Commuting nilpotent matrices and Artin algebras
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Fix an $n \times n$ nilpotent matrix B whose Jordan blocks are given by the partition P of n. Assume the field k is closed. Consider the irreducible variety N_B parametrizing nilpotent $n \times n$ matrices A that commute with B. What partition $Q(P)$ occurs for a generic A?

The ring $k[A, B]$ is an Artinian ring. V. Baranovsky, R. Basili, A. Premet and others explored the connection between the family $P(n)$ of pairs of commuting nilpotent matrices and the Hilbert scheme parametriizing Artin algebra quotients of $k[x, y]$. P. Oblak and T. Košir showed that when A is generic, then $k[A, B]$ is Gorenstein. However, the Hilbert function of this ring determines $Q(P)$. A result of F.H.S. Macaulay then shows that $Q(P)$ has parts that differ pairwise by at least two. P. Oblak has determined the largest part of $Q(P)$. We report on these results and others connecting the study of Artinian algebras and commuting nilpotent matrices. In work joint with R. Basili and L. Khatami, we give a criterion on A for $k[A, B]$ to be Gorenstein.