# Network Competition when Costs are Heterogeneous*

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*Abstract:*

*In this paper we study network competition when costs differ among interconnected networks. Such cost differences are observed in the mobile sector as well as in fixed networks. In the paper we find that cost based regulation will not result in first best market shares. The low cost firm will be too small in equilibrium. This is partly due to tariff mediated network externalities. This result is in contrast to the standard result in the literature on network competition where one assumes symmetric cost structure. In the present paper, the regulator can induce a first best market equilibrium by combining cost based regulation of termination rates with a tax based on the number of subscribers. If such a tax is not an available instrument, the regulator can improve welfare by granting a termination margin to the low cost firm as compared to cost based regulation.*

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## 1. Introduction

The telecommunication industry is deregulated in most countries and is becoming increasingly more competitive. Nevertheless it is expected that some services will still require regulatory scrutiny. Wholesale termination of calls is an example of a market where networks, even in competitive environments, is in a *de facto* monopoly position since they are exclusive providers of termination services to their own customer base. Accordingly regulators have implemented, or are considering implementing price regulation of termination rates. Starting with the papers by Laffont Rey and Tirole (1998a,b) and Armstrong (1998) there is a considerable body of papers analysing network competition under various assumptions. These papers provide guidance for regulators when determining termination rates. The implication of cost heterogeneity is however hardly addressed in the literature.

In the present paper we introduce cost heterogeneity in the, by now standard model of network competition. It is demonstrated that an optimal policy is not characterised by termination rates regulated at marginal costs because the low cost firm becomes too small in equilibrium. This is partly due to the assumed type of competition and partly due to tariff mediated externalities. Consumers choosing to subscribe to the high cost network do not take into account that by doing so the price other consumers have to pay to call them becomes high. By choosing to change subscription from the high cost to the low cost network, the cost of calling that particular subscriber would decrease for all other subscribers. When choosing network, subscribers do not take into account this tariff mediated externality. An optimal regulatory scheme can be implemented by introducing instruments in addition to regulated termination rates. By taxing inefficient firms and/or subsidising efficient firms based on the number of subscribers one obtains optimal market shares. To our knowledge such taxation and subsidisation are not available instruments to regulators. If regulators are restricted to using regulation of termination rates as the only regulatory instruments we demonstrate that the equilibrium market share of the efficient firm increases if this firm is granted a (small) margin on termination. The welfare gain from increased market share of the efficient firm will however have to be balanced against the deadweight loss due to increasing prices above marginal costs. In the paper we demonstrate that the positive market share effect dominates for small termination margins, thus it is welfare improving to grant a small termination margin to the most efficient firm. In special cases it may be optimal to set a reciprocal termination fee equal to the marginal cost of the high cost firm.

There is reason to believe that marginal cost of terminating calls will differ among some types of networks. In the current paper we have two particular cases in mind. The first case is a situation where a traditional fixed telephony network is competing with an IP based telephony network. The other case we have in mind is when two competing mobile networks based on different radio spectrum allocations compete.[1]

IP telephony comes in many varieties, some with characteristics very similar to traditional telephony, seen from the consumer side. By connecting an adapter to any broadband link with a standard interface (Ethernet), the customer can connect any traditional telephone terminal to the adapter. The customer will hear a dial tone, can use traditional phone numbers and reach any other phone (examples of providers of such services are Vonage in the US and Telio in Norway). The cost structure of IP based telephony is different from the cost structure of running a traditional telephony network. This is mainly due to economies of scope. A traditional telephony network is dedicated for one single service, whereas an IP network is multipurpose. The infrastructure cost is accordingly shared among several services. The cost of providing IP telephony to a customer that also consumes other services based on IP is accordingly low compared to the cost of providing traditional telephony. One might argue that since the cost of IP telephony is lower than the cost of providing traditional telephony, IP will rapidly replace traditional telephony. If so, the question of analysing interconnection of networks with different cost structures is only of interest in a transitory phase. The cost advantage is however based on economies of scope between a set of communication services. It is likely that a significant proportion of

[1] Another evident example of networks with different cost structure is when fixed and mobile networks interconnect. In the current paper the focus is on competing networks where consumers choose to connect to one, and only one of the networks. This is not necessarily the case for fixed/mobile. A large proportion of the mobile customer base is also subscribers to the fixed network (so-called multihoming).

the customers will only buy one single communication service; telephony. Thus it is likely that both traditional telephony and IP based telephony will have positive market shares and accordingly exist side by side over a period.

Mobile telephony is based on usage of the radio spectrum. The radio spectrum is a scarce resource, and competing mobile networks are typically based on different spectrum allocations resulting in different cost structures. In a European context, mobile networks are based on 900 MHz licenses, on 1800 MHz licences, and/or UMTS licences (a number of frequency blocks in the range 1900 to 2200 MHz). It is likely that this variation results in cost differences since the geographical area that can be covered by a single radio cell is a function of frequency.

There is some empirical evidence supporting the assertion that there are differences in marginal costs between telephony networks. Correa (2003) estimates a cost function for fixed line telephony providers in the UK and finds significant differences in marginal costs of providing local calls between firms based on traditional fixed line technology as compared to firms based on cable TV technology.[2] The Competition Commission in the UK, based on an engineering model, estimated the difference in long run incremental network cost of termination mobile calls between combined 900/1800 operators and 1800 operators to be in the range 12% -18% (see Competition Commission 2003, table 2.8). Another example is the cost calculations done by the Swedish regulator PTS where they also discovered cost differences (see PTS 2004). They did not however publish the exact cost differences.[3]

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[2] On the one hand, the number of TV subscribers connected to the network has a significant positive effect on the marginal cost of providing local calls in her study. On the other hand, the number of TV subscribers has a significant negative effect on the cost of providing telephony subscriptions. The implication is that cable TV networks, as compared to traditional telephony networks, can provide telephony subscriptions at lower marginal costs, whereas the marginal costs of providing calls are higher.

[3] It is not a trivial task to measure incremental cost, and it is likely that some fixed and/or common costs are included in the LRIC results referred above. Fixed and common costs are typically not relevant in pricing decisions. Due to the technical reasons listed above it is nevertheless likely that marginal costs also differ between networks.

Regulators determine termination rates in many countries. As indicated above the literature does not provide much guidance as to how one should deal with heterogeneous costs. Interestingly, the regulators in both Sweden and in the UK identified cost differences in the mobile sector, but they have chosen quite different approaches. In the UK, estimated cost differences are exactly reflected in the regulated termination rates, i.e. the termination rates differ among the networks. In Sweden the regulator chose to set the same termination fee for all the three regulated mobile networks and it was set at the highest estimated level.

The literature on network competition was initiated by Laffont Rey and Tirole (1998a,b) and Armstrong (1998). Introductions to this literature can be found in Laffont and Tirole (2000) as well as in Armstrong (2002), and an overview of some recent contributions is provided in Peitz et al. (2004). In most of these works it is assumed identical cost structure and reciprocal termination fees. Armstrong (1998) and Laffont et al. (1998a) demonstrate that under uniform pricing, a reciprocal termination fee above costs will serve as a collusive device. Under nonlinear pricing, this result changes. Laffont et al. (1998b) demonstrate that the two networks are indifferent with respect to the termination fee under two part tariffs, whereas Gans and King (2001) find that a reciprocal termination fee below cost will serve as a collusive device when networks are allowed to price discriminate between on- and off-net traffic.

