How to sketch a subset of $\mathbb{R}^n$

$\mathbb{R}^n$ = the set of all points $(x_1, x_2, \dots, x_n)$ in n-dimensional space

A subset of $\mathbb{R}^n$ is usually given as the set of points that satisfy one or more equalities or inequalities.

Ex: A) $\{(x,y): x^2+y^2 < 2\}$ (Problem 6.2 a))

B) $\{(x,y): xy \le 1\}$ (Problem 6.2 c))

C) $\{(x,y): x \ge 0, y \ge 0, xy \le 1\}$

First step: Compute the boundary and draw it.

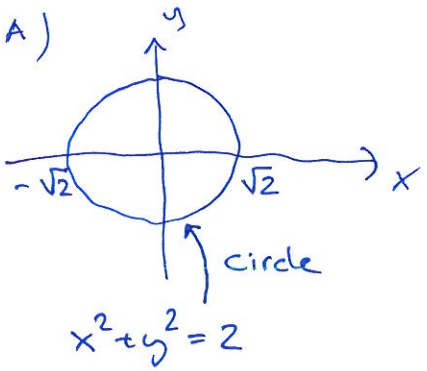
A) Boundary: $x^2+y^2 = 2$

B) Boundary: $xy = 1$

C) Boundary: $x=0$ or $y=0$ or $xy=1$

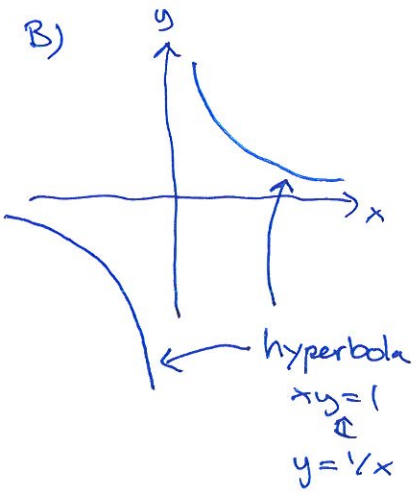
It is easier to draw a set given by an equation.

A)

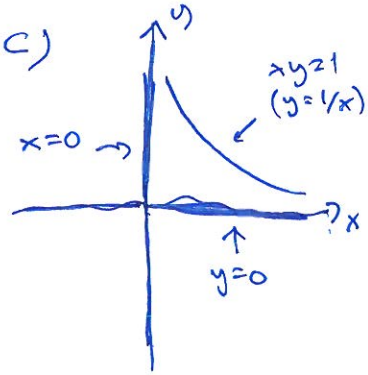


(We use that $x^2+y^2=r^2$ is a circle with radius r and center (0,0).)

B)

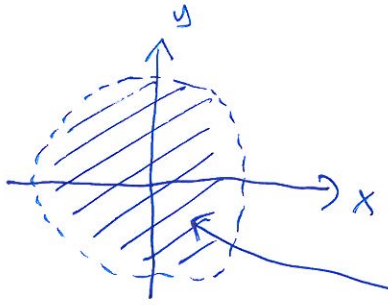


C)



Second step: We figure out which side of the boundary the set occupies, and if the boundary is included.

A)

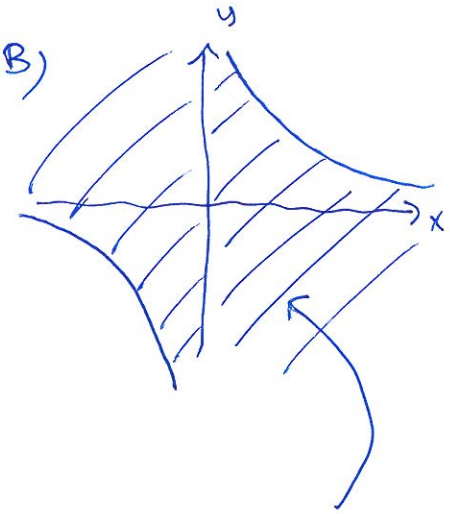


- Boundary not included since $x^2+y^2<2$
- Set is on the inside of the circle since $x^2+y^2<2$.

$\{(x,y): x^2+y^2<2\}$ is

- open, not closed
- bounded
- convex

B)

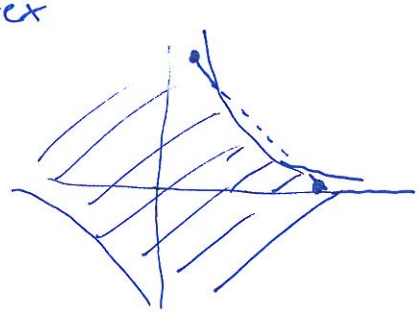


$\{(x,y): xy\leq 1\}$

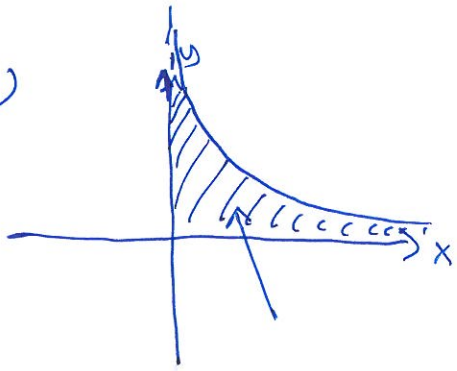
- closed, not open
- not bounded
- not convex

- Boundary included since $xy\geq 1$.
- When $x>0$, $xy \geq 1$ $\Rightarrow$ $\frac{xy}{x} \geq \frac{1}{x}$ $\Rightarrow$ $y \geq \frac{1}{x}$ — The set is under the boundary for $x>0$

- When $x<0$, $xy \leq 1$ $\Rightarrow$ $\frac{xy}{x} \geq \frac{1}{x}$ $\Rightarrow$ $y \geq 1/x$ — The set is over the boundary for $x<0$.



c)



- Boundary is included since $x \geq 0,\ y \geq 0,\ xy \leq 1$
- The set is over $y=0$ since $y \geq 0$ to the right of $x=0$ since $x \geq 0$ under $xy=1$ since $xy \leq 1$ (and $x \geq 0$)

$$\{(x,y) : x \geq 0, y \geq 0, xy \leq 1\}$$

- closed, not open
- not bounded
- not convex