Derivasjon – kjerneregelen (kap. 8.8)

(6) Kjerneregelen:

$$\boxed{\left(f(u(x))\right)' = f'(u) \cdot u'(x)}$$

derivert ytre

derivert av kjernen

Sammensatte funksjoner:

Eks: $f(x) = \sqrt{x^2+1}$ sammensatt funksjon

$u(x) = x^2+1$ $\begin{cases} \text{indre funksjon} \\ \text{kjerne} \end{cases}$

$f(u) = \sqrt{u}$ ytre funksjon

$$\left(\sqrt{x^2+1}\right)' = \frac{1}{2\sqrt{u}} \cdot 2x = \frac{\cancel{2}x}{\cancel{2}\sqrt{u}} = \frac{x}{\sqrt{u}}$$

$$= \frac{x}{\sqrt{x^2+1}}$$

Eks:

$$f(x) = \sqrt{2x}$$

$$f'(x) = f'(u) \cdot u'$$

$$= \frac{1}{2\sqrt{u}} \cdot 2$$

$$= \frac{\cancel{2}}{\cancel{2}\sqrt{u}} = \underline{\underline{\frac{1}{\sqrt{2x}}}}$$

$u = 2x$ kjernen

$u' = 2$ derivert av kjernen

$f(u) = \sqrt{u}$ ytre fn.

$f'(u) = \frac{1}{2\sqrt{u}}$ derivert

<u>Alt:</u>

$$f(x) = \sqrt{2x} = \sqrt{2} \cdot \sqrt{x}$$

$$f'(x) = (\sqrt{2} \cdot \sqrt{x})' = \sqrt{2} \cdot (\sqrt{x})'$$

$$= \sqrt{2} \cdot \frac{1}{2\sqrt{x}} = \frac{\sqrt{2}}{2\sqrt{x}} = \underline{\underline{\frac{1}{\sqrt{2} \cdot \sqrt{x}}}}$$

<u>Husk:</u> $\frac{2}{\sqrt{2}} = \frac{\cancel{\sqrt{2}} \cdot \sqrt{2}}{\cancel{\sqrt{2}}} = \sqrt{2}$

<u>Eks:</u>

$$f(x) = (x+1)^7$$

<u>Kjerneregelen:</u>

$u = x+1$

$\underline{u' = 1}$

$f(u) = u^7$

$f'(u) = \underline{7u^6}$

$$f'(x) = 7u^6 \cdot 1 = 7u^6 = \underline{\underline{7 \cdot (x+1)^6}}$$

$$\underline{g(x) = (1-x)^7}$$

$u = 1-x$

$u' = -1$

$g(u) = u^7$

$g'(u) = 7u^6$

$$g'(x) = 7u^6 \cdot (-1)$$

$$= -7u^6 = \underline{\underline{-7(1-x)^6}}$$

Skrivemåte: Leibniz - notasjon

$$f'(x) = (f(x))' = \frac{df}{dx} = \frac{dy}{dx}$$

Husk at: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$

Kjerneregelen med Leibniz - notasjon:

$$\boxed{\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}}$$ når $f(x) = f(u(x))$

Oppgaver:

a) $f(x) = (2x+3)^2$ $f'(x) = ?$

b) $f(x) = \sqrt{2x+3}$ $f'(x) = ?$

c) $f(x) = \frac{1}{x+1}$ $f'(x) = ?$

d) $f(x) = x^2 \cdot \sqrt{2x-1}$ $f'(x) = ?$

(a) $f(x) = (2x+3)^2$

$\boxed{\begin{array}{l} u = 2x+3 \\ u' = 2 \end{array}}$

$f'(x) = 2u \cdot 2$

$= 4u = 4(2x+3) = \underline{\underline{8x+12}}$

(b) $f(x) = \sqrt{2x+3}$

$\boxed{\begin{array}{l} u = 2x+3 \\ u' = 2 \end{array}}$

$f'(x) = \dfrac{1}{2\sqrt{2x+3}} \cdot 2$

$= \dfrac{\cancel{2}}{\cancel{2}\sqrt{2x+3}} = \underline{\underline{\dfrac{1}{\sqrt{2x+3}}}}$

(c) $f(x) = \dfrac{1}{x+1} = \dfrac{1}{u} = u^{-1}$

$\boxed{\begin{array}{l} u = x+1 \\ u' = 1 \end{array}}$

$f'(\cancel{x}) = -1 \cdot \dfrac{1}{u^2} \cdot 1$

$\boxed{(u^{-1})' = -1\, u^{-2}}$

$= \dfrac{-1}{u^2} = \underline{\underline{\dfrac{-1}{(x+1)^2}}}$

$f(x) = \dfrac{1}{x+1} = \dfrac{u}{v}$

$\boxed{\begin{array}{ll} u = 1 & v = x+1 \\ u' = 0 & v' = 1 \end{array}}$

$f'(x) = \dfrac{u' \cdot v - u \cdot v'}{v^2} = \dfrac{0 \cdot v - 1 \cdot 1}{(x+1)^2}$

$= \underline{\underline{\dfrac{-1}{(x+1)^2}}}$

d) $f(x) = x^2 \cdot \sqrt{2x-1}$
$= u \cdot v$

$f'(x) = u' \cdot v + u \cdot v'$
$= 2x \cdot \sqrt{2x-1}$
$+ x^2 \cdot \frac{1}{\sqrt{2x-1}}$

$= 2x\sqrt{2x-1} + \frac{x^2}{\sqrt{2x-1}}$

$= \frac{2x\sqrt{2x-1} \cdot \sqrt{2x-1} + x^2}{\sqrt{2x-1}}$

$= \frac{2x \cdot (2x-1) + x^2}{\sqrt{2x-1}}$

$= \frac{5x^2 - 2x}{\sqrt{2x-1}} = \frac{x \cdot (5x-2)}{\sqrt{2x-1}}$

produktregelen:

$u = x^2 \qquad v = \sqrt{2x-1}$
$u' = 2x \qquad v' = \frac{1}{\sqrt{2x-1}}$

$(\sqrt{2x-1})' = (\sqrt{w})'$ $\quad \boxed{\begin{array}{l} w = 2x-1 \\ w' = 2 \end{array}}$

$= \frac{1}{2\sqrt{w}} \cdot 2 = \frac{\cancel{2}}{\cancel{2}\sqrt{w}}$

$= \frac{1}{\sqrt{w}} = \frac{1}{\sqrt{2x-1}}$

$f(x) = x^2 \cdot \sqrt{2x-1}$,
$x \geq 1/2$