The literature accordingly indicates that reciprocal termination fees may serve as a collusive device in some cases. Furthermore, if the termination fee is determined unilaterally each network has an incentive to raise its termination fee well above marginal cost[4] resulting in a welfare loss (Gans and King 2001).

Regulators have recognised these results and thus they are attempting to regulate termination rates. When networks are symmetric, the advice to regulators from the literature is straightforward; it is optimal to set a reciprocal termination fee equal to

[4] Given that the consumer's willingness to pay does not increase with the number of incoming calls, the profit maximising termination fee is the monopoly price. Introducing willingness to pay for receiving calls will result in a downward correction to this price.

marginal cost. The picture is however not as straightforward if networks are asymmetric. Two classes of asymmetries have been studied in the literature:[5] 1) Vertical differentiation between the networks, and 2) asymmetric cost structures.

Vertically differentiated networks are studied in Carter and Wright (2003) where the source of the quality differential is motivated by the asymmetry between an entrant and an incumbent. They consider two part tariffs and reciprocal termination fees. The superior network (the incumbent) will then always prefer a termination fee at marginal cost whereas the newcomer may want a termination fee above costs. The termination fee preferred by the high quality network is also the welfare maximising termination fee. Peitz (2005) considers a model fairly identical to the model considered by Carter and Wright, Peitz however focuses on incentives for newcomers to entry.[6] Granting a termination mark-up to the entrant makes consumers better off, but the total welfare is reduced. Since the profits of the newcomer also increase, Peitz argues that entry is being stimulated. Peitz (2002) demonstrates that most of these results are also valid under price discrimination between on- and off-net traffic.

Armstrong (2004) introduces asymmetric costs and heterogeneous calling patterns into a model of network competition. In this model it is the low cost network that should be granted a termination mark-up in order to stimulate the low cost network to sign up the welfare maximising number of subscribers. The modelling in this paper is however different from the modelling framework in the other papers cited above. In particular, Armstrong analyses a case where demand is inelastic and where a dominant firm is being regulated in the downstream market, and a number of small firms are price takers (a competitive fringe). By assuming inelastic demand, modelling is simplified, but by assuming that usage is independent of marginal prices, there is no welfare loss from granting termination mark-ups for given market shares.

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[5] There is another strand of literature considering consumer heterogeneity and unbalanced calling patterns, demonstrating that reciprocal termination fees under two part tariffs is still profit neutral, see e.g. Dessein, (2003).

[6] In De Bijl and Peitz, 2002, the dynamics of asymmetric competition and entry is considered in a number of numerical simulations.

Furthermore, since the dominant firm is regulated in the downstream market, there is limited strategic interaction between the firms.

The contribution of the current paper is to take a cost asymmetry similar to the one considered by Armstrong into the standard Laffont Rey Tirole model where firms offer three part tariffs and compete à la Hotelling in the downstream market. Based on this model we are in a position to investigate welfare properties of some policies with respect to the regulation of termination fees.

The three major results in the current paper are: 1) cost based regulation will not result in first best market shares. The most efficient firm will be too small. 2) Taxation and subsidisation based on the number of subscribers can induce the first outcome. 3) As compared to cost based regulation of termination rates, granting a (small) termination mark up to the most efficient firm results in increased welfare.

The first results are in line with the results derived in Armstrong 2004. The model studied by Armstrong does however not allow distinguishing result 2 from result 3. In the Armstrong model a margin on termination services has the same effect as a subsidy based on the number of subscribers since demand is inelastic. In the current paper we demonstrate that the deadweight loss from regulating a price away from the underlying marginal cost is dominated by a positive market share effect for small termination margins. A regulator can accordingly increase welfare by granting (small) margins to low cost firms. This result lends support to the regulatory approach taken in Sweden where the regulation of termination rates in effect results in margins to the efficient firms.

The paper is organised as follows: In section 2 the model is presented, in section 3 the welfare properties of cost based regulations are considered. In section 4 optimal regulation is derived. In section 5 the effects of granting termination margins to the efficient firm are investigated and finally, in section 6 we conclude.

## 2. The model

We consider a two-stage game; in the first stage the regulator determines the interconnection fees and in the second stage the two networks compete à la Hotelling.

The market is covered, i.e. all consumers are signed up to one of the two networks. Thus prices are not affecting market size, but prices affect market shares and usage. For notational simplicity total market size is normalised to 1. It is important to have in mind that results from models on network competition depend on the contracts offered to consumers. There are four different types of contracts typically being discussed in the literature: uniform pricing, price discrimination, two part tariffs and two part tariffs with price discrimination.[7] In the current paper we consider the most general contract; two part tariffs with price discrimination. Network $i$ ($i$ = 1, 2) offers contracts: $\{F_i, p_i, \hat{p}_i\}$ where $F_i$ is the fixed fee (subscription fee), $p_i$ is the per minute price of calling other subscribers of the same network (on – net price), and $\hat{p}_i$ is the price of calling subscribers of the other network (off – net price).

### *2.1. Demand and market shares*

Let $y$ denote the sum of the value of income and the stand alone value of network subscription.[8] Consumer tastes are assumed to be uniformly distributed over a line of length 1. Given quantity of calls made $q$, a consumer located at $x$ joining network $i$ has utility:

$$y - t|x - x_i| + u(q).$$

The parameter $t$ is a measure of disutility from not consuming the most preferred brand (travelling cost). Our assumption of a fully covered market is fulfilled given that the utility from making calls on the network is sufficiently high. Define:

$$v(p) \equiv \max_q u(q) - pq .$$

Let $\alpha_i$ denote the market share of network $i$. The net value of being a subscriber of network $i$ can then be written:

$$V_i = \alpha_i v(p_i) + (1 - \alpha_i) v(\hat{p}_i) - F_i .$$

Throughout the paper we will focus on shared market equilibriums. Such equilibriums exist as long as the disutility parameter $t$ is sufficiently large and the difference in

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[7] In LRT 98a and LRT 98b the basic model is derived in all these four cases.

[8] For notational simplicity the two terms income and value of network subscription are added together since the market is covered and all consumers are connected to a network.

utility ($V_i - V_j$) is not too large. We will later return to the exact parameter restrictions under the different cases considered below. Given the existence of a shared equilibrium, market shares will be determined by the location of the consumer being indifferent between the two networks:

$$V_i - t\alpha_i = V_j - t(1-\alpha_i) \Leftrightarrow \alpha_i = \frac{1}{2} + \frac{1}{2t}(V_i - V_j)$$

By defining $\sigma = \frac{1}{2t}$, substituting for $V_i$ and $V_j$ and rearranging, market shares can be written:

(1.) $$\alpha_i = \frac{\frac{1}{2} + \sigma\left[v(\hat{p}_i) - v(p_j) - F_i + F_j\right]}{1 - \sigma\left[v(p_i) + v(p_j) - v(\hat{p}_i) - v(\hat{p}_j)\right]}$$

