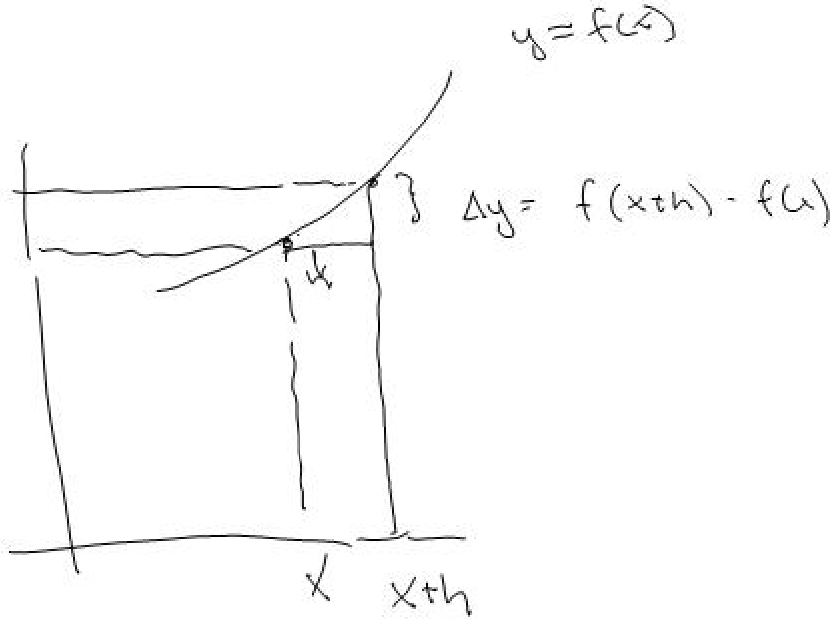
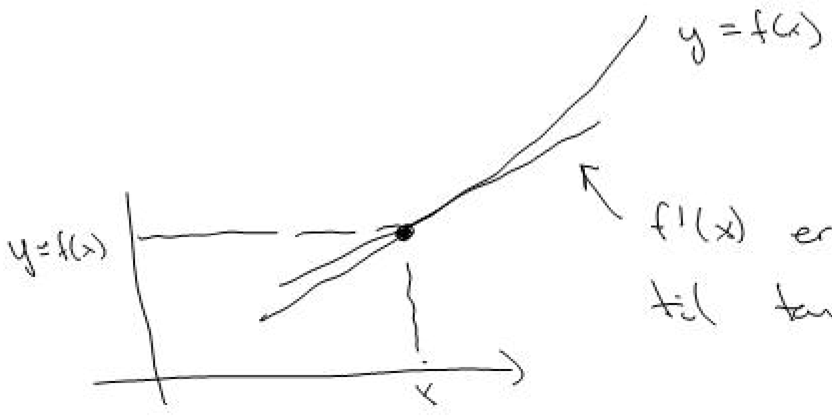
Derivasjon:

Den deriverte til en funksjon $f(x)$ er funksjonen

$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$



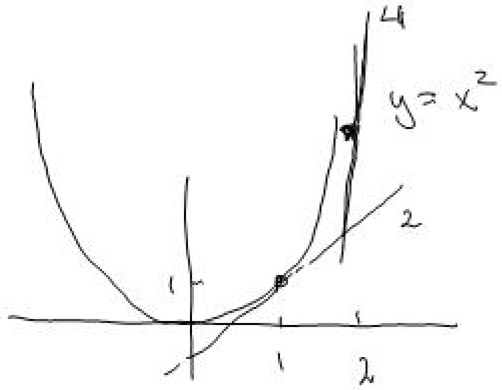
Geometrisk tolkning av $f'(x)$:



$f'(x)$ er stigningstallet til tangenten i punktet.

