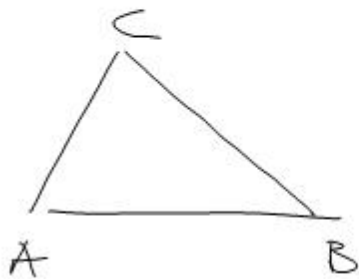


Geometri (kap. 9)

- trekkanter
- sirkler
- volum / overflate
av tredimensjonale figurer

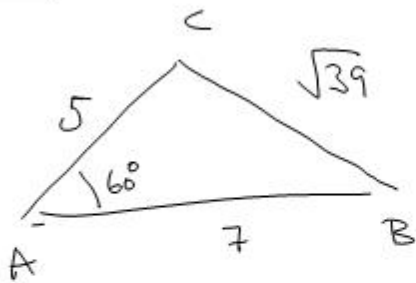
Trekkanter:

① Cosinus setningen



$$BC^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos(A)$$

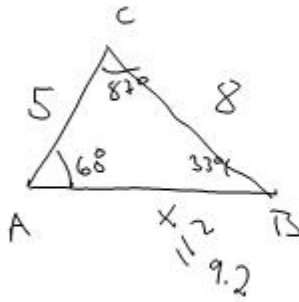
Eks:



$$\begin{aligned} BC^2 &= 7^2 + 5^2 - 2 \cdot 7 \cdot 5 \cdot \cos 60^\circ \\ &= 49 + 25 - 70 \cdot \frac{1}{2} \\ &= 74 - 35 = 39 \end{aligned}$$

$$BC = \underline{\underline{\sqrt{39}}}$$

EW:



$$BC^2 = AB^2 + AC^2 - 2 \cdot AB \cdot AC \cdot \cos A$$

$$8^2 = AB^2 + 5^2 - 2 \cdot AB \cdot 5 \cdot \frac{1}{2}$$

$$64 = x^2 + 25 - 5x$$

$$0 = x^2 - 5x - 39$$

$$x = \frac{5 \pm \sqrt{25 - 4 \cdot (-39)}}{2 \cdot 1}$$

$$= \frac{5 \pm \sqrt{181}}{2} \sim \frac{4.2}{2} \quad \underline{\underline{9.2}}$$

$$AB^2 = AC^2 + BC^2 - 2 \cdot AC \cdot BC \cdot \cos(C)$$

$$9.2^2 \approx 5^2 + 8^2 - 2 \cdot 5 \cdot 8 \cdot \cos C$$

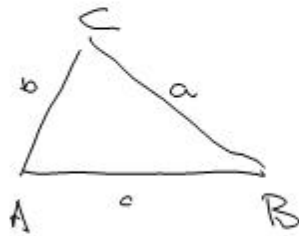
$$\frac{9.2^2 - 25 - 64}{-80} \approx \frac{-80 \cdot \cos C}{-80}$$

$$\cos(C) \approx \frac{9.2^2 - 89}{-80} \approx 0.0545$$

$$C \approx \underline{\underline{87^\circ}}$$

$$\angle B = 180^\circ - \angle A - \angle C \approx 180^\circ - 60^\circ - 87^\circ = \underline{\underline{33^\circ}}$$

② Sinussætninger



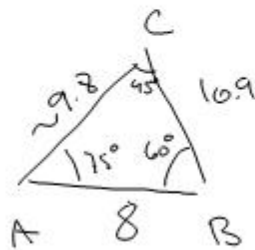
$$\frac{AB}{\sin C} = \frac{AC}{\sin B} = \frac{BC}{\sin A}$$

$$\boxed{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$$

Tre
Tre } forskellige ligninger:

$$\frac{a}{\sin A} = \frac{b}{\sin B} \quad \text{og} \quad \frac{b}{\sin B} = \frac{c}{\sin C} \quad \left(\frac{a}{\sin A} = \frac{c}{\sin C} \right)$$

Eks:



$$\angle A = 75^\circ$$

$$\angle C = 45^\circ$$

$$\angle B = 60^\circ$$

$$AB = 8$$

AC = b:

$$\frac{b}{\sin B} = \frac{c}{\sin C} \quad \frac{b}{\sin 60^\circ} = \frac{8}{\sin 45^\circ}$$

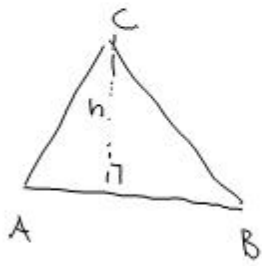
$$b = \frac{8 \cdot \sin 60^\circ}{\sin 45^\circ} = \frac{8 \cdot \frac{1}{2}\sqrt{3}}{\frac{1}{\sqrt{2}}} = 4\sqrt{6} \approx \underline{\underline{9.8}}$$

