NORWEGIAN
BUSINESS SCHOOL

| Multiple-choice exam: | GRA 60352 | Mathematics |
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## PLEASE READ THE FOLLOWING BEFORE YOU BEGIN!

- Students must themselves assure that the examination papers are complete.
- Students must provide the following information on the answer sheet:
- Examination code
- Personal initials
- ID-nr

The ID-nr must be recorded with both the appropriate numbers and by putting an " X " by the corresponding number in the columns below.

- Do not use pencils or pens with green ink when filling in answer sheets. Answer sheets must not be used for rough drafts.
- All answers must be recorded with an "X" under the letter you believe corresponds with the correct answer.
- Cancel an " $X$ " by filling in the box completely (boxes that are completely filled in will not be registered). " X " in two boxes for one question will be registered as a wrong answer.
- The attached example shows you how the answer sheet would be filled in if A were the correct answer for question 1, B correct for question 2, C correct for question 3 and D correct for question 4. An " X " under E indicates that you choose not to answer question 5.
- Your answers are to be recorded on the answer sheet. Answers written on the examination papers and not on the answer sheets will not be graded.
- There is only one right answer for each question. Because the questions are weighted equally, it can be to your advantage to answer the easiest questions first.
- Wrong answers are given -1 point, unanswered questions get 0 points (indicated by an "X" next to E") and correct answers are given 3 points.
- You can keep the examination papers.


## This exam has 8 questions

## Question 1.

Consider a linear system $A \cdot \mathbf{x}=\mathbf{0}$, where $A$ is a $3 \times 4$ matrix with rk $A=2$. Which statement is true?
(a) The linear system has a unique solution
(b) The linear system is inconsistent
(c) The linear system has one degree of freedom
(d) The linear system has two degrees of freedom
(e) I prefer not to answer.

## Question 2.

Consider the vectors $\mathbf{v}_{1}, \mathbf{v}_{2}, \mathbf{v}_{3}$, given by

$$
\mathbf{v}_{1}=\left(\begin{array}{l}
0 \\
h \\
1
\end{array}\right), \quad \mathbf{v}_{2}=\left(\begin{array}{c}
h-1 \\
1 \\
1
\end{array}\right), \quad \mathbf{v}_{3}=\left(\begin{array}{l}
h \\
1 \\
1
\end{array}\right)
$$

Which statement is true?
(a) The vectors $\left\{\mathbf{v}_{1}, \mathbf{v}_{2}, \mathbf{v}_{3}\right\}$ are linearly independent for all $h$
(b) The vectors $\left\{\mathbf{v}_{1}, \mathbf{v}_{2}, \mathbf{v}_{3}\right\}$ are linearly dependent for all $h$
(c) The vectors $\left\{\mathbf{v}_{1}, \mathbf{v}_{2}, \mathbf{v}_{3}\right\}$ are linearly independent exactly when $h \neq 1$
(d) The vectors $\left\{\mathbf{v}_{1}, \mathbf{v}_{2}, \mathbf{v}_{3}\right\}$ are linearly dependent exactly when $h \neq 1$
(e) I prefer not to answer.

## Question 3.

Compute the rank of the matrix

$$
A=\left(\begin{array}{cccc}
1 & 1 & -1 & -1 \\
1 & 3 & h & -1 \\
2 & 3 & 0 & h
\end{array}\right)
$$

## Which statement is true?

(a) $\operatorname{rk} A=2$ for all $h$
(b) $\operatorname{rk} A=3$ for $h \neq 3$ and $\operatorname{rk} A=2$ for $h=3$
(c) $\operatorname{rk} A=3$ for $h \neq-2$ and $\operatorname{rk} A=2$ for $h=-2$
(d) $\operatorname{rk} A=3$ for all $h$
(e) I prefer not to answer.

## Question 4.

Consider the matrix

$$
A=\left(\begin{array}{ccc}
5 & 0 & -1 \\
0 & 2 & 0 \\
4 & 0 & 0
\end{array}\right)
$$

## Which statement is true?

(a) $A$ has three positive eigenvalues
(b) $A$ has two positive and one negative eigenvalue
(c) $A$ has one positive and two negative eigenvalues
(d) $A$ has three negative eigenvalues
(e) I prefer not to answer.

Question 5.

Consider the matrix $A$ and the vector $\mathbf{v}$ given by

$$
A=\left(\begin{array}{cc}
1 & s \\
3 & -1
\end{array}\right), \quad \mathbf{v}=\binom{2}{1}
$$

Which statement is true?
(a) $\mathbf{v}$ is not an eigenvector of $A$
(b) $\mathbf{v}$ is an eigenvector of $A$ exactly when $s=5$
(c) $\mathbf{v}$ is an eigenvector of $A$ exactly when $s=8$
(d) $\mathbf{v}$ is an eigenvector of $A$
(e) I prefer not to answer.

Question 6.

Consider the quadratic form

$$
f\left(x_{1}, x_{2}, x_{3}, x_{4}\right)=x_{1}^{2}+3 x_{1} x_{4}+2 x_{2}^{2}+6 x_{2} x_{4}+3 x_{3}^{2}+7 x_{4}^{2}
$$

Which statement is true?
(a) $f$ is positive semidefinite but not positive definite
(b) $f$ is positive definite
(c) $f$ is indefinite
(d) $f$ is negative semidefinite
(e) I prefer not to answer.

## Question 7.

Consider the function $f(x, y, z)=x^{3}-3 x y^{2}-z^{4}-3 x+4 z$. Which statement is true?
(a) $f$ has a local maximum point
(b) $f$ has a local minimum point
(c) $f$ has stationary points, but all are saddle points
(d) $f$ has no stationary points
(e) I prefer not to answer.

## Question 8.

Consider the function $f$ given by

$$
f(x, y, z)=x^{2}+4 x z+3 y^{2}-2 y z+7 z^{2}+h x^{4}
$$

Which statement is true?
(a) $f$ is a convex function for all $h$
(b) $f$ is a convex function for $h \geq 0$, and concave for $h<0$
(c) $f$ is a convex function for $h \geq 0$, and neither convex nor concave for $h<0$
(d) $f$ is a concave function for all $h$
(e) I prefer not to answer.

