EXTRA LECTURE 4

GRA 6035

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PLAN:

① REVIEW MATRICES - CHECKING EIGENVECTORS
② DIFFERENTIAL EQUATIONS

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① Checking eigenvectors

When a matrix and a vector is given, it is <u>much</u> easier to check if the vector is an eigenvector directly!

Look at: $\boxed{A\underline{v} = \lambda \underline{v}}$ ($A, \underline{v}$ given)

Exe:

i) Is $\begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix}$ an eigenvector for $\begin{pmatrix} 3 & -4 & 8 \\ -2 & 1 & 4 \\ 2 & -2 & -5 \end{pmatrix}$?

What is the eigenvalue in that case?

ii) Is $\begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$ an eigenvector for $\begin{pmatrix} 1 & t & -2 \\ 2 & 4 & -t \\ -t & -4 & -4 \end{pmatrix}$ for any value of t?

If so, what is the corresponding eigenvalue?

Solution:

i) $$A\underline{v} = \begin{pmatrix} 3 & -4 & 8 \\ -2 & 1 & 4 \\ 2 & -2 & -5 \end{pmatrix} \cdot \begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$$

$$\lambda \underline{v} = \lambda \cdot \begin{pmatrix} -2 \\ -2 \\ 0 \end{pmatrix}$$

Equality hold is $\underline{\lambda = -1}$.

Yes, $\underline{v}$ is eigenvecter with eigenvalue $\lambda \underline{= -1}$

ii) $$A\underline{v} = \begin{pmatrix} 1 & t & -2 \\ 2 & 4 & -t \\ -t & -4 & -4 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} -1+2t+4 \\ -2+8+2t \\ t-8+8 \end{pmatrix} = \begin{pmatrix} 2t+3 \\ 2t+6 \\ t \end{pmatrix}$$

$$\lambda \underline{v} = \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} -\lambda \\ 2\lambda \\ -2\lambda \end{pmatrix}$$

Equality holds if $\begin{cases} 2t+3 = -\lambda \\ 2t+6 = 2\lambda \\ t = -2\lambda \end{cases}$ $\Rightarrow t = -2\lambda$ $\Rightarrow 2(-2\lambda)+3 = -\lambda$

$-4\lambda + 3 = -\lambda$

$3 = -3\lambda$

$\underline{\lambda = +1}$

$\underline{t = -2}$

Yes, $\underline{v}$ is eigenvecter for $t=-2$, and the eigenvalue is $\lambda = 1$. ($\underline{v}$ is not eigenvecter for $t \neq -2$).

How to solve differential equations

Use variable name $y = y(t)$.

$\Rightarrow$ We should find $y = y(t)$.

i) Find the general solution $y = y(t)$. It will in general be a function with 1 or 2 undetermined constants (first order – one
second order – two)

ii) Determine constants using initial conditions, if such conditions are present.

Methods: Find general solution

a) For linear equations: with constant coefficients

$$\begin{cases} \ddot{y} + a\dot{y} + by = f(t) \\ \dot{y} + ay = f(t) \end{cases}$$

– How do we know if the differential equation is linear?

– How to find the general solution?

$$\begin{cases} y = y_h + y_p \\ y_h \text{ is given by characteristic roots} \\ y_p \text{ is a special solution} \end{cases}$$

Ex:

$$\ddot{y} - 3\dot{y} + 2y = 4 \Rightarrow y = y_h + y_p$$

$r^2 - 3r + 2 = 0 \Rightarrow r = 1, 2$

$y_h = C_1 e^{1 \cdot t} + C_2 e^{2 \cdot t}$

$y_p = 2$

b) Separable differential equations

How do we recognize separable diff-eqn.'s ?

$$\boxed{\begin{gathered} f(y)\cdot y' = g(t) \\ \Updownarrow \\ f(y)\,dy = g(t)\,dt \end{gathered}}$$

How to solve separable diff-eqn.'s :

