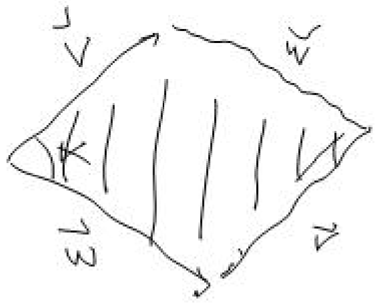
Arealberegning:



I rommet: $\vec{v} = (v_1, v_2, v_3)$, $\vec{w} = (w_1, w_2, w_3)$

$$\text{Areal} = |\vec{v} \times \vec{w}|$$

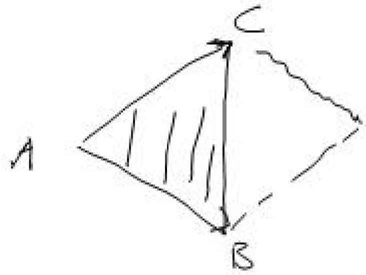
I planet: $\vec{v} = (v_1, v_2)$, $\vec{w} = (w_1, w_2)$

$$\text{Areal} = \left| \begin{vmatrix} v_1 & v_2 \\ w_1 & w_2 \end{vmatrix} \right|$$

Areal:

$$\boxed{A = |\vec{v}| \cdot |\vec{w}| \cdot \sin\alpha}$$

Eks:
$A = (-2, 1, 1)$
$B = (1, -1, 2)$
$C = (-1, 3, 3)$



Areal av trekanten $\triangle ABC$

$$A = \frac{1}{2} \cdot |\vec{AB} \times \vec{AC}|$$

$$\begin{cases} \vec{AB} = (3, -2, 1) \\ \vec{AC} = (1, 2, 2) \end{cases}$$

$$\vec{AB} \times \vec{AC} = (3, -2, 1) \times (1, 2, 2)$$

$$= \begin{vmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ 3 & -2 & 1 \\ 1 & 2 & 2 \end{vmatrix}$$

$$= \vec{e}_x \cdot \begin{vmatrix} -2 & 1 \\ 2 & 2 \end{vmatrix} - \vec{e}_y \cdot \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} + \vec{e}_z \cdot \begin{vmatrix} 3 & -2 \\ 1 & 2 \end{vmatrix}$$

$$= \vec{e}_x \cdot (-2 \cdot 2 - 1 \cdot 2) - \vec{e}_y \cdot (3 \cdot 2 - 1 \cdot 1) + \vec{e}_z \cdot (3 \cdot 2 - (-2) \cdot 1)$$

$$= \vec{e_x} \cdot (-6) - \vec{e_y} \cdot 5 + \vec{e_z} \cdot 8$$

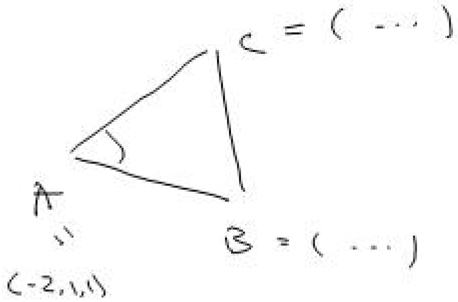
$$= -6\vec{e_x} - 5\vec{e_y} + 8\vec{e_z} = \underline{(-6,-5,8)}$$

$$\vec{AB} \times \vec{AC} = (-6,-5,8)$$

$$\text{Areal} = \frac{1}{2} \left| \vec{AB} \times \vec{AC} \right| = \frac{1}{2} \cdot \sqrt{(-6)^2 + (-5)^2 + 8^2}$$

$$= \frac{1}{2} \cdot \sqrt{36 + 25 + 64} = \frac{\sqrt{125}}{2} = \underline{\underline{\frac{5 \cdot \sqrt{5}}{2}}}$$

Eks: Hva er $\angle A$?



