<u>Repetisjon</u> $\begin{cases} \text{Vektorer i planet} \\ \text{med koordinater} \end{cases}$

① Addisjon / subtraksjon

$\vec{v} = (v_1, v_2)$ $\vec{w} = (w_1, w_2)$

$$\boxed{\begin{aligned} \vec{v} + \vec{w} &= (v_1 + w_1, v_2 + w_2) \\ \vec{v} - \vec{w} &= (v_1 - w_1, v_2 - w_2) \end{aligned}}$$

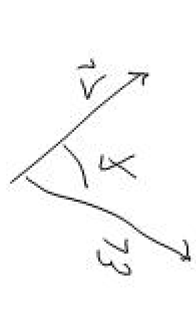
② Skalarmultiplikasjon

c : skalar $\vec{v} = (v_1, v_2)$

$$\boxed{c \cdot \vec{v} = (c \cdot v_1, c \cdot v_2)}$$

③ Prikkprodukt / skalarprodukt

$\vec{v} = (v_1, v_2)$ $\vec{w} = (w_1, w_2)$



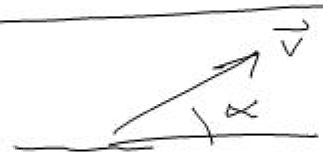
$$\boxed{\begin{aligned} \vec{v} \cdot \vec{w} &= v_1 \cdot w_1 + v_2 \cdot w_2 \\ \vec{v} \cdot \vec{w} &= |\vec{v}| \cdot |\vec{w}| \cdot \cos \alpha \end{aligned}}$$

(4) Lengden til en vektor

$\vec{v} = (v_1, v_2)$

$$\boxed{|\vec{v}| = \sqrt{v_1^2 + v_2^2}}$$

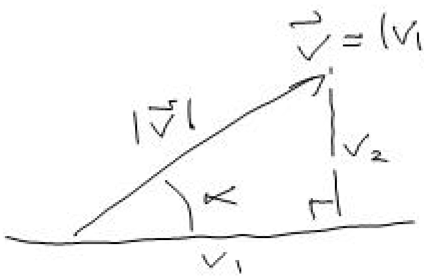
Vinkelen til en vektor ift. horisontalen



$\vec{v} = (v_1, v_2)$

$$\boxed{\alpha = \tan^{-1}(v_2/v_1)}$$

(5) Finne koordinatene til en vektor



Oppgitt: α
$|\vec{v}|$

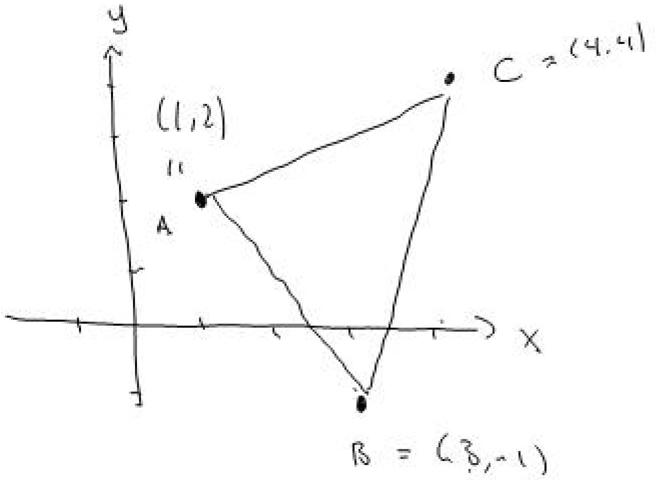
$$\boxed{\begin{aligned} v_1 &= |\vec{v}| \cdot \cos \alpha \\ v_2 &= |\vec{v}| \cdot \sin \alpha \end{aligned}}$$

Eks: $\vec{v}$ har $\begin{cases} |\vec{v}| = 7 \\ \alpha = 30^\circ \end{cases}$ $\quad (6, 3{,}5)$

$$\left.\begin{aligned} v_1 &= 7 \cdot \cos 30^\circ = \frac{7 \cdot \sqrt{3}}{2} \approx \underline{6} \\ v_2 &= 7 \cdot \sin 30^\circ = \underline{3.5} = 7/2 \end{aligned}\right\} \quad \underline{\underline{\vec{v} \approx (6, 3.5)}}$$

Eks: ΔABC der $A = (1,2)$, $B = (3,-1)$, $C = (4,4)$.

