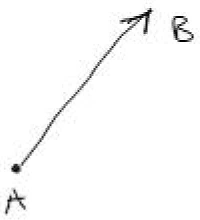
Vektorregning

- vektorer (kap. 12)
- Vektorer i planet (kap. 13)
- Vektorer i rommet (kap. 14)

Defn: En vektor er et rettet linjestykke



$\vec{v} = \vec{AB}$

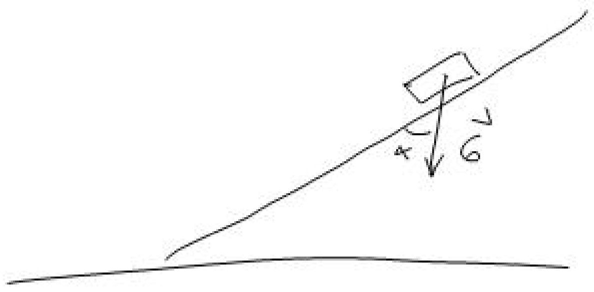
Kjennetegn: både en størrelse og en retning

størrelse : lengden til vektoren

$|\vec{v}|$ (et tall)

retning : angis ofte som en vinkel

Eks:



$|\vec{G}| = 50N$

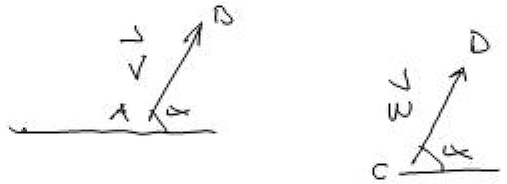
α angir retning

(1) <u>Parallelle vektorer</u>

$\vec{v}$ og $\vec{w}$ er parallelle om linjestykkene er parallelle.

(2)

To vektorer er like hvis de har samme <u>størrelse</u> og samme <u>retning</u>.