### 2.2. Cost structure

There is a fixed cost for connecting customers *f*. Furthermore the marginal costs of on-net traffic for network *i* is assumed to be $c_i$. This cost can be decomposed into two parts, origination and termination, each assumed to be 50% of the total cost. The cost is assumed to differ between the two networks. Network *i* is assumed to charge $a_i$ for termination services. In order to simplify notation we define true and perceived marginal cost for off-net traffic:

$$\bar{c} = \frac{1}{2}(c_i + c_j)$$

$$\hat{c}_i = \frac{1}{2}c_i + a_j$$

### 2.3. Benchmark, welfare-maximising solution

As a reference point we start by deriving the welfare maximising solution, i.e. maximising the welfare function given by:

$$\begin{aligned} W = y - f - \frac{1}{4\sigma}\left(2\alpha_i^2 - 2\alpha_i + 1\right) \\ + \alpha_i\left(\alpha_i\left(u(q_1) - q_1c_1\right) + (1-\alpha_i)\left(u(\hat{q}_1) - \hat{q}_1\bar{c}\right)\right) \\ + (1-\alpha_i)\left((1-\alpha_i)\left(u(q_2) - q_2c_2\right) + \alpha_i\left(u(\hat{q}_2) - \hat{q}_2\bar{c}\right)\right) \end{aligned}$$

Recall that the total number of subscribers is normalised to one. The interpretation of the welfare function is then straightforward. For market shares $\alpha_i$ (and 1 - $\alpha_i$) the third term is average disutility from not consuming the most preferred variety, whereas the last to terms give the difference between generated utility and costs for a given number of calls. This function is to be maximised with respect to $q_1, \hat{q}_1, q_2, \hat{q}_2, \alpha_i$. It is straightforward to see from the expression above, that as long as the function *u*() is increasing and concave, optimal usage is given by:

$$q_i^* = \{ q_i(c_i) | u'_{q_i} = c_i \}$$
$$\hat{q}_i^* = \{ \hat{q}_i(\bar{c}) | u'_{q_i} = \bar{c} \}$$

Define $v_i = u(q_i^*) - c_i q_i^*, \quad \bar{v} = u(\hat{q}_i^*) - \bar{c}\hat{q}_i^*$, then the welfare function can be written:

$$\text{(2.)} \quad W^*(\alpha_i) = y - f - \frac{1}{4\sigma}\left(2\alpha_i^2 - 2\alpha_i + 1\right) + \alpha_i^2 v_i + (1-\alpha_i)^2 v_j + 2\alpha_i(1-\alpha_i)\bar{v}$$

Differentiating with respect to market share yields:

$$\frac{\partial W}{\partial \alpha_i} = -\frac{1}{2\sigma}(2\alpha_i - 1) + 2\alpha_i v_i - 2v_j + 2\alpha_i v_j + 2\bar{v} - 4\alpha_i \bar{v}$$

An interior solution satisfies:

$$\text{(3.)} \quad \frac{\partial W}{\partial \alpha_i} = 0 \Leftrightarrow \alpha^* = \frac{\frac{1}{2} - 2\sigma(v_j - \bar{v})}{1 - 2\sigma(v_i + v_j - 2\bar{v})}$$

This is an interior solution to the maximisation problem iff $\frac{1}{2\sigma} > 2v_i - 2\bar{v}$, when network *i* is the low cost network. We will throughout the paper assume that this condition is fulfilled.[9]

## 2.4. Market equilibrium

The firms will maximise their profits by determining an optimal contract $\{F_i, p_i, \hat{p}_i\}$. The profits of each firm can be written:

[9] The condition for existence and stability is that the networks are sufficiently differentiated, i.e. σ small, and that the differences in costs are not too large and that the termination margins are not too large. One of the firms will corner the market if either of these conditions are violated. This condition is discussed in appendix A.

$$\pi_i = \alpha_i (F_i - f) \\ + \alpha_i^2 (p_i - c_i) q(p_i) \\ + \alpha_i \alpha_j (\hat{p}_i - \hat{c}_i) q(\hat{p}_i) \\ + \alpha_i \alpha_j (a_i - \tfrac{1}{2} c_i) q(\hat{p}_j)$$

The first line is the profits on subscription, the second line is profits from on-net traffic, the third is profits on off-net traffic, and the last line is the profits in the wholesale market. As demonstrated in LRT 98a it is convenient to consider profit maximisation as if firms offer a net surplus $V_i = \alpha_i v(p_i) + (1-\alpha_i) v(\hat{p}_i) - F_i$, and some usage prices[10], thus the firms solve:

$$\max_{V_i, p_i, \hat{p}_i} \left[ \alpha_i (\alpha_i v(p_i) + \alpha_j v(\hat{p}_i) - V_i - f) \\ + \alpha_i^2 (p_i - c_i) q(p_i) \\ + \alpha_i \alpha_j (\hat{p}_i - \hat{c}_i) q(\hat{p}_i) \\ + \alpha_i \alpha_j (a_i - \tfrac{1}{2} c_i) q(\hat{p}_j) \right]$$

Note that, for given net surplus *V*, market shares are independent of usage prices. Recall that $v'(p_i) = -q(p_i)$ and consider the first order conditions for optimal on- and off-net prices:

$$p_i: \quad \alpha_i^2 (v'(p_i) + q(p_i) + (p_i - c_i) q(p_i)) = 0 \Leftrightarrow p_i = c_i$$
$$\hat{p}_{i:} \quad \alpha_i \alpha_j (v'(\hat{p}_i) + q(\hat{p}_i) + (\hat{p}_i - \hat{c}_i) q(\hat{p}_i)) = 0 \Leftrightarrow \hat{p}_i = \hat{c}_i$$

This is a well-known result (see LRT 1998a). The firms determine usage prices by maximising the sum of producer and consumer surplus, and then they will extract as much consumer surplus as possible via the fixed fee. Since on-net traffic is always priced at marginal cost, we can save notation by defining: $v_i \equiv v(c_i)$. Let $m_i$ be the margin on termination services defined by: $m_i \equiv a_i - \tfrac{1}{2} c_i$. Then we can write: $\hat{c}_i = \bar{c} + m_j$ since: $\hat{c}_i = \tfrac{1}{2} c_i + a_j = \tfrac{1}{2} c_i + \tfrac{1}{2} c_j + m_j = \bar{c} + m_j$.

Consider now the optimal fixed fees:

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[10] Note however that games where net utilities are the strategic variables yield equilibria different from the equilibria one obtains when firms use prices as strategic variables. The result with respect to usage prices is however identical in the two types of games.

(4.) $\max_{F_i}\left[\alpha_i\left(F_i - f\right) + \alpha_i\alpha_j m_i q\left(\overline{c} + m_i\right)\right]$

Inserting for $\alpha_j = 1-\alpha_i$ and differentiating yields the following set of first order conditions:

$$0 = \alpha_i + \frac{\partial\alpha_i}{\partial F_i}\left(F_i - f\right) + \frac{\partial\alpha_i}{\partial F_i}\left(1-2\alpha_i\right)m_i q\left(\overline{c}+m_i\right)$$

Inserting for market shares, and rearranging yields:

$$(5.)\quad \begin{aligned} F_i = &\left(\frac{1}{2} + \frac{\sigma m_i q\left(\overline{c}+m_i\right)}{2\left(1-\sigma\left(v_i + v_j - \hat{v}_i - \hat{v}_j\right) + \sigma m_i q\left(\overline{c}+m_i\right)\right)}\right)F_j \\ &+ \frac{f}{2} + \frac{1}{4\sigma} + \frac{\hat{v}_i - v_j}{2} - \frac{m_i q\left(\overline{c}+m_i\right)\left[1+2\sigma\left(f - v_i + \hat{v}_j\right)\right]}{4\left(1-\sigma\left(v_i + v_j - \hat{v}_i - \hat{v}_j\right) + \sigma m_i q\left(\overline{c}+m_i\right)\right)} \end{aligned}$$

For given termination margins, $m_i$, the system of first order conditions (5.) is a system of linear equations. Equilibrium will typically exist and be stable for sufficiently differentiated networks with not too large cost asymmetries and not too large termination margins. For each of the special cases considered below we will provide conditions for the existence of a shared market equilibrium as well as conditions for stability.[11]

## 3. Cost based termination fees

As indicated above the literature suggests that regulating termination services to marginal costs yields a socially optimal outcome.[12] In this section of the paper we will investigate whether this result is valid when costs differ among the two networks. When termination fees are regulated down to marginal cost, i.e. $m_i = 0$, the best response functions in (5.) simplifies to:

$$F_i = \frac{F_j}{2} + \frac{f}{2} + \frac{1}{4\sigma} + \frac{\overline{v} - v_j}{2}$$

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[11] The best response function above is a generalised version of the response functions in e.g. Gans and King (2001), and Laffont Rey and Tirole (1998b). They provide conditions for existence under cost symmetry.