Metode I:

$$\cos A = \frac{\vec{AB} \cdot \vec{AC}}{AB \cdot AC}$$

$$= \frac{(3,-2,1) \cdot (1,2,2)}{\sqrt{14} \cdot 3} = \frac{3 \cdot 1 + (-2) \cdot 2 + 1 \cdot 2}{3\sqrt{14}} = \frac{1}{3\sqrt{14}}$$

$AB = |\vec{AB}|$

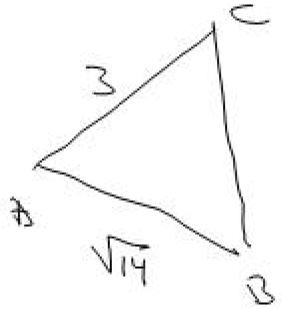
$AC = |\vec{AC}|$

$$AB = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{14}$$

$$AC = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{9} = \underline{3}$$

$$\angle A = \cos^{-1}\left(\frac{1}{3\sqrt{14}}\right) \simeq \underline{85^\circ}$$

Metode 2:



$$\text{Areal} = \frac{5 \cdot \sqrt{5}}{2}$$

$$\text{Areal} = \frac{1}{2} \cdot AB \cdot AC \cdot \sin A$$

$$\frac{5\sqrt{5}}{2} = \frac{AB \cdot AC \cdot \sin A}{2}$$

$$\frac{5\sqrt{5}}{\cancel{2}} = \frac{\sqrt{14} \cdot 3 \cdot \sin A}{\cancel{2}} \quad \Big| \cdot 2$$

$$\frac{5\sqrt{5}}{3\sqrt{14}} = \frac{\cancel{\sqrt{14}\,3} \cdot \sin A}{\cancel{3\sqrt{14}}} \quad \Big| : 3\sqrt{14}$$

$$\sin A = \frac{5\sqrt{5}}{3\sqrt{14}} \implies \angle A = \sin^{-1}\left(\frac{5\sqrt{5}}{3\sqrt{14}}\right)$$

$$\simeq \underline{\underline{85^\circ}}$$

Eks:

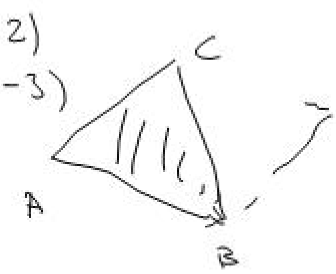
$A = (3,5)$

$B = (-1,7)$

$C = (2,2)$

$\vec{AB} = (-4, 2)$

$\vec{AC} = (-1, -3)$



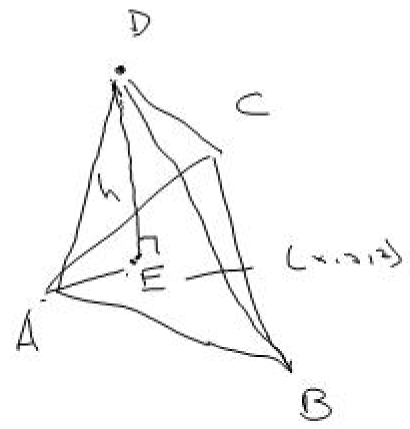
$$\text{Areal av } \Delta ABC = \frac{1}{2}\left|\begin{vmatrix} \vec{AB} \\ \vec{AC} \end{vmatrix}\right| = \frac{1}{2}\left|\begin{vmatrix} -4 & 2 \\ -1 & -3 \end{vmatrix}\right|$$

$$= \frac{1}{2}\Big| (-4) \cdot (-3) - 2 \cdot (-1) \Big| = \frac{1}{2} \cdot 14 = \underline{\underline{7}}$$

Volumberegning

Eks:

$$\begin{cases} A = (-2,1,1) \\ B = (1,-1,2) \\ C = (-1,3,3) \\ D = (3,7,1) \end{cases}$$



Trekantet pyramide med topp-pkt D og grunnflate $\triangle ABC$.

$$V = \frac{1}{3} \cdot A \cdot h$$

$$= \frac{1}{3} \cdot \underbrace{A(\triangle ABC)}_{= \frac{5\cdot\sqrt{5}}{2}} \cdot h$$

Metode I: Regn ut E og $h = DE$.

$$\begin{cases} \vec{AE} = s \cdot \vec{AB} + t \cdot \vec{AC} \\ \vec{AE} \cdot \vec{DE} = 0 \end{cases}$$ (s og t ukjente)