Compute the integrals

$$\int f(y)\,dy = \int g(t)\,dt$$

Ex: $\dot{y} = yt$

$$\frac{1}{y}\cdot\dot{y} = t \iff \frac{1}{y}\,dy = t\,dt$$

$$\int \frac{1}{y}\,dy = \int t\,dt$$

$$\ln|y| = \frac{1}{2}t^2 + C$$

$$|y| = e^{\frac{1}{2}t^2 + C} = e^{\frac{1}{2}t^2}\cdot e^C$$

$$y = \underline{\underline{K e^{\frac{1}{2}t^2}}} \quad \left(K = \pm e^C\right)$$

c) Linear first order diff. eqn.'s with non-constant coefficient.

How to recognize this kind of diff. eqn. ?

$$\boxed{\dot{y} + a(t) \cdot y = f(t)}$$

How to solve it: Integrating factor

$$u(t) = e^{\int a(t)\, dt}$$

$$\dot{y} + a(t)\, y = f(t)$$

$$\underbrace{u \cdot (\dot{y} + a(t) \cdot y)} = u \cdot f(t)$$

$$(u \cdot y)' = u \cdot f(t)$$

$$u \cdot y = \int u \cdot f(t)\, dt$$

$$y = \frac{1}{u} \cdot \int u(t) \cdot f(t)\, dt$$

Ex: $\dot{y} + 3t^2 \cdot y = t^2$

$$\boxed{u = e^{\int 3t^2 dt} = e^{t^3}}$$

$$e^{t^3} \cdot (\dot{y} + 3t^2 y) = e^{t^3} \cdot t^2$$

$$(e^{t^3} \cdot y)' = t^2 \cdot e^{t^3}$$

$$e^{t^3} \cdot y = \int t^2 e^{t^3}\, dt$$

$$y = \frac{1}{e^{t^3}} \cdot \int t^2 e^{t^3}\, dt$$

$$= \frac{1}{e^{t^3}} \cdot \left(\frac{1}{3} e^{t^3} + C \right) = \frac{1}{3} + \frac{C}{e^{t^3}}$$

d) Exact differential equations:

How to recognize exact differential eqn's.?

$$p(y,t) + q(y,t)\cdot \dot{y} = 0$$

such that

$$\boxed{\frac{\partial p(y,t)}{\partial y} = \frac{\partial q(y,t)}{\partial t}}$$

How to solve exact diff. eqn.'s:

Find a function $u(y,t)$ such that

$$\boxed{\frac{\partial u}{\partial t} = p \,, \quad \frac{\partial u}{\partial y} = q}$$

The general solution is

$$\boxed{u(y,t) = C}$$

Ex: $\underbrace{1-2y}_{p} + \underbrace{(2y-2t)}_{q}\cdot\dot{y} = 0$

Check if exact:

$$\left.\begin{array}{l} \dfrac{\partial p}{\partial y} = -2 \\[2ex] \dfrac{\partial q}{\partial t} = -2 \end{array}\right\} \text{ ok, exact.}$$

Find $u = u(y,t)$ such that

① $\frac{\partial u}{\partial t} = 1-2y$

② $\frac{\partial u}{\partial y} = 2y-2t$

Start with ①:

$$\frac{\partial u}{\partial t} = 1-2y \Rightarrow u = \int (1-2y)\,dt$$

$$u = t - 2yt + C(y)$$

Then ②:

$$\frac{\partial u}{\partial y} = 2y-2t \Rightarrow \cancel{-2t} + C'(y) = 2y \cancel{-2t}$$

$$C(y) = y^2$$

$u = t - 2yt + C(y)$

$u = \underline{t - 2yt + y^2}$

General solutcón:

$u = C$

$$\boxed{t - 2yt + y^2 = C}$$

$y^2 - 2t \cdot y + (t - C) = 0$

$$y = \frac{2t \pm \sqrt{(2t)^2 - 4 \cdot 1 \cdot (t - C)}}{2 \cdot 1}$$

$$y = t \pm \frac{1}{2}\sqrt{4t^2 - 4(t - C)}$$

② Differential equations:

Exc:

i) $\dot{y} = y - t^2$

ii) $\ddot{y} = 3\dot{y} - 2y + t^2$

iii) $\dot{y} = \dfrac{y^3}{t^3}$

iv) $\dot{y} = \dfrac{t^3}{y^3}$

v) $\dot{y} = y^2 (t^2 + t)$.

vi) $\dot{y} = e^{y+t}$

vii) $\dot{y} = \dfrac{y+1}{t} = \dfrac{1}{t} \cdot y + \dfrac{1}{t}$

viii) $\dot{y} = y^2 + 1$

ix) $\dot{y} = t^2 y$

x) $t\dot{y} + (1-t)y = e^{2t}$

xi) $\ddot{y} - 2\dot{y} - 3y = 9t^2$

xii) $2\ddot{y} + 3\dot{y} - 2y = 0$

xiii) $\ddot{y} + 6\dot{y} + 9y = 0$

xiv) $\ddot{y} + \dot{y} - 2y = 6t$

xv) $\ddot{y} - y = e^t$

xvi) $2yt^3\dot{y} + 3y^2t^2 - 1 = 0$

xvii) $3y^2t\,\dot{y} + y^3 - 3t^2 = 0$

xviii) $\dfrac{1}{y} \cdot \dot{y} + \dfrac{1}{t} = 1$

xix) $\dfrac{1}{y+t} \cdot \dot{y} + \dfrac{1}{y+t} = 2t$

xx) $(t+1)\dot{y} + (y-1) = 0$

Find the general solution $y = y(t)$ in i) – xx).

Solution:

i) $\dot{y} = y - t^2$

$\dot{y} - y = -t^2$ linear (also possible to use integrating factor)

$y = y_h + y_p$

$= \underline{\underline{Ce^t + t^2 + 2t + 2}}$

$\underline{y_h}$: $r - 1 = 0$

$r = 1 \Rightarrow y_h = Ce^{1 \cdot t} = \underline{Ce^t}$

$\underline{y_p}$:

$$\left.\begin{aligned} y &= At^2 + Bt + C \\ \dot{y} &= 2At + B \\ \ddot{y} &= 2A \end{aligned}\right\} \quad (2At + B) - (At^2 + Bt + C) = -t^2$$

$$\left.\begin{aligned} -At^2 &= -t^2 \\ 2At - Bt &= 0 \\ B - C &= 0 \end{aligned}\right\} \quad \begin{aligned} A &= 1 \\ B &= 2 \\ C &= 2 \end{aligned}$$

$y_p = \underline{t^2 + 2t + 2}$

Hint: How to guess y_p.

Ex: $\dot{y} - y = -t^2$ Start with $f(t) = -t^2$, Compute $f'(t) = -2t$, $f''(t) = -2$ $\Big\}$ Choose: $\underline{At^2 + Bt + C}$

$\dot{y} - y = e^{-t}$ — " — $f(t) = e^{-t}$, $f'(t) = -e^{-t}$, $f''(t) = e^{-t}$ $\Big\}$ Choose: $\underline{Ae^{-t}}$

$\dot{y} - y = te^t$ — " — $f(t) = te^t$, $f'(t) = 1 \cdot e^t + te^t = e^t + te^t$, $f''(t) = e^t + 1 \cdot e^t + t \cdot e^t = 2e^t + te^t$ $\Big\}$ Choose: $Ate^t + Be^t = \underline{(At+B)e^t}$

Hint: How to find solution that satisfy initial condition:

$$\dot{y} - y = -t^2, \quad y(1) = 1$$

a) General solution: $y = Ce^t + t^2 + 2t + 2$ (see above)

b) Substitute $y(1) = 1$: $1 = C \cdot e^1 + 1^2 + 2 \cdot 1 + 2$
$\Updownarrow$
$t = 1, y = 1$