[12] Even in the Peitz (2005) model, total welfare is reduced when one network (the entrant) is granted a termination mark-up. Peitz however argues that there may be a dynamic gain from allowing termination margins for newcomers because entry is stimulated.

where $\bar{v} \equiv v(\bar{c})$. The slope of the best response functions is the same for the two firms, whereas the intercepts differ. Let firm *i* be the low cost firm. We impose the following parameter restriction[13] in order to obtain an interior solution:

$\frac{1}{2\sigma} > \left[\frac{2}{3}v_i + \frac{1}{3}v_j - \bar{v}\right]$. The equilibrium is illustrated below:

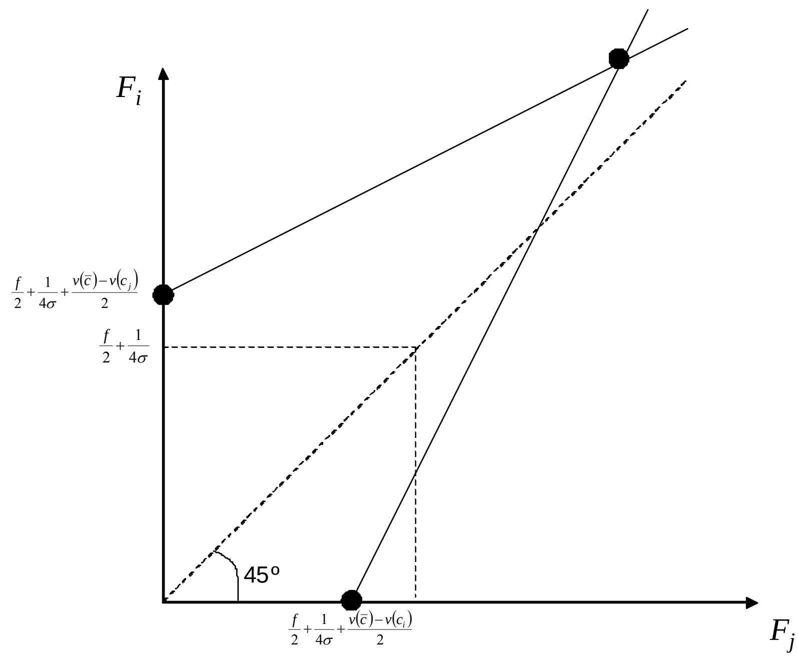


**Figure 1, Equilibrium under cost based regulation**

From figure 1 we can see directly that the most efficient firm is charging a fixed fee that is higher than the less efficient firm. By combining the two best response functions we can calculate equilibrium fixed fees:

$$F_i = \frac{1}{2\sigma} + f - \frac{1}{3}\left(v_i + 2v_j - 3\bar{v}\right)$$

PROPOSITION 1. *Under cost based regulation, the market share of the most efficient firm is too small compared to the welfare maximising market share*.

PROOF: Let firm *i* be the low cost firm. The equilibrium difference in fixed fees is:

$$F_i - F_j = \frac{1}{3}\left(v_i - v_j\right) > 0.$$

[13] The condition is discussed in appendix B.

Consider now the difference in fixed fees that would have induced welfare maximising market shares, $\Delta F^*$. Combining the condition for first best market shares (3.), with the expression for market shares as a function of fixed fees (1.), yields:

$$\frac{\frac{1}{2}-2\sigma\left(v_j-\bar{v}\right)}{1-2\sigma\left(v_i+v_j-2\bar{v}\right)}=\frac{\frac{1}{2}+\sigma\left[\bar{v}-v_j-\Delta F^*\right]}{1-\sigma\left[v_i+v_j-2\bar{v}\right]}\Leftrightarrow \Delta F^*=\frac{v_j-v_i}{2\left(1-2\sigma\left(v_i+v_j-2\bar{v}\right)\right)}<0$$

Thus: $\left(F_i-F_j\right)>0>\Delta F^*$. From the market share function (1.) we readily see that the market share of the most efficient firm then becomes too small in equilibrium. QED.

Equilibrium market shares can be calculated by inserting the difference in equilibrium fixed fees, calculated above, into the market share function (1.):

(6.) $$\alpha_i=\frac{\frac{1}{2}+\sigma\left[\bar{v}-v_j-\frac{v_i-v_j}{3}\right]}{1-\sigma\left(v_i+v_j-2\bar{v}\right)}=\frac{3+2\sigma\left(3\bar{v}-v_i-2v_j\right)}{6\left(1-\sigma\left(v_i+v_j-2\bar{v}\right)\right)}$$

As demonstrated above, the most efficient firm is too small in equilibrium. This result is partly due to externalities. Since prices differ in equilibrium, the model exhibits tariff mediated network externalities. Consumers on both networks are affected by the network choice made by the indifferent consumer. If one consumer switches from the high cost to the low cost network, the price of making calls to this consumer falls, both for customers in the high cost and customers in the low cost network. Thus, if a consumer switches from the high cost to the low cost network, everybody else is better off. The firms do not however have any incentives to let this externality be reflected in the fixed fees. The two firms compete for the marginal customer taking into consideration the profit contribution from this consumer and without having a mechanism to extract (a fraction of) the increased willingness to pay from all the customers already on the network.

The result above is however only partly due to network externalities. One obtains the same qualitative result in a Hotelling model with differences in marginal costs in the absence of network externalities. The low cost firm does not have incentives to compete sufficiently aggressively for consumers. It becomes too small in equilibrium. The externality effect comes however in addition.

The conditions for having two networks as a welfare maximising solution (A1) and getting a shared market equilibrium (A2) are respectively:

(A1) $\frac{1}{2\sigma} > 2v_i - 2\bar{v}$

(A2) $\frac{1}{2\sigma} > \left[\frac{2}{3}v_i + \frac{1}{3}v_j - \bar{v}\right]$

There exist parameter combinations where the second, but not the first condition is satisfied. Thus we may have equilibrium under cost based regulation where two firms are active, but where a welfare maximising market structure is to only have one network. Following Peitz (2005), one of the networks can be considered as a newcomer. The implication of the result above is then that cost based regulation may stimulate inefficient entry. This case may be relevant in the mobile sector where the licenses to the most cost effective frequencies are allocated first, implying that the last entrant to the market has cost disadvantages relative to the established firms. In the fixed sector we may have the opposite case. Newcomers to the fixed sector are typically based on IP technology which is expected to be more cost efficient. The results above indicate that we may end up in a situation where it would be welfare maximising to switch off the old telephony network, but where market equilibrium yields the opposite result.

