$



lokalt topp:

$x = -1$

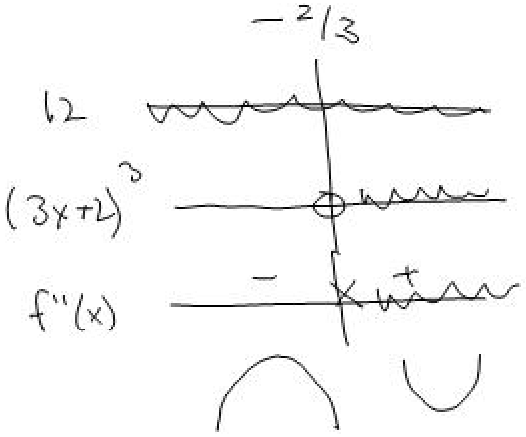
$\underline{\underline{y = -3}}$

lokalt bunn:

$x = -\frac{1}{3}$

$\underline{\underline{y = -\frac{1}{3}}}$

g) $f''(x) = \dfrac{12}{(3x+2)^3}$



<u>Ingen vendepkt.</u>

Eneste mulige vendepkt er $x = -2/3$

Men $x = -2/3$ er ikke med i D_f.