$E = (x,y,z)$:

$$\vec{AE} = (x+2,\ y-1,\ z-1)$$

$$\vec{DE} = (x-3,\ y-7,\ z-1)$$

$$\vec{AE} \cdot \vec{DE} = (x+2)(x-3) + (y-1)(y-7) + (z-1)^2 = 0$$

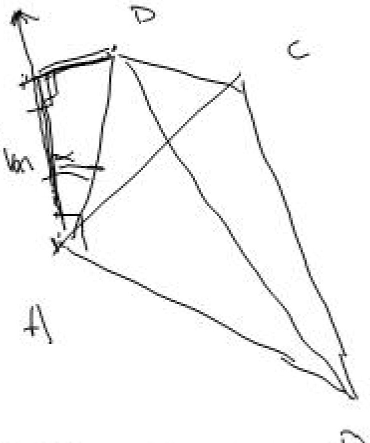
$\vdots$

Mulig, men vanskelig.

Metode 2:

$\vec{AB} \times \vec{AC}$

$|\vec{AB} \times \vec{AC}| = \text{Areal}(\triangle ABC) \cdot 2$



$\boxed{h = AD \cdot \cos\alpha}$

Volum:

$$V = \frac{1}{6} \left| (\vec{AB} \times \vec{AC}) \cdot \vec{AD} \right|$$

$$= \frac{1}{6} \cdot \left| \underbrace{|\vec{AB} \times \vec{AC}|} \cdot \underbrace{|\vec{AD}| \cdot \cos\alpha} \right|$$

$$= \frac{1}{6} \left(\text{Areal}(\triangle ABC) \cdot 2 \right) \cdot h$$

$$= \frac{1}{3} \cdot \text{Areal}(\triangle ABC) \cdot h$$

$\vec{AB} \times \vec{AC} = (-6, -5, 8)$

$\vec{AD} = (5, 6, 0)$

$A = (-2, 1, 1)$

$D = (3, 7, 1)$

$$(\vec{AB} \times \vec{AC}) \cdot \vec{AD} = (-6, -5, 8) \cdot (5, 6, 0)$$

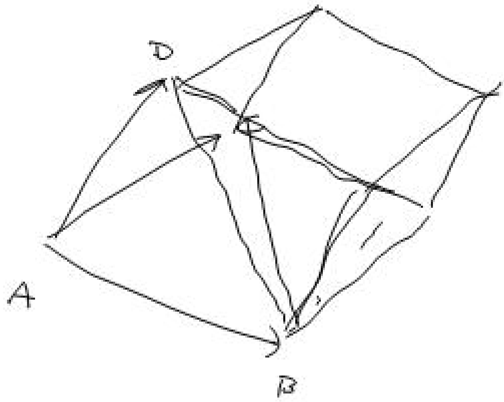
$$= -6 \cdot 5 + (-5) \cdot 6 + 8 \cdot 0$$

$$= \underline{-60}$$

$$V = \frac{1}{6} \left| (\vec{AB} \times \vec{AC}) \cdot \vec{AD} \right| = \frac{1}{6} \cdot |-60| = \frac{60}{6} = \underline{\underline{10}}$$

$$\left|\left(\vec{AB}\times\vec{AC}\right)\cdot\vec{AD}\right|$$

trippelprodukt



Volumet av <u>parallellepipedet</u> utspent av $\vec{AB}$, $\vec{AC}$, $\vec{AD}$.

$$\frac{1}{3}\cdot\frac{5\sqrt{5}}{2}\cdot h = 10$$

$$\frac{5\sqrt{5}}{6}\cdot h = 10$$

$$h = \frac{60}{5\sqrt{5}} = \underline{\underline{\frac{12}{\sqrt{5}}}}$$

<u>Volum av pyramiden:</u>

$$\frac{1}{3}\cdot\left(\frac{1}{2}\cdot \text{Grunnflate}\right)\cdot \text{høyde}$$

$$= \frac{1}{6}\cdot \text{Volum av parallellepiped}$$

$$\Downarrow$$

<u>Volum av pyramiden:</u>

$$V = \frac{1}{6}\cdot\left|\left(\vec{AB}\times\vec{AC}\right)\cdot\vec{AD}\right|$$