## 4. Optimal regulation

In the section above we demonstrated that cost based regulation of termination fees on the one hand resulted in optimal usage prices, but on the other hand in market shares deviating from the welfare maximising level. If termination rates are altered in order to induce optimal market shares, the result will be that usage prices deviate from the optimal level. The regulator is in a situation where the number of objectives exceeds the number of available instruments. The regulator accordingly needs more instruments in order to induce a welfare maximising outcome.

One obvious alternative for introducing more regulatory instruments is to consider a tax $\tau_i$ per subscriber in order to induce a first best market equilibrium. Then profits become:

$$\pi_i = \alpha_i(F_i - f - \tau_i)$$

and best response functions are:

$$F_i = \frac{F_j}{2} + \frac{f}{2} + \frac{\tau_i}{2} + \frac{1}{4\sigma} + \frac{v(\bar{c}) - v(c_j)}{2}$$

PROPOSITION 2. *The regulator can induce a welfare maximising market equilibrium by setting cost based termination rates and:*

*a) Subsidise the efficient firm per subscriber, or*

*b) Impose a tax per subscriber on the inefficient firm, or*

*c) A combination*

PROOF:

Equilibrium fixed fees are a function of the pair of per subscriber taxes:

$$F_i = f + \frac{\tau_j + 2\tau_i}{3} + \frac{1}{2\sigma} - \frac{1}{3}(v_i + 2v_j - 3\bar{v})$$

Optimal taxes must be determined such that they induce welfare maximising market shares, thus they must satisfy:

$$F_i = F_j - \frac{v_i - v_j}{2(1 - 2\sigma(v_i + v_j - 2\bar{v}))}$$

Inserting for equilibrium fixed fees and solving with respect to the tax difference yields:

$$\tau_i - \tau_j = -(v_i - v_j)\left[1 + \frac{3}{2(1 - 2\sigma(v_i + v_j - 2\bar{v}))}\right] < 0$$

Under our parameter restrictions (A1), the square bracket is positive. Assuming that firm *i* is the most efficient firm we have $v_i - v_j > 0$. Thus a pair of taxes implementing first best is characterized by $\tau_i - \tau_j < 0$. QED

From the result above we see that it is sufficient to impose a tax on the inefficient firm or to introduce a subsidy to the efficient firm. This result may seem to oppose the regulatory objectives of providing a "level playing field". Instead efficient technologies should be "favoured".

## 5. Effects of granting a termination margin to the efficient firm

There are, to our knowledge, no examples of regulators having introduced per subscriber taxes and subsidies of the type discussed above. In this section we will consider "second best" regulation, i.e. analyse whether allowing termination margins will result in market equilibrium where market shares are closer to the optimum level.[14]

Firm *i* is still assumed to be the efficient firm. Assume that the regulator determines a positive termination margin, *m*, for the efficient firm and applies cost based regulation for the inefficient firm.

In this case usage prices are:

$$p_i = c_i \qquad p_j = c_j$$
$$\hat{p}_i = \bar{c} \qquad \hat{p}_j = \bar{c} + m$$

The best response functions simplify to:

$$F_i = \left(\frac{1}{2} + \frac{\sigma m \hat{q}_j}{2\left(1 - \sigma\left(v_i + v_j - \bar{v} - \hat{v}_j\right) + \sigma m \hat{q}_j\right)}\right) F_j + \frac{f}{2} + \frac{1}{4\sigma} + \frac{\bar{v} - v_j}{2} - \frac{m\hat{q}_j\left[1 + 2\sigma\left(f - v_i + \hat{v}_j\right)\right]}{4\left(1 - \sigma\left(v_i + v_j - \bar{v} - \hat{v}_j\right) + \sigma m \hat{q}_j\right)}$$

and:

$$F_j = \frac{F_i}{2} + \frac{f}{2} + \frac{1}{4\sigma} + \frac{\hat{v}_j - v_i}{2}$$

By combining the best response functions we obtain equilibrium fixed fees. Explicit expressions are provided in appendix A.3. These equilibrium fixed fees can be inserted into the market share function (1.). Then we obtain market shares as a function of the termination margin:

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[14] Note that the regulation of Swedish mobile termination can be seen as a special case of such regulation since the regulated reciprocal termination rate is cost based for the least efficient firm. Thus the other two firms, being more efficient, are granted a termination mark-up.

$$\alpha(m) = \frac{3 - 2\sigma(v_i + 2v_j - 2\overline{v}_i - \hat{v}_j) + 2m\sigma\hat{q}_j}{6(1 - \sigma(v_i + v_j - \overline{v}_i - \hat{v}_j)) + 4m\sigma\hat{q}_j}$$

Under our assumptions there does exist an interior solution for $m = 0$, i.e. under cost based regulation. For sufficiently high termination margins, this may change. Differentiation of the market share function with respect to the termination margin yields:

$$\text{(7.)} \quad \alpha'(m) = 2\sigma \frac{2\sigma\left(-m\frac{\partial \hat{q}_j}{\partial m}\right)(v_i + \overline{v} - v_j - \hat{v}_j) + \hat{q}_j(3 - 2\sigma(v_i + 2v_j - 2\overline{v}_i - \hat{v}_j) + 2\sigma m\hat{q}_j)}{(6(1 - \sigma(v_i + v_j - \overline{v}_i - \hat{v}_j)) + 4\sigma m\hat{q}_j)^2}$$

PROPOSITION 3. *The market share of the efficient firm is monotonously increasing in the termination margin granted to the efficient firm for any non negative termination margin.*

PROOF

A sufficient condition is that the expression in (7.) is strictly positive. The denominator is always positive. A non negative termination margin implies $m \geq 0$. The first bracket in the numerator is then positive (= 0 for $m = 0$) since the demand function has a negative slope. The second bracket is also positive because $v_i > v_j$ by assumption, and $\overline{v} > \hat{v}_j$ for $v_i > v_j$ and $m \geq 0$. Finally, the third bracket is positive by assumption since it is a necessary condition for a shared market equilibrium to exist.

QED

The implication of the result above is that the regulator, starting from a cost based equilibrium, can bring the market closer to the optimal market shares by introducing a termination margin to the efficient firm. However, this gain has to be balanced against the deadweight loss resulting from increased usage prices due to the termination margin.

PROPOSITION 4. *Total welfare is increasing in the termination margin at m = 0*

The proof of this proposition is provided in the appendix A.4.

By granting a (small) termination margin to the efficient firm, the regulator can accordingly increase welfare.

Note that the regulatory regime in Sweden, where calculated costs differ and where the regulator is imposing a reciprocal termination fee equal to the cost of the least efficient network can be seen as a way of approximating the second best solution of the type we are discussing. Whether the termination margins granted to the more efficient firms in Sweden are too small, exactly equal to, or above the welfare maximising level is however not possible to evaluate based on the model presented here.

The introduction of a termination margin has two effects on the market outcome. A termination margin of the type discussed here results in increased vertical differentiation since the price paid for off-net traffic by customers of the high cost firm increases. We will denote this effect the retail effect. Furthermore, the termination margin has a direct effect on profits in the wholesale market since the low cost firm makes profits on incoming traffic due to the termination margin. In the following we will decompose the effect of introducing a termination margin into the retail effect and a wholesale effect.

