Revision Problems
GRA 6035 Mathematics

BI Norwegian Business School

Revision Problems

1. Consider the optimization problem

$$\max x^2y^2z^2 \text{ subject to } x^2+y^2+z^2=3$$

a) Write down the Lagrangian $\mathscr{L}$ and the first order conditions for this problem.
b) Find all admissible points that satisfy the first order conditions. Hint: Try to find such points with $x \neq 0, y \neq 0, z \neq 0$ first, these are the most important solutions since $f = 0$ if one of the coordinates are zero.
c) Check that that point $(x,y,z) = (1,1,1)$ is an admissible point that satisfy the first order conditions, and use the bordered Hessian at this point to determine if it is a local maximum points for the constrained optimization problem.
d) Is the set of admissible points closed and bounded? Use this to solve the optimization problem.

2. Consider the optimization problem

$$\max f(x,y,z) = 2z \text{ subject to } x^2+y^2=2,\ x+y+z=1$$

a) Write down the Lagrangian $\mathscr{L}$ and the first order conditions for this problem.
b) Solve the optimization problem. What is the maximum value?
c) Write down the NDCQ for this problem. It NDCQ satisfied for all admissible points (x,y,z)? It is necessary to check NDCQ to solve this optimization problem?
d) Change the last constraint to $x+y+z=b$. Show that the problem has a solution, a maximal value, for each value of b. How does this maximum value change if you increase b?

3. Consider the Kuhn-Tucker optimization problem

$$\max f(x,y,z) = 2z \text{ subject to } x^2+y^2 \leq 2,\ x+y+z \leq 1$$

a) Write down the Lagrangian $\mathscr{L}$ and the first order conditions for this problem. Also, write down the complementary slackness conditions.
b) Solve the optimization problem. What is the maximum value?
c) Write down the NDCQ for this problem. It NDCQ satisfied for all admissible points (x,y,z)? It is necessary to check NDCQ to solve this optimization problem?