$\vec{v} = \vec{w}$

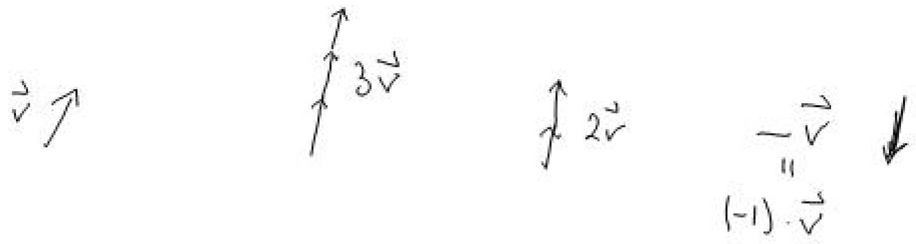
(3) <u>Noen operasjoner:</u>

(a) <u>Vektoraddisjon</u> : $\vec{v} + \vec{w}$

(b) <u>Skalarmultiplikasjon:</u> $\qquad c \cdot \vec{v}$

Skalar = tall $\quad - \; c$

Vektor $\quad - \; \vec{v}$



<u>Regneregler for vektorregning</u>

a) $\vec{u} + \vec{v} = \vec{v} + \vec{u}$

b) $\vec{u} + (\vec{v} + \vec{w}) = (\vec{u} + \vec{v}) + \vec{w}$

c) $\vec{u} + \vec{0} = \vec{0} + \vec{u} = \vec{u}$

d) $\vec{u} + (-\vec{u}) = \vec{0}$

e) $c \cdot (\vec{u} + \vec{v}) = c \cdot \vec{u} + c \cdot \vec{v}$

f) $(c+d) \cdot \vec{u} = c \cdot \vec{u} + d \cdot \vec{u}$

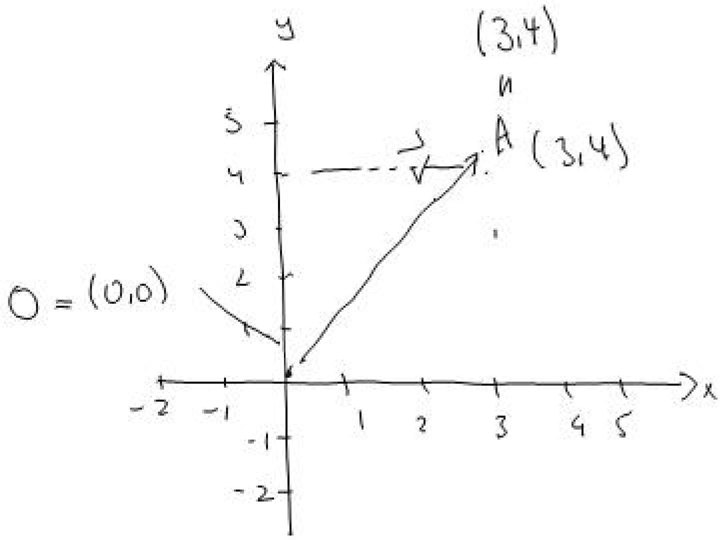
g) $c \cdot (d\vec{u}) = (cd) \cdot \vec{u}$

h) $1 \cdot \vec{u} = \vec{u}$

$\vec{u}, \vec{v}, \vec{w}$ er vektorer

c, d er skalarer

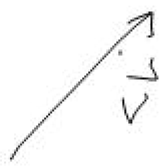
Vektorer i planet



To-dimensjonalt koordinatsystem

x-aksen og y-aksen står normalt på hverandre

= Kartesisk koordinatsystem



$\vec{v} = (3,4)$ $\Longleftrightarrow$ Når vi parallellforskyver $\vec{v}$ slik at startpunktet er origo = (0,0), så er sluttpunktet (3,4)

vektoren $\vec{v}$ med koordinater

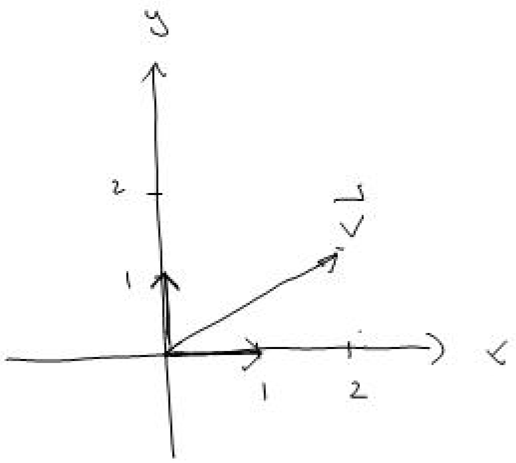
$[3,4]$

$\langle 3,4 \rangle$

$\vec{OA} = (3,4)$ | $A = (3,4)$

$\vec{OA} = [3,4]$

Vektorer | punkter



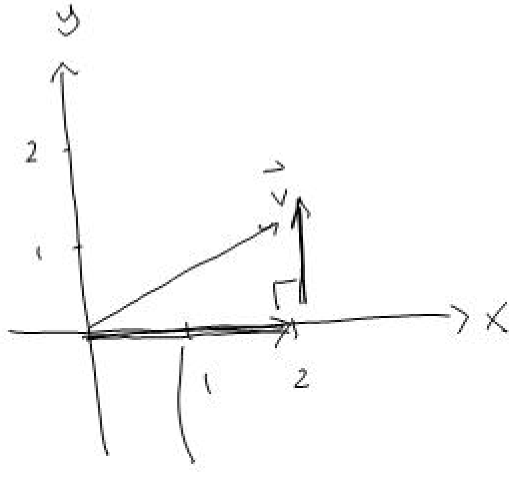
Enhetsvektorer:

$\vec{i} = \vec{e_x}$ $\longrightarrow$

$\vec{j} = \vec{e_y}$ $\uparrow$

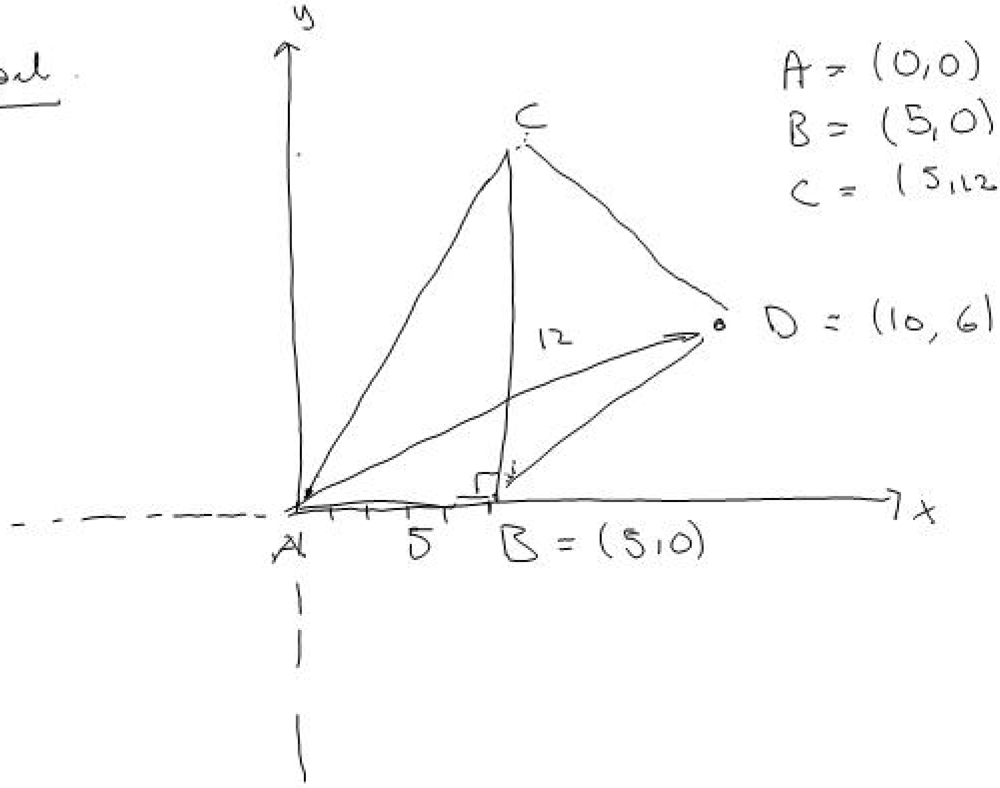
Dekomponering av $\vec{v}$:

$$\vec{v} = a \cdot \vec{e_x} + b \cdot \vec{e_y} = (a, b)$$



$$\vec{v} = 2 \cdot \vec{e_x} + 1 \cdot \vec{e_y} = (2, 1)$$

Eksempel



A = (0,0)
B = (5,0)
C = (5,12)

$\vec{AB} = (5,0)$
$\vec{AC} = (5,12)$
$\vec{BC} = (0,12)$

$\vec{AB} = (x_B - x_A, \; y_B - y_A)$
$= (5 - 0, \; 0 - 0)$
$= \underline{(5,0)}$

$\vec{AC} = (x_C - x_A, \; y_C - y_A)$
$= (5 - 0, \; 12 - 0)$
$= \underline{(5,12)}$

$\vec{BD} = (10 - 5, \; 6 - 0)$
$= (5,6)$

$\vec{AB} + \vec{BC} = \vec{AC}$

$\vec{AD} = \vec{AB} + \vec{BD}$
$= (5,0) + (5,6)$
$= (5+5, \; 0+6) = \underline{\underline{(10,6)}}$

Vektorregning i koordinater

$\vec{v} = (v_1, v_2)$ $\qquad$ c skalar

$\vec{w} = (w_1, w_2)$

① Vektoraddisjon / subtraksjon

$$\vec{v} + \vec{w} = (v_1, v_2) + (w_1, w_2) = (v_1 + w_1, v_2 + w_2)$$

$$\vec{v} - \vec{w} = (v_1, v_2) - (w_1, w_2) = (v_1 - w_1, v_2 - w_2)$$

Eks:

$$(2,3) + (1,-2) = (2+1, 3+(-2)) = (3,1)$$

$$(2,3) - (1,-2) = (2-1, 3-(-2)) = (1,5)$$

② Skalarmultiplikasjon

$$c \cdot \vec{v} = c \cdot (v_1, v_2) = (c \cdot v_1, c \cdot v_2)$$

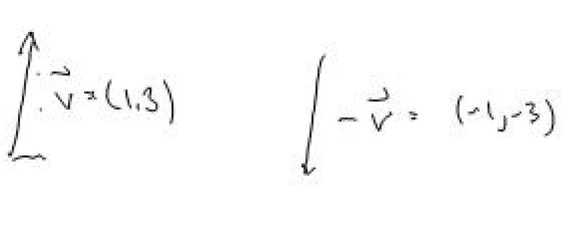
$$-\vec{v} = (-1) \cdot \vec{v} = (-v_1, -v_2)$$

$$\vec{0} = 0 \cdot \vec{v} = (0,0)$$

Eks:

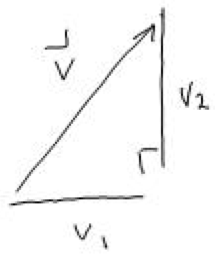
$$2 \cdot (1,3) = (2 \cdot 1, 2 \cdot 3) = (2,6)$$

$$-(1,3) = (-1,-3)$$



(3) Lengden til en vektor

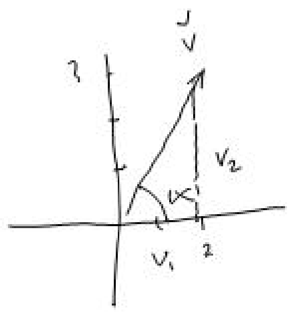
$\vec{v} = (v_1, v_2)$



$|\vec{v}| = \sqrt{v_1^2 + v_2^2}$

Eks: $|(2,3)| = \sqrt{2^2 + 3^2} = \underline{\underline{\sqrt{13}}}$

(4) Vinkelen mellom en vektor og horisontalplanet



Eks: $\vec{v} = (2,3)$

$\tan \alpha = 3/2$

$\alpha = \tan^{-1}(3/2) \sim \underline{56^\circ}$

$\vec{v} = (v_1, v_2)$

$\tan \alpha = v_2/v_1$

$\alpha = \underline{\tan^{-1}(v_2/v_1)}$

(5) Skalarprodukt (prikkprodukt)

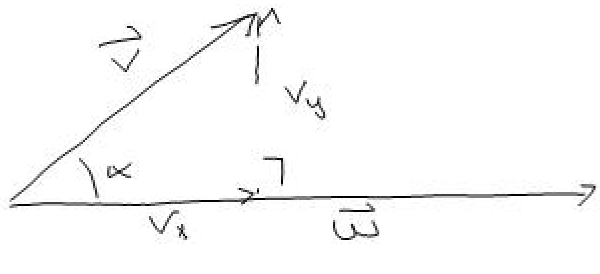
$\vec{v} = (v_1, v_2)$
$\vec{w} = (w_1, w_2)$

$$\vec{v} \cdot \vec{w} = (v_1, v_2) \cdot (w_1, w_2)$$
$$= \underline{v_1 \cdot w_1 + v_2 \cdot w_2}$$

Eks:

$$(2,3) \cdot (1,4) = 2 \cdot 1 + 3 \cdot 4 = \underline{\underline{14}}$$

Kan tenke:
$\vec{v} = \vec{F}$
$\vec{w} = \vec{s}$



$\vec{v}$'s komponent langs $\vec{w}$:

$$\left.\begin{aligned} v_x &= |\vec{v}| \cdot \cos(\alpha) \\ v_y &= |\vec{v}| \cdot \sin(\alpha) \end{aligned}\right\} \text{tall}$$

$$\boxed{v_x \cdot |\vec{w}| = |\vec{v}| \cdot |\vec{w}| \cdot \cos\alpha}$$

Skalarprodukt:

$$\begin{cases} \vec{v} \cdot \vec{w} = |\vec{v}| \cdot |\vec{w}| \cdot \cos(\alpha) \\ \vec{v} \cdot \vec{w} = v_1 \cdot w_1 + v_2 \cdot w_2 \end{cases}$$

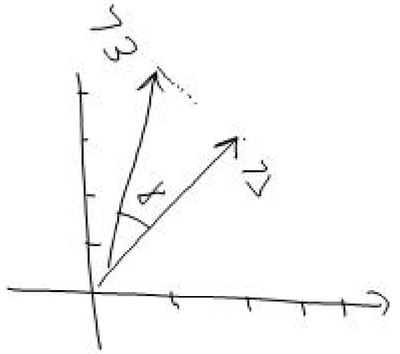
Eks:

$\vec{v} = (2,3)$ $\quad |\vec{v}| = \sqrt{2^2+3^2} = \sqrt{13}$

$\vec{w} = (1,4)$ $\quad |\vec{w}| = \sqrt{1^2+4^2} = \sqrt{17}$

$|\vec{v}| \cdot |\vec{w}| = \sqrt{13} \cdot \sqrt{17} \approx \underline{14.9}$

$\vec{v} \cdot \vec{w} = \underline{14}$



Bruk av prikkprodukt:

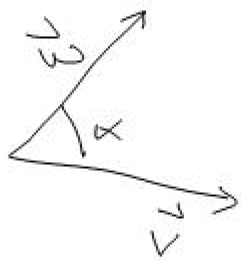
* I fysikk, for eksempel for å beregne arbeid.
* For å regne ut vinkelen mellom to vektorer.