Finn AB, AC, BC, $\angle A$, $\angle B$, $\angle C$, areal.



$\underline{AB:}$

$\vec{AB} = (2,-3)$

$AB = \sqrt{2^2 + (-3)^2} = \underline{\sqrt{13}} \cong \underline{3.6}$

$$\vec{AB} = (2,-3) = (\underset{3}{x_B} - \underset{1}{x_A}, \underset{-1}{y_B} - \underset{2}{y_A})$$

$$\vec{AC} = (3,2)$$

$$\vec{BC} = (1,5) = (x_C - x_B, \underset{4}{y_C} - \underset{-1}{y_B})$$

$\underline{\text{Avstands formel:}}$

$$\left.\begin{aligned} A &= (x_A, y_A) \\ B &= (x_B, y_B) \end{aligned}\right\} \quad \boxed{AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}}$$

$$AB = \sqrt{13} \simeq 3.6$$

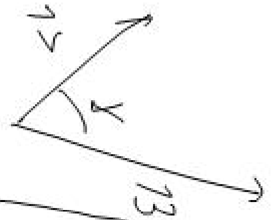
$$AC = \sqrt{3^2 + 2^2} = \sqrt{13} \cong \underline{3.6}$$

$$BC = \sqrt{1^2 + 5^2} = \sqrt{26} \cong \underline{5.0}$$

6 Vinkelen mellom to vektorer

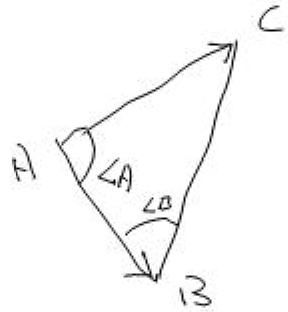
$\vec{v} = (v_1, v_2)$

$\vec{w} = (w_1, w_2)$



$$\boxed{\cos\alpha = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}|\cdot|\vec{w}|} = \frac{v_1w_1 + v_2w_2}{|\vec{v}|\cdot|\vec{w}|}}$$

Eks: $\vec{BA} = (-2,3)$



$\vec{AB} = (2,-3)$ $\quad AB = \sqrt{13}$

$\vec{AC} = (3,2)$ $\quad AC = \sqrt{13}$

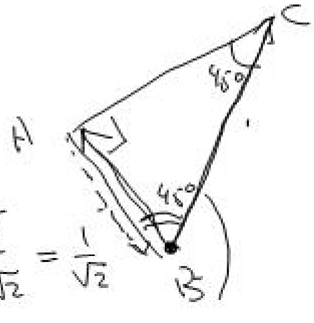
$\vec{BC} = (1,5)$ $\quad BC = \sqrt{26}$

∠A: $\cos A = \frac{\vec{AB}\cdot\vec{AC}}{AB\cdot AC} = \frac{(2,-3)\cdot(3,2)}{\sqrt{13}\cdot\sqrt{13}}$

$\cos A = \frac{2\cdot3 + (-3)\cdot2}{13} = \frac{0}{13} = 0$

$\angle A = \underline{\underline{90^\circ}}$

∠B: $\cos B = \frac{\vec{BA}\cdot\vec{BC}}{BA\cdot BC}$

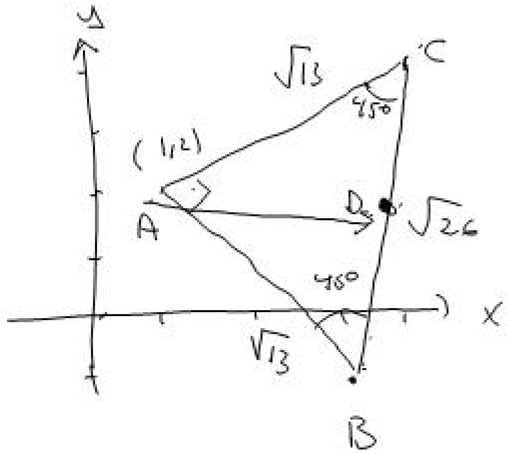


$= \frac{(-2,3)\cdot(1,5)}{\sqrt{13}\cdot\sqrt{26}} = \frac{-2\cdot1 + 3\cdot5}{13\cdot\sqrt{2}} = \frac{\cancel{13}}{\cancel{13}\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\angle B = \cos^{-1}(1/\sqrt{2}) = \underline{\underline{45^\circ}}$

Eks fts.

