Derivasjon (kap. 8.8 – 8.9)

Regneregler:

(1) $(x^n)' = n \cdot x^{n-1}$ for alle n

(2) $(u \pm v)' = u' \pm v'$ for alle uttrykk u og v

(3) $(c \cdot u)' = c \cdot u'$ for alle tall c og alle uttrykk

(4) $(u \cdot v)' = u' v + u \cdot v'$ for alle uttrykk u og v

(5) $\left(\frac{u}{v}\right)' = \frac{u' v - u \cdot v'}{v^2}$ for alle uttrykk u og v

(4) kalles <u>produktregelen</u>

(5) kalles <u>brøkregelen</u> eller <u>kvosientregelen</u>

$$\boxed{\begin{array}{ll} u = x+1 & v = x-3 \\ u' = 1 & v' = 1 \end{array}}$$

<u>Eks:</u>

$$f(x) = \underbrace{(x+1)}_{u} \cdot \underbrace{(x-3)}_{v}$$

$$f'(x) = u' \cdot v + u \cdot v'$$

$$= 1 \cdot (x-3) + (x+1) \cdot 1$$

$$= x - 3 + x + 1 = \underline{2x - 2}$$

<u>Alt:</u>

$$f(x) = (x+1) \cdot (x-3) = x^2 - 2x - 3$$

$$f'(x) = \underline{2x - 2}$$

Ec5:

$$f(x) = \overset{u}{x} \cdot \overset{v}{\sqrt{x}}$$

$$\boxed{\begin{array}{ll} u = x & v = \sqrt{x} \\ u' = 1 & v' = \frac{1}{2\sqrt{x}} \end{array}}$$

$$f'(x) = u' \cdot v + u \cdot v'$$

$$= 1 \cdot \sqrt{x} + x \cdot \frac{1}{2\sqrt{x}}$$

$$= \sqrt{x} + \frac{x}{2\sqrt{x}} = \sqrt{x} + \frac{1}{2}\sqrt{x} = \underline{\underline{\frac{3}{2}\sqrt{x}}}$$

E6:

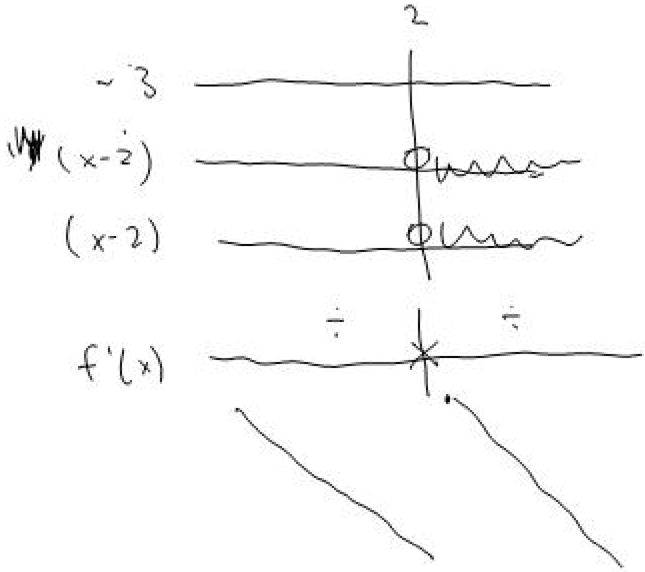
$$f(x) = \frac{x+1}{x-2} = \frac{u}{v}, \quad x \neq 2$$

$$\boxed{\begin{array}{ll} u = x+1 & v = x-2 \\ u' = 1 & v' = 1 \end{array}}$$

$$f'(x) = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$= \frac{1 \cdot (x-2) - (x+1) \cdot 1}{(x-2)^2}$$

$$= \frac{(\cancel{x}-2) - (\cancel{x}+1)}{(x-2)^2} = \underline{\underline{\frac{-3}{(x-2)^2}}}$$



Res:

$$f(x) = \frac{x^2 - 3x + 2}{x+3} = \frac{u}{v}$$

$$\boxed{\begin{array}{ll} u = x^2 - 3x + 2 & v = x+3 \\ u' = 2x - 3 & v' = 1 \end{array}}$$

$$f'(x) = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$= \frac{(2x-3)(x+3) - (x^2-3x+2)\cdot 1}{(x+3)^2}$$