Assume that the high cost firm has to pay a margin *m* on outgoing traffic, and assume that the low cost firm is exposed to a tax on incoming traffic such that the regulator exactly confiscates the revenues from the termination margin. Thus the retail effect is present whereas the regulator is cancelling out the wholesale effect due to taxation. By analysing equilibrium in this case we can highlight the effects due to the retail effect.

In this case the best response functions simplify to:

$$F_i = \frac{F_j}{2} + \frac{f}{2} + \frac{1}{4\sigma} + \frac{\bar{v} - v_j}{2}$$

$$F_j = \frac{F_i}{2} + \frac{f}{2} + \frac{1}{4\sigma} + \frac{\hat{v}_j - v_i}{2}$$

Equilibrium fixed fees become:

$$F_i = \frac{1}{2\sigma} + f - \frac{v_i + 2v_j - 2\overline{v} - \hat{v}_j}{3}$$
$$F_j = \frac{1}{2\sigma} + f - \frac{2v_i + v_j - \overline{v} - 2\hat{v}_j}{3}$$

Thus the market share of the low cost firm (firm *i*) becomes:

$$\alpha^r(m) = \frac{3 - 2\sigma(v_i + 2v_j - 2\overline{v} - \hat{v}_j)}{6(1 - \sigma(v_i + v_j - \overline{v} - \hat{v}_j))}$$

PROPOSITION 5, *As compared to cost based regulation, introducing a termination margin results in the following effects on the market share of the most efficient firm:*

*a) A positive retail effect*

*b) A negative wholesale effect.*

Proof:

A sufficient condition for the result above to hold is that the market share of the most efficient firm is larger when we only take the retail effect into consideration as compared to a situation where we consider both effects, i.e. $\alpha(m) < \alpha^r(m)$:

$$\alpha^r(m) - \alpha(m) = \frac{3 - 2\sigma(v_i + 2v_j - 2\overline{v}_i - \hat{v}_j) + 2m\sigma\hat{q}_j}{6(1 - \sigma(v_i + v_j - \overline{v}_i - \hat{v}_j)) + 4m\sigma\hat{q}_j} - \frac{3 - 2\sigma(v_i + 2v_j - 2\overline{v} - \hat{v}_j)}{6(1 - \sigma(v_i + v_j - \overline{v} - \hat{v}_j))}$$
$$= \frac{m\sigma^2\hat{q}_j(v_i - v_j + \overline{v} - \hat{v}_j)}{3(1 - \sigma(v_i + v_j - \overline{v} - \hat{v}_j))(3(1 - \sigma(v_i + v_j - \overline{v} - \hat{v}_j)) + 2m\sigma\hat{q}_j)} > 0$$

The numerator is positive and the two terms in the denominator are identical to the numerators in the respective market share functions, both positive. The total effect of introducing a termination margin is defined as the sum of the retail and the wholesale effect. We have calculated that the retail effect is larger than the total effect, thus the wholesale effect has to be negative. QED.

The implication of proposition 5 is that the regulator can induce a given market share for the low cost firm at a lower deadweight loss due to distorted prices when the wholesale effect is neutralised. Thus it is welfare improving to introduce taxation in order to extract all net revenues due to the termination margin.

The negative wholesale effect is driven by the fact that the volume of incoming traffic is given by: $\alpha_i(1-\alpha_i)\hat{q}_j$. For a given termination margin, this volume is maximised for market shares equal to 0.5. We have already demonstrated that market shares for the low cost firm under cost based regulation are characterised by $\alpha_i(0) > 0.5$. Thus, there is an adverse effect for the low cost firm from increasing market shares. In a richer model where we have three or more competing networks and where market shares are below 0.5 for the low cost firm we can expect the wholesale effect to be positive as well (in addition to the positive retail effect). Furthermore, networks typically receive a significant volume of incoming traffic from networks operating in other markets (e.g. incoming traffic from abroad and/or incoming traffic from fixed to a mobile network or *vice versa*). The share of this traffic being received by a network is monotonously increasing in market share. Thus it is likely that the wholesale effect is positive in a richer environment with more than two competing networks and where the competing networks also receive traffic from other markets.

## 6. Conclusions

In this paper we have studied network competition when costs differ among the interconnected networks. We have analysed the implications of three different principles for regulating termination fees when marginal costs differ. The first case we have analysed is cost based in the sense that termination fees exactly reflect marginal costs. It is a standard result in the literature that marginal prices then are determined at the optimal level. In the current paper we have demonstrated that with cost differences equilibrium market shares are not optimal in this regime. The most efficient network is too small compared to a welfare maximising solution. The reason is partly that with cost differences there is a tariff mediated network externality. There is however no mechanism in the market that enables the efficient firm to internalise this effect.

In the second regulatory regime we introduce taxation and subsidisation, of the two firms based on the number of subscribers as an addition to the cost based regulation of termination rates. By subsidising the low cost firm and/or imposing a tax on the high cost firm, the regulator can implement first best.

In the third regime we investigate whether granting a termination mark-up to the low cost firm can improve the situation as compared to cost based regulation. We have demonstrated that the mark-up has the desired effect on market shares; the low cost firm becomes bigger. Furthermore, we have demonstrated that, starting from cost based regulation, welfare increases as a termination mark-up granted to the low cost firm is introduced. Thus it is welfare improving to let the efficient firm enjoy a (small) mark-up. The effect of granting a termination mark-up to the low cost firm can be decomposed into two parts, a retail effect and a wholesale effect. The retail effect is positive whereas the wholesale effect is negative. This is partly due to the fact that the volume of terminating traffic, where the firm enjoys a mark-up, is maximised for market shares equal to 0.5. Thus wholesale revenues decrease as market shares increase.

## 7. Appendix

**A.1. Conditions for interior solution to the welfare maximisation**

The solution:

$$\frac{\partial W}{\partial \alpha} = 0 \Leftrightarrow \alpha^* = \frac{\frac{1}{2} - 2\sigma\left(v_j - \overline{v}\right)}{1 - 2\sigma\left(v_i + v_j - 2\overline{v}\right)}$$

is an optimum if second order conditions are fulfilled and $\alpha^* \in [0,1]$. Consider first the second order conditions:

$$\frac{\partial^2 W}{\partial \alpha^2} = -\frac{1}{\sigma} + 2v_i + 2v_j - 4\overline{v} \leq 0 \Leftrightarrow \frac{1}{2\sigma} \geq v_i + v_j - 2\overline{v}$$

The function $w(c) = u(q^*(c)) - c\, q^*(c)$ is decreasing and convex in $c$. Then $v(c_i) + v(c_j) > 2\, v((c_i + w_j)/2)$. Thus the right hand side is positive and increasing in the cost difference $c_i - c_j$. The second order condition is accordingly fulfilled as long as the two networks are sufficiently differentiated ($\sigma$ small) and the cost difference is not too large. $\alpha^* \in [0,1]$ if the following two conditions are fulfilled

$$\left.\frac{\partial W}{\partial \alpha}\right|_{\alpha=0} > 0 \Leftrightarrow 0 < \frac{1}{2\sigma} - 2v_j + 2\overline{v} \Leftrightarrow \frac{1}{2\sigma} > 2v_j - 2\overline{v}$$

$$\left.\frac{\partial W}{\partial \alpha}\right|_{\alpha=1} < 0 \Leftrightarrow 0 > -\frac{1}{2\sigma} + 2v_i - 2\overline{v} \Leftrightarrow \frac{1}{2\sigma} > 2v_i - 2\overline{v}$$

Assume, without loss of generality that $c_i < c_j$, then $v_i > \overline{v} > v_j$, the right hand side of the first condition above is then always negative. The first condition is accordingly always fulfilled. The right hand side of the second condition is always positive. If the cost difference is large, then this condition is violated. Finally, note that when this condition is fulfilled the second order condition above is also fulfilled. Thus a necessary and sufficient condition for an interior solution is:

(A.1) $\frac{1}{2\sigma} > 2v_i - 2\overline{v}$

### A.2. Conditions for existence and stability under cost based regulation

Note first that the slope of the best response functions is ½, thus if equilibrium exists, it will be stable.