$1 = Ce + 5$

$Ce = 1 - 5 = -4$

$C = \underline{-4/e}$ $\Rightarrow$ $\underline{y = -\frac{4}{e}e^t + t^2 + 2t + 2}$

ii) $\ddot{y} = 3\dot{y} - 2y + t^2$

$\ddot{y} - 3\dot{y} + 2y = t^2$ linear

$y = y_h + y_p$

$$y = \underline{\underline{C_1 e^{t} + C_2 e^{2t} + \tfrac{1}{2}t^2 + \tfrac{3}{2}t + \tfrac{7}{4}}}$$

y_h: $r^2 - 3r + 2 = 0$

$r = 1, 2 \Rightarrow y_h = \underline{C_1 e^{t} + C_2 e^{2t}}$

y_p: $y = At^2 + Bt + C$

$\dot{y} = 2At + B$

$\ddot{y} = 2A$

$(2A) - 3(2At + B) + 2(At^2 + Bt + C) = t^2$

$2At^2 = t^2 \Rightarrow A = \underline{1/2}$

$-6At + 2Bt = 0 \Rightarrow B = \underline{3/2}$

$2A - 3B + 2C = 0 \Rightarrow C = \frac{3B - 2A}{2} = \frac{9}{4} - \frac{2}{4}$

$= \underline{7/4}$

$$y_p = \underline{\tfrac{1}{2}t^2 + \tfrac{3}{2}t + \tfrac{7}{4}}$$

iii) $\dot{y} = \dfrac{y^3}{t^3}$

$\dfrac{1}{y^3} \cdot \dot{y} = \dfrac{1}{t^3}$ separable

$$\int \frac{1}{y^3}\,dy = \int \frac{1}{t^3}\,dt$$

$-\frac{1}{2} \cdot y^{-2} = -\frac{1}{2}t^{-2} + C$

$y^{-2} = t^{-2} - 2C$

$\dfrac{1}{y^2} = \dfrac{1}{t^2} - 2C$

$$y^2 = \frac{1}{\frac{1}{t^2} - 2c} = \underline{\underline{\frac{t^2}{1 - 2ct^2}}}$$

iv) $\dot{y} = \frac{t^3}{y^3}$

$y^3 \cdot \dot{y} = t^3$ (seperable)

$\int y^3 dy = \int t^3 dt$

$\frac{1}{4} y^4 = \frac{1}{4} t^4 + C$

$y^4 = t^4 + 4c \Rightarrow \underline{y = \pm\sqrt[4]{t^4 + 4C}}$

v) $\dot{y} = y^2 (t^2 + t)$

$\frac{1}{y^2} \cdot \dot{y} = t^2 + t$ (seperable)

$\int \frac{1}{y^2} dy = \int t^2 + t \, dt$

$-\frac{1}{y} = \frac{1}{3} t^3 + \frac{1}{2} t^2 + C$

$y = -\underline{\frac{1}{\frac{1}{3} t^3 + \frac{1}{2} t^2 + C}}$

vi) $\dot{y} = e^{y+t}$

$\dot{y} = e^y \cdot e^t$

$e^{-y} \cdot \dot{y} = e^t$ (separable)

$\int e^{-y} dy = \int e^t dt$

$-e^{-y} = e^t + C$

$e^{-y} = -e^t - C$

$-y = \ln|-e^t - c|$

$y = -\ln \underline{|-e^t - c|}$

vii) $\dot{y} = \frac{y+1}{t}$

$\frac{1}{y+1} \cdot \dot{y} = \frac{1}{t}$ (separable) — Also possible to use integrating factor (linear)

$\int \frac{1}{y+1}\, dy = \int \frac{1}{t}\, dt$

$\ln|y+1| = \ln|t| + C$

$|y+1| = e^{\ln|t| + C} = e^{C} \cdot |t|$

$y+1 = \pm e^{C} \cdot t = kt$

$\underline{\underline{y = kt - 1}}$

viii) $\dot{y} = y^2 + 1$

$\frac{1}{y^2+1} \dot{y} = 1$ (separable)