D er midtpunktet på BC



$$\vec{AD} = \vec{AB} + \vec{BD}$$
$$= \vec{AB} + \frac{1}{2}\vec{BC}$$
$$= (2,-3) + \frac{1}{2}\cdot(1,5) = (2,-3) + (1/2, 5/2)$$
$$= (5/2, -1/2) = \underline{(2.5, -0.5)}$$

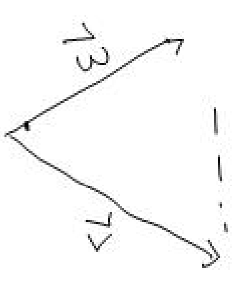
Finn koordinatene til D:

$$\vec{OD} = \vec{OA} + \vec{AD}$$
$$= (1,2) + (2.5, -0.5)$$
$$= (3.5, 1.5)$$

$$D = \underline{\underline{(3.5, 1.5)}}$$

Areal: $A = \frac{\sqrt{13}\cdot\sqrt{13}}{2} = \frac{13}{2} = \underline{\underline{6.5}}$

(7) Areal vha vektorregning



Trekanten utspent av $\vec{v}$ og $\vec{w}$.

$\vec{v} = (v_1, v_2)$

$\vec{w} = (w_1, w_2)$

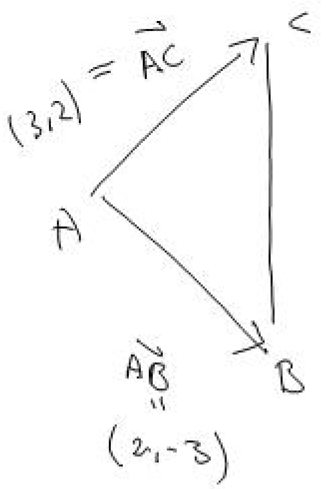
$$A = \frac{1}{2}\cdot\left|\begin{vmatrix} v_1 & v_2 \\ w_1 & w_2 \end{vmatrix}\right|$$
$$= \underline{\underline{\frac{1}{2}\left|(v_1\cdot w_2 - v_2\cdot w_1)\right|}}$$

$$\begin{vmatrix} v_1 & v_2 \\ w_1 & w_2 \end{vmatrix} = v_1 w_2 - v_2 w_1$$

kalles en 2×2 - determinant.

$$A = \frac{1}{2} \cdot \left\| \begin{matrix} v_1 & v_2 \\ w_1 & w_2 \end{matrix} \right\|$$

Eks:



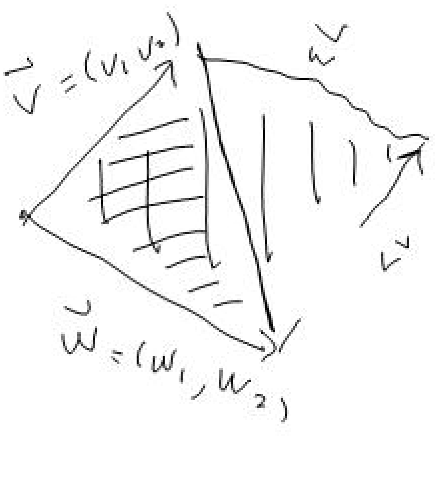
$$\text{Areal} = \frac{1}{2} \cdot \begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix}$$

$$= \frac{1}{2}\left(2 \cdot 2 - (-3) \cdot 3\right)$$

$$= \frac{1}{2}\left(4 + 9\right) = \frac{1}{2} \cdot 13$$

$$= \underline{\underline{6.5}}$$

Husk: $\left\| \begin{matrix} v_1 & v_2 \\ w_1 & w_2 \end{matrix} \right\|$ = arealet av parallello-grammet



Absolutt verdi

$$\|x\| = \begin{cases} x & , x \geq 0 \\ -x & , x < 0 \end{cases}$$

$$|2| = 2$$

$$|-2| = 2$$