Under cost based regulation equilibrium, market shares are:

$$\alpha_i^{cb} = \frac{\frac{1}{2} + \sigma\left[\bar{v} - v_j - \frac{1}{3}(v_i - v_j)\right]}{1 - \sigma[v_i + v_j - 2\bar{v}]} = \frac{\frac{1}{2} + \sigma\left[v(\bar{c}) - \frac{2}{3}v_j - \frac{1}{3}v_i\right]}{1 - \sigma[v_i + v_j - 2\bar{v}]}$$

a shared market equilibrium exists provided that $\alpha_i^{CB} \in (0,1)$. Without loss of generality we assume that firm *i* is the most efficient firm. Then we have the following parameter restriction:

$$1 > \frac{\frac{1}{2} + \sigma\left[v(\bar{c}) - \frac{2}{3}v(c_j) - \frac{1}{3}v(c_i)\right]}{1 - \sigma[v(c_i) + v(c_j) - 2v(\bar{c})]}$$

$$\frac{1}{2\sigma} > \left[\frac{2}{3}v_i + \frac{1}{3}v_j - \bar{v}\right]$$

Consider then the second order conditions for profit maximisation:

$$\frac{\partial^2 \pi}{\partial F_i^2} = \frac{\partial \alpha_i}{\partial F_i} + \frac{\partial \alpha_i}{\partial F_i} + \frac{\partial^2 \alpha_i}{\partial F_i^2}(F_i - f)$$

$$= \frac{-2\sigma}{1 - \sigma(v_i + v_j - 2\bar{v})}$$

The sign of this expression is negative when the denominator is positive. This condition can be written: $\frac{1}{2\sigma} > \frac{1}{2}(v_i + v_j - 2\bar{v})$ and is fulfilled when the condition above is fulfilled.

The binding condition is accordingly:

(A.2. $\frac{1}{2\sigma} > \left[\frac{2}{3}v_i + \frac{1}{3}v_j - \bar{v}\right]$

This condition can be compared to the condition for an interior solution to the problem of welfare maximisation: $\alpha_i^{CB} \in (0,1)$ is fulfilled.

$$\frac{1}{2\sigma} > 2v_i - 2\bar{v}$$

$$\frac{1}{2\sigma} > \left[\frac{2}{3}v_i + \frac{1}{3}v_j - \bar{v}\right] = 2v_i - 2\bar{v} \underbrace{-\left(\frac{4}{3}v_i - \frac{1}{3}v_j + \bar{v}\right)}_{-}$$

This condition is accordingly always satisfied when the condition $\frac{1}{2\sigma} > 2v_i - 2\bar{v}$ is satisfied. Thus if it is welfare maximising to have two networks then there will always exist an interior equilibrium under cost based regulation. Note that the opposite not is true. There exist parameter combinations such that the welfare maximising outcome is to have only one network but where we obtain shared equilibrium under cost based regulation.

### A.3. Deriving market shares when the most efficient firm is granted a margin

In this case usage prices are:

$$p_i = c_i \qquad p_j = c_j$$
$$\hat{p}_i = \bar{c} \qquad \hat{p}_j = \bar{c} + m$$

The best response functions simplify to:

$$F_i = \left(\frac{1}{2} + \frac{\sigma m \hat{q}_j}{2\left(1 - \sigma\left(v_i + v_j - \bar{v} - \hat{v}_j\right) + \sigma m \hat{q}_j\right)}\right)F_j + \frac{f}{2} + \frac{1}{4\sigma} + \frac{\bar{v} - v_j}{2} - \frac{m\hat{q}_j\left[1 + 2\sigma\left(f - v_i + \hat{v}_j\right)\right]}{4\left(1 - \sigma\left(v_i + v_j - \bar{v} - \hat{v}_j\right) + \sigma m \hat{q}_j\right)}$$

and:

$$F_j = \frac{F_i}{2} + \frac{f}{2} + \frac{1}{4\sigma} + \frac{\hat{v}_j - v_i}{2}$$

Solving the system of equations above yields the following equilibrium fixed fees:

$$F_i = \frac{1}{2\sigma} + f - \frac{\left(v_i + 2v_j - 2\bar{v} - \hat{v}_j\right)\left(1 - \sigma\left(v_i + v_j - \bar{v} - \hat{v}_j\right)\right) - 2\sigma m \hat{q}_j\left(\bar{v} - v_j\right)}{3\left(1 - \sigma\left(v_i + v_j - \bar{v} - \hat{v}_j\right)\right) + 2\sigma m \hat{q}_j}$$

and:

$$F_j = \frac{1}{2\sigma} + f - \frac{\left(2v_i + v_j - \overline{v} - 2\hat{v}_j\right)\left(1 - \sigma\left(v_i + v_j - \overline{v} - \hat{v}_j\right)\right) + \sigma m \hat{q}_j\left(v_i + v_j - \overline{v} - \hat{v}_j\right)}{3\left(1 - \sigma\left(v_i + v_j - \overline{v} - \hat{v}_j\right)\right) + 2\sigma m \hat{q}_j}$$

The equilibrium fixed fees calculated above can be inserted into the market share function (1.). Then we obtain market shares as a function of the termination margin:

$$\alpha(m) = \frac{3 - 2\sigma\left(v_i + 2v_j - 2\overline{v}_i - \hat{v}_j\right) + 2m\sigma\hat{q}_j}{6\left(1 - \sigma\left(v_i + v_j - \overline{v}_i - \hat{v}_j\right)\right) + 4m\sigma\hat{q}_j}$$

The only terms being functions of *m* are *m*, $\hat{v}_j$ and $\hat{q}_j$, thus we can define constants $K_1$ and $K_2$: in order to simplify the expression:

$$\alpha(m) = \frac{3 - 2\sigma\left(v_i + 2v_j - 2\overline{v}_i - \hat{v}_j\right) + 2m\sigma\hat{q}_j}{6\left(1 - \sigma\left(v_i + v_j - \overline{v}_i - \hat{v}_j\right)\right) + 4m\sigma\hat{q}_j} = \frac{\overbrace{3 - 2\sigma\left(v_i + 2v_j - 2\overline{v}_i\right)}^{K_1} + 2\sigma\left(\hat{v}_j + m\hat{q}_j\right)}{\underbrace{6\left(1 - \sigma\left(v_i + v_j - \overline{v}_i\right)\right)}_{K_2} + 2\sigma\left(3\hat{v}_j + 2m\hat{q}_j\right)}$$

$$= \frac{K_1 + 2\sigma\left(\hat{v}_j + m\hat{q}_j\right)}{K_2 + 2\sigma\left(3\hat{v}_j + 2m\hat{q}_j\right)}$$