Regner ut vinkelen mellom $\vec{v}$ og $\vec{w}$:

$$v_1 \cdot w_1 + v_2 \cdot w_2 = |\vec{v}| \cdot |\vec{w}| \cdot \cos\alpha$$

$$\cos\alpha = \frac{v_1 w_1 + v_2 w_2}{|\vec{v}| \cdot |\vec{w}|}$$

$$\cos\alpha = \frac{\vec{v} \cdot \vec{w}}{|\vec{v}| \cdot |\vec{w}|}$$

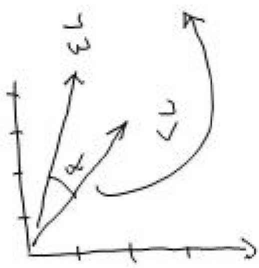


Eks: $\vec{v} = (2,3)$, $\vec{w} = (1,4)$

$$\vec{v} \cdot \vec{w} = (2,3) \cdot (1,4) = 2 \cdot 1 + 3 \cdot 4 = \underline{14}$$

$$|\vec{v}| \cdot |\vec{w}| = \sqrt{13} \cdot \sqrt{17} \sim 14.9$$

$$\cos\alpha \cong \frac{14}{14.9} \rightsquigarrow \alpha \cong 20^\circ$$

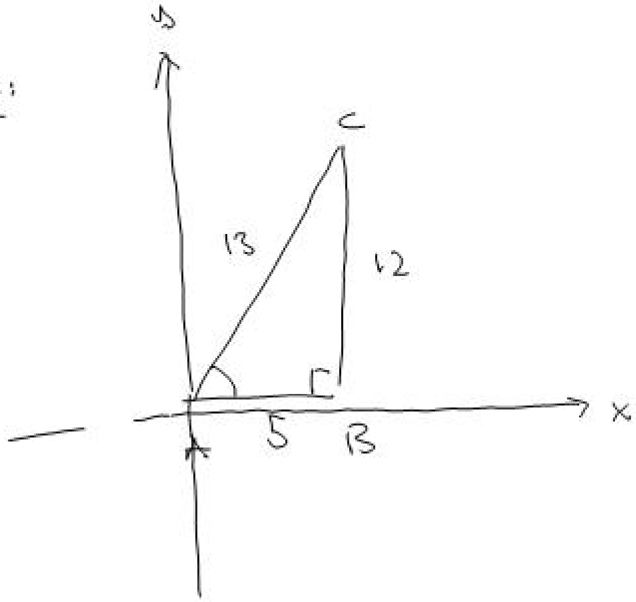


Viktig:

$$\vec{v} \perp \vec{w} \iff \vec{v} \cdot \vec{w} = 0$$

(vinkelen $\alpha = 90^\circ$)

Eks:



$A = (0,0)$
$B = (5,0)$
$C = (5,12)$

$\vec{AB} = (5,0)$
$\vec{AC} = (5,12)$
$\vec{BC} = (0,12)$

$AB = |\vec{AB}| = |(5,0)|$
$= \sqrt{5^2 + 0^2} = \underline{\underline{5}}$

$AC = |\vec{AC}| = |(5,12)|$
$= \sqrt{5^2 + 12^2} = \sqrt{169} = \underline{\underline{13}}$

$\underline{\angle A:}$ $\quad \cos(A) = \dfrac{\vec{AB} \cdot \vec{AC}}{AB \cdot AC} = \dfrac{(5,0) \cdot (5,12)}{5 \cdot 13}$

$= \dfrac{5 \cdot 5 + 0 \cdot 12}{5 \cdot 13} = \dfrac{\cancel{5} \cdot 5}{\cancel{5} \cdot 13} = \dfrac{5}{13}$

$\angle A = \cos^{-1}(5/13) \cong \underline{\underline{67^\circ}}$