Difficult integral, <u>not</u> basic problem

$\int \frac{1}{y^2+1}\, dy = \int 1\, dt$

$\arctan(y) = t + C$

$y = \underline{\underline{\tan(t+C)}}$

ix) $\dot{y} = t^2 \cdot y$

$\frac{1}{y} \cdot \dot{y} = t^2$ (separable) — Also possible to use int. factor (linear)

$\int \frac{1}{y}\, dy = \int t^2\, dt$

$\ln|y| = \frac{1}{3}t^3 + C$

$|y| = e^{\frac{1}{3}t^3 + C} = e^{\frac{1}{3}t^3} \cdot e^{C}$

$y = \pm e^{C} \cdot e^{\frac{1}{3}t^3} = \underline{k e^{\frac{1}{3}t^3}}$

x) $t\dot{y} + (1-t)y = e^{2t}$

$\dot{y} + \frac{1-t}{t}\cdot y = \frac{e^{2t}}{t}$

linear, first order, non-const. coeff

$\Rightarrow$ int. factor

$u = e^{\int \frac{1-t}{t} dt}$

$\frac{t}{e^t}\dot{y} + \frac{t}{e^t}\cdot\frac{1-t}{t}y = \frac{\cancel{t}}{e^t}\cdot\frac{e^{2t}}{\cancel{t}}$

$\int \frac{1-t}{t} dt = \int \frac{1}{t} - 1 \, dt = \ln|t| - t$

$\left(\frac{t}{e^t}\cdot y\right)' = e^t$

$u = e^{\ln|t| - t} = te^{-t} = \frac{t}{e^t}$

$\frac{t}{e^t}\cdot y = \int e^t dt = e^t + C$

$y = (e^t + C)\cdot\frac{e^t}{t} = \frac{e^{2t} + Ce^t}{t}$

xi) $\ddot{y} - 2\dot{y} - 3y = 9t^2$ linear

$y = y_h + y_p$

$= C_1e^{3t} + C_2e^{-t} - 3t^2 + 4t - 14/3$

y_h: $r^2 - 2r - 3 = 0$

$r = 3, -1$

$y_h = C_1e^{3t} + C_2e^{-t}$

y_p: $y = At^2 + Bt + C$

$\dot{y} = 2At + B$

$\ddot{y} = 2A$

$(2A) - 2(2At + B) - 3(At^2 + Bt + C) = 9t^2$

$-3At^2 = 9t^2 \Rightarrow A = -3$

$-4At - 3Bt = 0$ $\quad B = 4$

$2A - 2B - 3C = 0$ $\quad C = \frac{2A - 2B}{3}$

$= -14/3$

$y_p = -3t^2 + 4t - 14/3$

xii) $2\ddot{y} + 3\dot{y} - 2y = 0$

$\ddot{y} + \frac{3}{2}\dot{y} - y = 0$ (linear)

$r^2 + \frac{3}{2}r - 1 = 0$

$r = \dfrac{-3/2 \pm \sqrt{9/4 - 4(-1)}}{2}$

$= -\frac{3}{4} \pm \frac{1}{2}\sqrt{25/4} = -\frac{3}{4} \pm \frac{1}{2}\cdot\frac{5}{2}$

$= -\frac{3}{4} \pm \frac{5}{4} = \frac{1}{2}, -2$

$y = y_h = C_1 e^{\frac{1}{2}t} + C_2 e^{-2t}$

xiii) $\ddot{y} + 6\dot{y} + 9y = 0$ (linear)

$r^2 + 6r + 9 = 0$

$r = -3$ (double root)

$y = y_h = C_1 e^{-3t} + C_2 t e^{-3t}$

$= (C_1 + C_2 t)\, e^{-3t}$

xiv) $\ddot{y} + \dot{y} - 2y = 6t$ (linear)

