NORWEGIAN SCHOOL

## PLEASE READ THE FOLLOWING BEFORE YOU BEGIN!

- Students must themselves assure that the examination papers are complete.
- Students must provide the following information on the answer sheet:
- Examination code
- Personal initials
- ID number

The student registration number must be recorded with both the appropriate numbers and by putting an " X " by the corresponding number in the columns below.

- Pens with green ink and pencils cannot be used in filling in answer sheets. Answer sheets must not be used for writing rough drafts.
- All answers must be recorded with an "X" under the letter you believe corresponds with the correct answer.
- Cancel an "X" by filling in the box completely (boxes that are completely filled in will not be registered). " $X$ " in two boxes for one question will be registered as a wrong answer.
- The attached example shows you how the answer sheet would be filled in if A were the correct answer for question 1, B correct for question 2, C correct for question 3 and D correct for question 4. An " X " under E indicates that you choose not to answer question 5.
- Your answers are to be recorded on the answer sheet. Answers written on the examination papers and not on the answer sheets will not be graded.
- There is only one right answer for each question. Because the questions are weighted equally, it can be to your advantage to answer the simplest questions first.
- Wrong answers are given -1 point, unanswered questions get 0 points (indicated by an "X" next to E") and correct answers are given 3 points.
- You can keep the examination papers.


## Good luck!

## This exam has 8 questions

Question 1.

Consider the linear system

$$
\left(\begin{array}{ccccc}
1 & 2 & -3 & -1 & 0 \\
0 & 1 & 7 & 3 & -4 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0
\end{array}\right) \cdot\left(\begin{array}{l}
x_{1} \\
x_{2} \\
x_{3} \\
x_{4} \\
x_{5}
\end{array}\right)=\left(\begin{array}{l}
3 \\
0 \\
2 \\
1
\end{array}\right)
$$

## Which statement is true?

(A) The linear system is inconsistent
(B) The linear system has a unique solution
(C) The linear system has one degree of freedom
(D) The linear system has two degrees of freedom
(E) I prefer not to answer.

## Question 2.

Consider the set of vectors $\mathcal{B}=\left\{\mathbf{v}_{1}, \mathbf{v}_{2}, \mathbf{v}_{3}\right\}$, where

$$
\mathbf{v}_{1}=\left(\begin{array}{c}
2 \\
3 \\
-1
\end{array}\right), \quad \mathbf{v}_{2}=\left(\begin{array}{l}
1 \\
2 \\
1
\end{array}\right), \quad \mathbf{v}_{3}=\left(\begin{array}{c}
h+1 \\
h \\
h-2
\end{array}\right)
$$

and $h$ is a parameter. Which statement is true?
(A) $\mathcal{B}$ is a linearly independent set of vectors for all $h$
(B) $\mathcal{B}$ is a linearly independent set of vectors exactly when $h=0$
(C) $\mathcal{B}$ is a linearly independent set of vectors exactly when $h \neq 5$
(D) $\mathcal{B}$ is a linearly independent set of vectors exactly when $h \neq-1$
(E) I prefer not to answer.

## Question 3.

Compute the rank of the matrix

$$
A=\left(\begin{array}{cccc}
2 & 10 & 6 & 8 \\
1 & 5 & 4 & 11 \\
3 & 15 & 7 & -2
\end{array}\right)
$$

Which statement is true?
(A) $\operatorname{rk} A=1$
(B) $\operatorname{rk} A=2$
(C) $\operatorname{rk} A=3$
(D) $\operatorname{rk} A=4$
(E) I prefer not to answer.

## Question 4.

Consider the matrix

$$
A=\left(\begin{array}{cc}
3 & 1 \\
-2 & 6
\end{array}\right)
$$

Which statement is true?
(A) $A$ has eigenvalues $\lambda=2$ and $\lambda=-1$
(B) $A$ has eigenvalues $\lambda=2$ and $\lambda=7$
(C) $A$ has a single eigenvalue $\lambda=3$
(D) $A$ has eigenvalues $\lambda=4$ and $\lambda=5$
(E) I prefer not to answer.

Question 5.

Consider the matrix $A$ and the vector $\mathbf{v}$ given by

$$
A=\left(\begin{array}{cc}
2 & 1 \\
-1 & 3
\end{array}\right), \quad \mathbf{v}=\binom{1}{b}
$$

Which statement is true?
(A) The vector $\mathbf{v}$ is not an eigenvector for $A$
(B) The vector $\mathbf{v}$ is an eigenvector for $A$ when $b=-1$
(C) The vector $\mathbf{v}$ is an eigenvector for $A$ when $b \geq 0$
(D) The vector $\mathbf{v}$ is an eigenvector for $A$ for all $b$
(E) I prefer not to answer.

## Question 6.

Consider the quadratic form

$$
Q\left(x_{1}, x_{2}\right)=-2 x_{1}^{2}+12 x_{1} x_{2}+2 x_{2}^{2}
$$

## Which statement is true?

(A) $Q$ is positive semidefinite but not positive definite
(B) $Q$ is negative semidefinite but not negative definite
(C) $Q$ is indefinite
(D) $Q$ is positive definite
(E) I prefer not to answer.

Question 7.

Consider the function

$$
f\left(x_{1}, x_{2}, x_{3}\right)=-x_{1}^{2}+2 x_{1} x_{2}-3 x_{2}^{2}-x_{3}^{2}-x_{1}-x_{3}
$$

defined on $\mathbb{R}^{3}$. Which statement is true?
(A) $f$ is a convex function but not a concave function
(B) $f$ is a convex function and a concave function
(C) $f$ is not a convex function but a concave function
(D) $f$ is neither a convex nor a concave function
(E) I prefer not to answer.

## Question 8.

Consider the matrix power $A^{7}$ of the matrix

$$
A=\left(\begin{array}{cc}
1 & 2 \\
0 & -1
\end{array}\right)
$$

Which statement is true?
(A) $A^{7}=\left(\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right)$
(B) $A^{7}=\left(\begin{array}{cc}1 & 2 \\ 0 & -1\end{array}\right)$
(C) $A^{7}=\left(\begin{array}{cc}3 & -2 \\ 4 & 1\end{array}\right)$
(D) $A^{7}=\left(\begin{array}{ll}1 & 128 \\ 0 & -1\end{array}\right)$
(E) I prefer not to answer.