$$= \frac{(2x^2 - 3x + 6x - 9) - (x^2 - 3x + 2)}{(x+3)^2}$$

$$= \frac{x^2 + 6x - 11}{(x+3)^2}$$

$$= \frac{(x-x_1)(x-x_2)}{(x+3)^2}$$

$$\simeq \frac{(x-1.46)(x+7.46)}{(x+3)^2}$$

$$x^2 + 6x - 11 = 0$$

$$x = \frac{-6 \pm \sqrt{36+44}}{2}$$

$$= \frac{-6 \pm \sqrt{80}}{2}$$

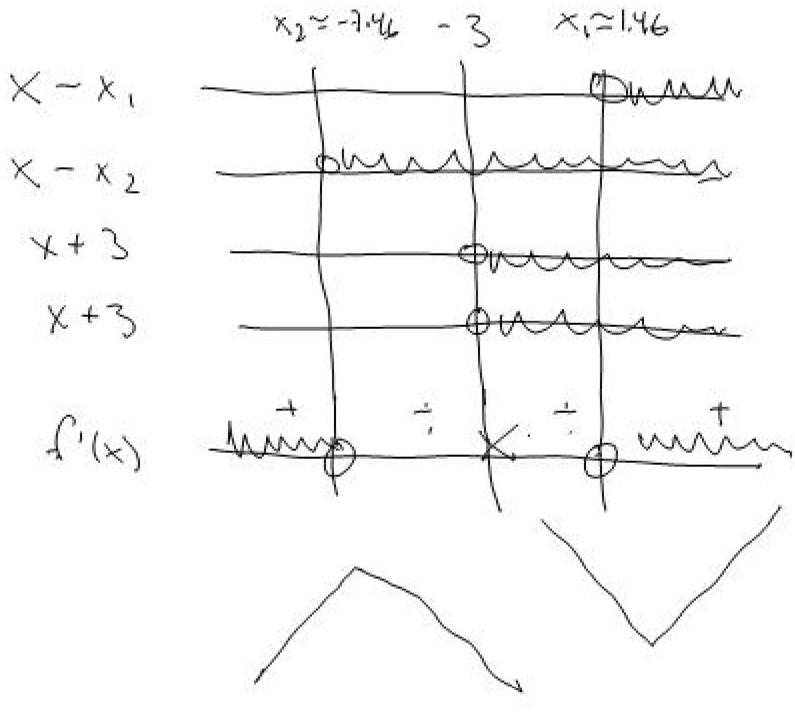
$$= -3 \pm \sqrt{20}$$

$$x = x_1 = -3 + \sqrt{20}$$

$$x = x_2 = -3 - \sqrt{20}$$

$$x_1 \simeq 1.46$$

$$x_2 \simeq -7.46$$



Om bevis for produktregelen

$$f(x) = u \cdot v = u(x) \cdot v(x)$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{u(x+h) \cdot v(x+h) - u(x) \cdot v(x)}{h}$$

$$= \lim_{h \to 0} \frac{u(x+h) \cdot v(x+h) - u(x) \cdot v(x+h) + u(x) \cdot v(x+h) - u(x) \cdot v(x)}{h}$$

$$= \lim_{h \to 0} \frac{\left(u(x+h) - u(x)\right) \cdot v(x+h)}{h} + \lim_{h \to 0} \frac{u(x) \cdot \left(v(x+h) - v(x) \right)}{h}$$

$$= \underline{u'(x) \cdot v(x) + u(x) \cdot v'(x)}$$

$$= \underline{\underline{u' \cdot v + u \cdot v'}}$$

$$f(x) = \frac{x^2-3x+2}{x+3}, \quad x \neq -3$$

$$f'(x) = \frac{\overbrace{x^2+6x-11}^{=u}}{\underbrace{(x+3)^2}_{=v}} \simeq \frac{\overbrace{(x-1.46)\cdot(x+7.46)}^{=u}}{\underbrace{(x+3)^2}_{=v}}$$

$$f''(x) = \frac{u' \cdot v - u \cdot v'}{v^2}$$

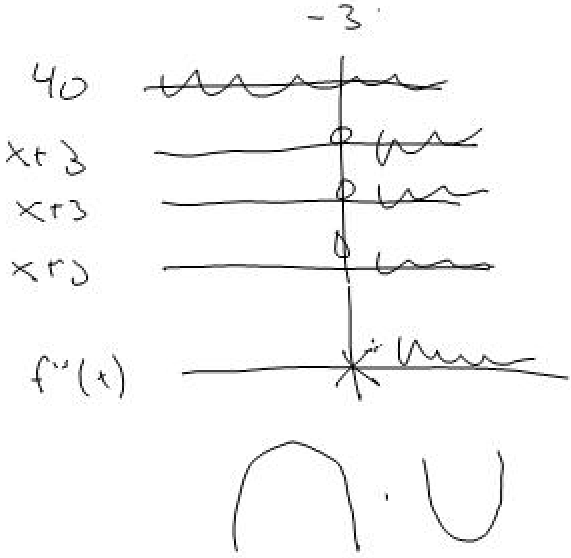
$$\boxed{\begin{array}{ll} u = x^2+6x-11 & v = (x+3)^2 \\ u' = 2x+6 & v' = 2x+6 \\ \hline v = (x+3)^2 = x^2+6x+9 & \end{array}}$$

$$= \frac{(2x+6)(x+3)^2 - (x^2+6x-11)(2x+6)}{((x+3)^2)^2}$$

$$= \frac{\cancel{(x+3)}\cdot\left((2x+6)(x+3) - (x^2+6x-11)\cdot 2\right)}{(x+3)^{\cancel{4}\,3}}$$

$$= \frac{(\cancel{2x^2}+\cancel{6x}+\cancel{6x}+18) - (\cancel{2x^2}+\cancel{12x}-22)}{(x+3)^3}$$

$$= \frac{40}{(x+3)^3}$$



ingen vendepkt.