BC = a:

$$\frac{a}{\sin A} = \frac{c}{\sin C} \quad \frac{a}{\sin 75^\circ} = \frac{8}{\sin 45^\circ}$$

$$a = \frac{8 \sin(75^\circ)}{\sin(45^\circ)} \approx \underline{\underline{10.9}}$$

③ Arealsetninger

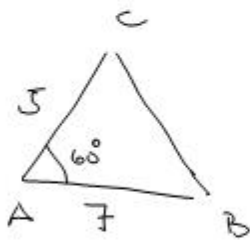


$$\text{Areal} = \frac{1}{2} \cdot AB \cdot AC \cdot \sin A$$

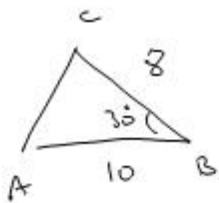
$$h = AC \cdot \sin A$$

$$\text{Areal: } \frac{1}{2} \cdot \text{grundlinje} \cdot \text{h\u00f8yde} = \frac{1}{2} \cdot AB \cdot \underbrace{AC \cdot \sin A}_h$$

Eks.

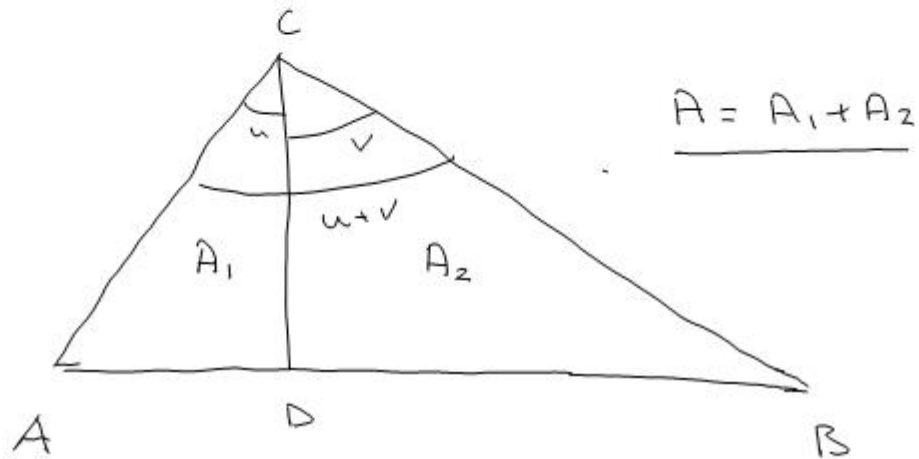


$$\begin{aligned} \text{Areal} &= \frac{1}{2} \cdot 5 \cdot 7 \cdot \sin 60^\circ \\ &= \frac{1}{2} \cdot 35 \cdot \frac{\sqrt{3}}{2} = \underline{\underline{\frac{35}{4}\sqrt{3}}} \approx 15 \end{aligned}$$



$$\begin{aligned} \text{Areal} &= \frac{1}{2} \cdot 10 \cdot 8 \cdot \sin 30^\circ \\ &= 40 \cdot \frac{1}{2} = \underline{\underline{20}} \end{aligned}$$

Eks: Formel for $\sin(u+v)$:



$$A = \frac{1}{2} \cdot AC \cdot BC \cdot \sin(u+v)$$

$$A_1 = \frac{1}{2} \cdot AC \cdot CD \cdot \sin(u)$$

$$A_2 = \frac{1}{2} \cdot BC \cdot CD \cdot \sin(v)$$

$$\frac{\frac{1}{2} AC \cdot BC \cdot \sin(u+v)}{A} = \frac{\frac{1}{2} AC \cdot CD \cdot \sin(u)}{A_1} + \frac{\frac{1}{2} BC \cdot CD \cdot \sin(v)}{A_2}$$

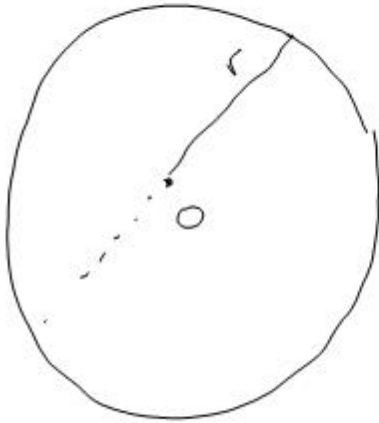
$$\sin(u+v) = \frac{AC \cdot CD \cdot \sin(u)}{AC \cdot BC} + \frac{BC \cdot CD \cdot \sin(v)}{AC \cdot BC}$$

$$= \left(\frac{CD}{BC} \right) \sin(u) + \left(\frac{CD}{AC} \right) \sin(v)$$

$$= \cos(v) \cdot \sin(u) + \cos(u) \cdot \sin(v)$$

$$\underline{\sin(u+v) = \sin(u) \cdot \cos(v) + \cos(u) \cdot \sin(v)}$$

Sirkler



Sirkel med radius r med senter i O

alle punkter med avstand r til O

Radius: r

Diameter: $d = 2r$

Areal og omkrets:

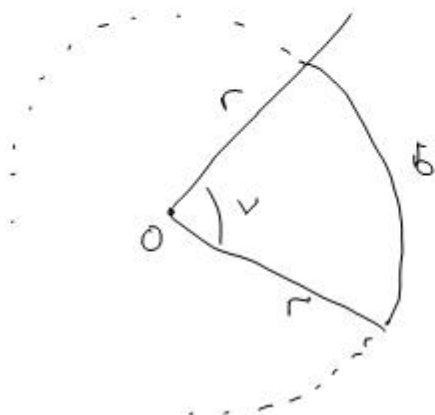
$$A = \pi r^2$$

$$O = 2\pi r = \pi \cdot d$$

Defn. av π :

$$\pi = O/d$$

Sirkelsektor:



Vinkelen v måles i grader:

$$A = \pi r^2 \cdot \frac{v}{360^\circ} \quad (\text{areal})$$

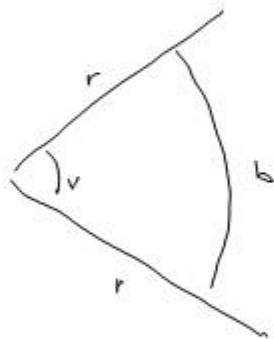
$$b = 2\pi r \cdot \frac{v}{360^\circ} \quad (\text{buelengde})$$

Vinkelen v mätes i radianer:

$$A = \pi r^2 \cdot \frac{v}{2\pi} = \cancel{\frac{1}{2}} \frac{1}{2} r^2 \cdot v$$

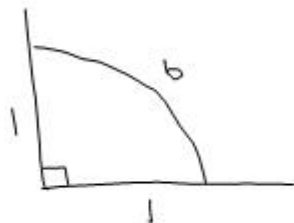
$$b = 2\pi r \cdot \frac{v}{2\pi} = r \cdot v \quad \Rightarrow v = b/r$$

Defn av vinkel i radianer



$$v = \frac{b}{r}$$

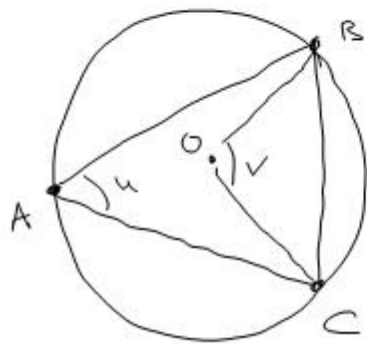
Exempel: Rett vinkel



$$\left. \begin{array}{l} b = \frac{2\pi \cdot 1}{4} = \frac{\pi}{2} \\ r = 1 \end{array} \right\} v = b/r = \frac{\pi}{2}$$

Altså: $\underline{90^\circ = \pi/2}$

Sentralvinkel og periferivinkel:

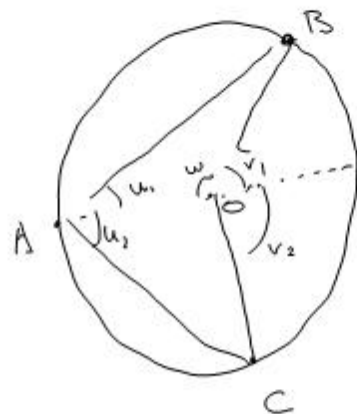


Periferivinkel: u
 Sentralvinkel: v

Resultat: $v = 2u$

Sirkel med sentr i O
 Trekant $\triangle ABC$ på
 sirkelen:

Hvorfor:



$$\left. \begin{aligned} \angle w &= 180^\circ - v_1 \\ \angle w &= 180^\circ - 2u_1 \end{aligned} \right\} \underline{v_1 = 2u_1}$$

$v_2 = 2u_2$ pga samme regning

⇓

$$v = v_1 + v_2 = 2u_1 + 2u_2 = 2(u_1 + u_2)$$

$$\underline{\underline{v = 2u}}$$

Periferivinkel: $u = u_1 + u_2$

Sentralvinkel: $v = v_1 + v_2$