$y = y_h + y_p$

$= C_1 e^{t} + C_2 e^{-2t} - 3t - 3/2$

y_h: $r^2 + r - 2 = 0$

$r = \dfrac{-1 \pm \sqrt{1 - 4(-2)}}{2} = \dfrac{-1 \pm 3}{2}$

$= 1, -2$

$y_h = C_1 e^{t} + C_2 e^{-2t}$

y_p: $y = At + B$

$\dot{y} = A$

$\ddot{y} = 0$

$0 + (A) - 2(At + B) = 6t$

~~At+B~~

$-2At = 6t \qquad A = -3$

$A - 2B = 0 \qquad B = -3/2$

$y_p = -3t - 3/2$

xv) $\ddot{y} - y = e^{t}$ (linear)

$y = y_h + y_p$

y_h: $r^2 - 1 = 0$

$r = \pm 1 \qquad y_h = C_1 e^{t} + C_2 e^{-t}$

y_p: $y = Ae^{t}$, $\dot{y} = Ae^{t}$, $\ddot{y} = Ae^{t}$

$\ddot{y} - y = e^{t}$

$0 = e^{t}$

no solution

try $y_p = Ate^{t}$

Hint: If y_p is chosen well but still doesn't work, try to multiply with t!

$y_p = Ae^t$ doesn't work

$\Downarrow$

try $y_p = Ate^t$

$y = Ate^t$

$\dot{y} = A \cdot 1e^t + At \cdot e^t = Ae^t + Ate^t$

$\ddot{y} = Ae^t + Ate^t + Ae^t$

$= Ate^t + 2Ae^t$

$\ddot{y} - y = e^t$

$(\cancel{Ate^t} + 2Ae^t) - \cancel{Ate^t} = e^t$

$2Ae^t = e^t$

$A = 1/2$

$y_p = \underline{\frac{1}{2}te^t}$

Solution:

$y = y_h + y_p$

$= \underline{C_1e^t + C_2e^{-t} + \frac{1}{2}te^t}$

xvi) $2yt^3 \cdot \dot{y} + 3y^2t^2 - 1 = 0$

$\underbrace{(3y^2t^2 - 1)}_{\frac{\partial u}{\partial t}} + \underbrace{(2yt^3)}_{\frac{\partial u}{\partial y}} \dot{y} = 0$ (exact?)

$\frac{\partial u}{\partial t} = 3y^2t^2 - 1 \Rightarrow u = \int 3y^2t^2 - 1 \, dt = y^2t^3 - t + C(y)$

$\frac{\partial u}{\partial y} = (y^2t^3 - t + C(y))'_y = \underline{2yt^3 + C'(y) = 2yt^3}$

$C'(y) = 0 \Rightarrow C(y) = 0$

Yes, it is ~~exact~~. Solution is:

$u = y^2t^3 - t = c$

$y^2t^3 = c + t$

$y^2 = \frac{c+t}{t^3} \Rightarrow \underline{y = \pm\sqrt{\frac{c+t}{t^3}}}$

xvii) $3y^2 t \dot{y} + y^3 - 3t^2 = 0$

$$\underbrace{(y^3 - 3t^2)}_{= \frac{\partial u}{\partial t}} + \underbrace{(3y^2 t)}_{= \frac{\partial u}{\partial y}} \cdot \dot{y} = 0$$ (exact?)

