Challenging Matrix Problems for Advanced Students

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These matrix problems are quite challenging and are meant for advanced students. Do not try these problems until you master the problems in the Lecture Notes.

Hint: The problems can be simplified a lot by using a *smart* approach rather than straight-forward calculations.

I will make solutions to these problems available later if there are students working with them.

Question 1

Solve the equation

$$\begin{vmatrix} x & 2 & 3 \\ 2 & x & 3 \\ 2 & 3 & x \end{vmatrix} = 0$$

Question 2

Solve the equation

$$\begin{vmatrix} x+1 & 0 & x & 0 & x-1 & 0 \\ 0 & x & 0 & x-1 & 0 & x+1 \\ x & 0 & x-1 & 0 & x+1 & 0 \\ 0 & x-1 & 0 & x+1 & 0 & x \\ x-1 & 0 & x+1 & 0 & x & 0 \\ 0 & x+1 & 0 & x & 0 & x-1 \end{vmatrix} = 9$$

Question 3

Solve the linear system

$$\begin{array}{ccccccccccccc} & & x_2 & + & x_3 & + & \ldots & + & x_{n-1} & + & x_n & = & 2 \\ x_1 & & & + & x_3 & + & \ldots & + & x_{n-1} & + & x_n & = & 4 \\ x_1 & + & x_2 & & & + & \ldots & + & x_{n-1} & + & x_n & = & 6 \\ \vdots & & \vdots & & \vdots & & \ddots & & \vdots & & \vdots & & \vdots \\ x_1 & + & x_2 & + & x_3 & + & \ldots & + & x_{n-1} & & & = & 2n \end{array}$$