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Review:

(1) Linear systems:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \vdots \qquad\qquad\qquad\qquad \vdots \qquad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases}$$

can be written as $\boxed{A \cdot \underline{x} = \underline{b}}$ with

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \qquad \underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \qquad \underline{b} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}$$

coefficient matrix

Solve the system:
(Gauss elimination)

$$(A \mid \underline{b}) \rightsquigarrow (* \mid *)$$

augmented matrix — row operations — echelon form

If $m=n$:
(square A)

$\det(A) \neq 0 \rightsquigarrow$ one solution $\underline{x} = A^{-1} \cdot \underline{b}$

$\det(A) = 0 \rightsquigarrow \begin{cases} \text{no solutions} \\ \quad \text{or} \\ \text{infinitely many sol's} \end{cases}$

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(2) Linear independence:

$$\underline{a}_1 = \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix}, \quad \underline{a}_2 = \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix}, \ldots, \quad \underline{a}_n = \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix}$$

n vectors in $\mathbb{R}^m$ (m-vectors)

$\text{span}\{\underline{a}_1, \underline{a}_2, \ldots, \underline{a}_n\}$ = all linear combinations of $\underline{a}_1, \underline{a}_2, \ldots, \underline{a}_n$

$$= \{x_1\underline{a}_1 + x_2\underline{a}_2 + \ldots + x_n\underline{a}_n : \underbrace{x_1, x_2, \ldots, x_n \in \mathbb{R}}_{\text{numbers}}\}$$

Ex: $\underline{a}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $\underline{a}_2 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $\underline{a}_3 = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$

In this case, $\boxed{1 \cdot \underline{a}_1 - 2\underline{a}_2 + 1 \cdot \underline{a}_3 = \underline{0}}$

$\Updownarrow$

$\underline{a}_3 = -\underline{a}_1 + 2\underline{a}_2$

so $\text{span}\{\underline{a}_1, \underline{a}_2 \underline{a}_3\} = \text{span}\{\underline{a}_1, \underline{a}_2\}$

Defn:

$\{\underline{a}_1, \underline{a}_2, \ldots, \underline{a}_n\}$ linearly independent:

no vector can be expressed as a linear combination of the others

$\Updownarrow$

$x_1\underline{a}_1 + x_2\underline{a}_2 + \ldots + x_n\underline{a}_n = \underline{0}$ has only the solution $x_1 = x_2 = \ldots = x_n = 0$ (trivial solution)

$\{\underline{a}_1, \underline{a}_2, \ldots, \underline{a}_n\}$ linearly dependent:

at least one vector is linear combination of the others

$\Updownarrow$

$x_1\underline{a}_1 + x_2\underline{a}_2 + \ldots + x_n\underline{a}_n = \underline{0}$ has non-trivial solutions