$$\frac{\partial u}{\partial t} = y^3 - 3t^2 \Rightarrow u = \int y^3 - 3t^2 \, dt = y^3 t - t^3 + C(y)$$

$$\frac{\partial u}{\partial y} = \left(y^3 t - t^3 + C(y) \right)'_y = 3y^2 t + C'(y) = 3y^2 t$$

$$C'(y) = 0 \Rightarrow C(y) = 0$$

Solution:

$$u = y^3 t - t^3 = C$$

$$y^3 t = C + t^3$$

$$y^3 = \frac{C + t^3}{t} \Rightarrow y = \sqrt[3]{\frac{C + t^3}{t}}$$

xviii) $\frac{1}{y} \cdot y' + \frac{1}{t} = 1 \quad | \cdot yt$

$ty' + y = yt$ (seperable) Also possible with int. factor (linear)

$$t\dot{y} = yt - y = y(t-1)$$

$$\dot{y} = \frac{y \cdot (t-1)}{t}$$

$$\frac{1}{y} \cdot y' = \frac{t-1}{t} = 1 - \frac{1}{t}$$

$$\int \frac{1}{y} \, dy = \int 1 - \frac{1}{t} \, dt$$

$$\ln|y| = t - \ln|t| + C$$

$$|y| = e^{t - \ln|t| + C} = \frac{e^t}{|t|} \cdot e^C \Rightarrow y = \pm e^C \cdot \frac{e^t}{t} = K \cdot \frac{e^t}{t}$$

xix) $\frac{1}{y+t} \cdot \dot{y} + \frac{1}{y+t} = 2t \quad | \cdot (y+t)$

$\dot{y} + 1 = 2t \cdot (y+t)$ linear, non-const. coeff. $\Rightarrow$ int. factor

$\dot{y} - 2t \cdot y = 2t^2 - 1$ $\quad u = e^{\int -2t\,dt} = e^{-t^2}$

$e^{-t^2} \cdot (\dot{y} - 2ty) = e^{-t^2} \cdot (2t^2 - 1)$

$\left(e^{-t^2} \cdot y\right)' = \left(2t^2 - 1\right) e^{-t^2}$

$e^{-t^2} \cdot y = \int (2t^2 - 1) e^{-t^2}\,dt$

Difficult integral not basic problem

try exact instead!

$\frac{1}{y+t} \cdot \dot{y} + \frac{1}{y+t} = 2t$

$$\underbrace{\left(\frac{1}{y+t} - 2t\right)}_{= \frac{\partial u}{\partial t}} + \underbrace{\left(\frac{1}{y+t}\right)}_{= \frac{\partial u}{\partial y}} \cdot \dot{y} = 0$$

$\frac{\partial u}{\partial t} = \frac{1}{y+t} - 2t \Rightarrow u = \int \frac{1}{y+t} - 2t\,dt = \ln|y+t| - t^2 + C(y)$

$\frac{\partial u}{\partial y} = \left(\ln|y+t| - t^2 + C(y)\right)'_y = \underline{\frac{1}{y+t} + C'(y) = \frac{1}{y+t}}$

$C'(y) = 0 \Rightarrow C(y) = 0$

<u>Solution:</u>

$u = \ln|y+t| - t^2 = c$

$\ln|y+t| = c + t^2$

$|y+t| = e^{c+t^2} = e^c \cdot e^{t^2}$

$y + t = \pm e^c e^{t^2} = Ke^{t^2} \Rightarrow \underline{y = Ke^{t^2} - t}$

Can be solved as:

- Separable
- linear (non-const. coeff.)
- exact

We try separable.

xx) $(t+1)\dot{y} + (y-1) = 0$

$$(t+1)\dot{y} = -(y-1)$$

$$\dot{y} = \frac{1-y}{t+1}$$

$$\frac{1}{1-y}\cdot\dot{y} = \frac{1}{t+1}$$

$$\int \frac{1}{1-y}\,dy = \int \frac{1}{t+1}\,dt$$

$$-\ln|1-y| = \ln|t+1| + C$$

$$\ln|1-y| = -\ln|t+1| - C$$

$$|1-y| = e^{-\ln|t+1| - C} = \frac{1}{|t+1|}e^{-C}$$

$$1-y = \pm e^{C}\cdot\frac{1}{t+1} = k\cdot\frac{1}{t+1}$$

$$y = 1 - k\cdot\frac{1}{t+1} = \frac{t+1-k}{t+1}$$