Utregning av den deriverte

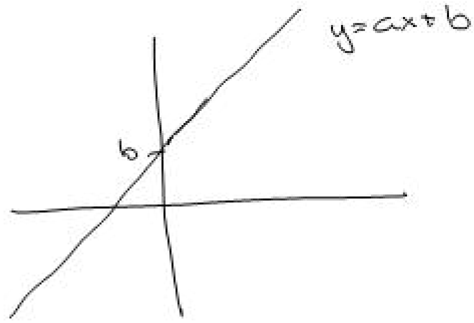
Defn: $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



Eks: $f(x) = x^2$

$f'(x) = 2x$

$f'(1) = \underline{2}$ $\qquad$ $f'(2) = \underline{4}$



Eks: $f(x) = ax + b$

$f'(x) = a$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(a \cdot (x+h) + b) - (ax+b)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{ax} + ah \cancel{+b} - \cancel{ax} - \cancel{b}}{\cancel{h}}$$

$$= \lim_{h \to 0} a = \underline{\underline{a}}$$

Regneregler for derivasjon:

$$\boxed{\left.\begin{aligned} f(x) &= x^n \\ f'(x) &= n \cdot x^{n-1} \end{aligned}\right\} \quad \text{for alle tall } n.}$$

Eks:

$f(x) = x^2$ $\quad n=2$

$f'(x) = 2x$

$f(x) = x^3$ $\quad n=3$

$f'(x) = 3x^2$

$f(x) = x$ $\quad n=1$

$f'(x) = 1$

Skrivemåte: $f'(x) = (f(x))'$

Eks:

$(x^3)' = 3x^2$

$(x^n)' = nx^{n-1}$

Regneregel: $\boxed{(x^n)' = n \cdot x^{n-1} \quad \text{for alle tall } n}$

Eks: $n=-1$ $\qquad (x^{-1})' = -1 \cdot x^{-2}$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$n=-3$: $\qquad (x^{-3})' = -3 \cdot x^{-4}$

$$\left(\frac{1}{x^3}\right)' = -\frac{3}{x^4}$$

$n=1/2$: $\qquad (x^{1/2})' = \frac{1}{2} \cdot x^{-1/2}$

$$\left(\sqrt{x}\right)' = \frac{1}{2}\,\frac{1}{x^{1/2}} = \underline{\frac{1}{2\sqrt{x}}}$$

$n=1/4$: $\qquad (x^{1/4})' = \frac{1}{4} x^{-3/4}$

$$\left(\sqrt[4]{x}\right)' = \underline{\frac{1}{4 \cdot \sqrt[4]{x^3}}}$$

Hvorfor blir $(x^n)' = n x^{n-1}$?

$$f(x) = x^n$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h \to 0} \frac{\left(\cancel{x^n} + n \cdot x^{n-1} \cdot \cancel{h} + \frac{n \cdot (n-1)}{2} x^{n-2} \cdot h^{\overset{h}{\cancel{2}}} + \dots + h^{\overset{n-1}{\cancel{n}}}\right) - \cancel{x^n}}{\cancel{h}}$$

$$= \lim_{h \to 0} \left(n x^{n-1} + \frac{n(n-1)}{2} x^{n-2} \cdot h + \dots \right)$$

$$= \underline{\underline{n x^{n-1}}}$$

Regneregler for derivasjon:

(1) $(x^n)' = n \cdot x^{n-1}$ for alle n

(2) $(u \pm v)' = u' \pm v'$ for alle uttrykk u og v

(3) $(c \cdot u)' = c \cdot u'$ for alle tall c og alle uttrykk u.

Eks:

$$(x^2 + x^3)' = (x^2)' + (x^3)' = 2x + 3x^2$$

$$(x - x^4) = (x)' - (x^4)' = 1 - 4x^3$$

$$f(x) = x^2 + x^3$$

$$f'(x) = 2x + 3x^2$$

$$f'(1) = 2 + 3 = 5$$

$$(x^2 + 4x + 7)' = (x^2)' + (4x)' + (7)'$$

$$= 2x + 4 \cdot (x)' + 0$$

$$= \underline{2x + 4}$$

Eks:

$$(x^3 - 3x^2 + 4x - 12)'$$

$$= (x^3)' - (3x^2)' + (4x)' - (12)'$$

$$= 3x^2 - 3 \cdot 2x + 4 \cdot 1 - 0$$

$$= \underline{\underline{3x^2 - 6x + 4}}$$

$$(12)' = 12 \cdot (1)' = 0$$

$$3x^2 - 6x + 4 = 0$$

$$x = \frac{6 \pm \sqrt{36 - 48}}{6}$$

ingen løsning