Differentiation of the market share function with respect to the termination margin yields:

$$\alpha'(m) = \frac{2\sigma\left(\frac{\partial\hat{v}_j}{\partial m} + \hat{q}_j + m\frac{\partial\hat{q}_j}{\partial m}\right)\left(K_2 + 2\sigma\left(3\hat{v}_j + 2m\hat{q}_j\right)\right) - 2\sigma\left(3\frac{\partial\hat{v}_j}{\partial m} + 2\hat{q}_j + 2m\frac{\partial\hat{q}_j}{\partial m}\right)\left(K_1 + 2\sigma\left(\hat{v}_j + m\hat{q}_j\right)\right)}{\left(K_2 + 2\sigma\left(3\hat{v}_j + 2m\hat{q}_j\right)\right)^2}$$

After rearranging we obtain:

$$\alpha'(m) = 2\sigma\frac{2\sigma\left(-m\frac{\partial\hat{q}_j}{\partial m}\right)\left(v_i + \overline{v} - v_j - \hat{v}_j\right) + \hat{q}_j\left(K_1 + 2\sigma\left(\hat{v}_j + m\hat{q}_j\right)\right)}{\left(K_2 + 2\sigma\left(3\hat{v}_j + 2m\hat{q}_j\right)\right)^2}$$

$$= 2\sigma\frac{2\sigma\left(-m\frac{\partial\hat{q}_j}{\partial m}\right)\left(v_i + \overline{v} - v_j - \hat{v}_j\right) + \hat{q}_j\left(3 - 2\sigma\left(v_i + 2v_j - 2\overline{v}_i - \hat{v}_j\right) + 2\sigma m\hat{q}_j\right)}{\left(6\left(1 - \sigma\left(v_i + v_j - \overline{v}_i - \hat{v}_j\right)\right) + 4\sigma m\hat{q}_j\right)^2}$$

**A.4. Proof of proposition 4**

Total welfare as a function of the termination margin can be written:

$$W(m) = v_0 - f - \frac{1}{4\sigma}\left(2\alpha^2 - 2\alpha + 1\right) + \alpha^2 v_i + (1-\alpha)^2 v_j + \alpha(1-\alpha)\left(\bar{v} + \hat{v}_j + m\hat{q}_j\right)$$

We will call this function *the second best welfare* function. In (2.) we have defined first best welfare as a function of market shares as:

$$W^*(\alpha_i) = v_0 - f - \frac{1}{4\sigma}\left(2\alpha_i^2 - 2\alpha_i + 1\right) + \alpha_i^2 v_i + (1-\alpha_i)^2 v_j + 2\alpha(1-\alpha)\bar{v}$$

With our parameter restrictions, this is a concave function with maximum for $\alpha = \alpha^*$. We can substitute for the first best welfare function in the second best function such that:

$$W(m) = W^*(\alpha_i(m)) - \alpha_i(m)(1-\alpha_i(m))\left(\bar{v} - \hat{v}_j - m\hat{q}_j\right)$$

Let $g(m) \equiv \bar{v} - \hat{v}_j - m\hat{q}_j$ denote the per subscriber deadweight loss. Note that:

$$\frac{\partial g}{\partial m} = -m\frac{\partial \hat{q}_j}{\partial m} > 0 \quad \text{for} \quad m > 0.$$

Differentiation of the second best welfare yields:

$$\frac{\partial W}{\partial m} = \frac{\partial W^*}{\partial \alpha_i}\frac{\partial \alpha_i}{\partial m} - \frac{\partial \alpha_i}{\partial m}(1-2\alpha_i)g(m) - \alpha_i(1-\alpha_i)\frac{\partial g}{\partial m}$$

Consider first the derivative at the point where *m* = 0, where *g* = 0 and *g'* = 0. Then the two last terms become zero and we have:

$$\left.\frac{\partial W}{\partial m}\right|_{m=0} = \underbrace{\frac{\partial W^*}{\partial \alpha_i}}_{+}\underbrace{\frac{\partial \alpha_i}{\partial m}}_{+} > 0$$

Thus, in the point where the termination margin is zero, the second best welfare function is an increasing function of the termination margin.

Letting the termination margin increase above zero has several effects. In particular the per subscriber deadweight loss, *g*(*m*) increases, whereas the number of subscribers being exposed to the increased price $(\alpha_i(1-\alpha_i))$ decreases due to the fact that the market share of the low cost firm is an increasing function of the termination margin.

Thus there is opposing effects on total welfare from increasing the termination margin. Consider the interval where $m > 0$, and smaller than $\tilde{m} = \{m | \alpha(m) = \alpha^*\}$. Then we have the following signs on the various terms in the welfare function:

$$\frac{\partial W}{\partial m} = \underbrace{\underbrace{\frac{\partial \alpha_i}{\partial m}}_{+}\left(\underbrace{\frac{\partial W^*}{\partial \alpha_i}}_{+} - \underbrace{(1-2\alpha_i)}_{-}\underbrace{g(m)}_{+}\right)}_{+} \underbrace{- \alpha_i(1-\alpha_i)\underbrace{\frac{\partial g}{\partial m}}_{+}}_{-}$$

i.e. the sum of one positive and one negative term. We do however know that close to $m = 0$, the positive terms dominate. Numerical simulations indicate that for reasonable parameter values, the negative dead weight loss effect dominates for sufficiently high termination margins.

## 8. References


Armstrong, M. 1998, Network interconnection in telecommunications, The Economic journal, 108, (May), 545 – 564.

Armstrong, M. 2002, The theory of access pricing and interconnection, in Cave et al. (eds) Handbook of Telecommunication economics, Vol 1, pp 295-384 (Elsevier science, Amsterdam).

Armstrong, M. 2004, Network interconnection with asymmetric networks and heterogeneous calling patterns, information Economics and policy 16, 375 – 390.

Carter, M. and J. Wright, 2003, Asymmetric Network Interconnection, Review of Industrial Organization, 22, 27 – 46.

Competition Commission, 2003, Vodafone, O2, Orange and T-Mobile: Reports on references under section 13 of the Telecommunications Act 1984 on the charges made by Vodafone, O2, Orange and T-Mobile for terminating calls from fixed and mobile networks, available at: http://www.competition-commission.org.uk/rep_pub/reports/2003/index.htm.

Correa, L, 2003, Natural or Unnatural Monopolies in UK Telecommunications? Queen Mary University, Dept of economics, working paper no 501, Available at: http://www.econ.qmul.ac.uk/papers/doc/wp501.pdf.

De Bijl, P and M. Peitz, 2002, Regulation and Entry into Telecommunications Markets, Cambridge University Press.

Dessein, W 2003, Network competition in nonlinear pricing, Rand Journal of Economics, 34, (2), 593 – 611.

Gans, JS. and SP. King, 2001, Using 'bill and keep' to interconnect arrangements to soften network competition, Economic Letters, 71, 413 – 420.

Laffont, J.-J., P. Rey, J. Tirole, 1998a, Network Competition: I. Overview and non-discriminatory pricing, RAND Journal of Economics, 29, (1) 1 – 37.

Laffont, J.-J., P. Rey, J. Tirole, 1998b, Network Competition: II. Price discrimination, RAND Journal of Economics, 29, (1) 38 – 58.

Laffont, J-J and J. Tirole, 2000, Competition in telecommunications, The MIT press, Cambridge, Mass.

Peitz, M. 2002, Asymmetric access price regulation in telecommunications markets, mimeo, available at http://yoda.eco.auckland.ac.nz/iep/papers/peitz_2002.pdf.

Peitz, M. 2005, Asymmetric access price regulation in telecommunications markets, European Economic Review, 49, 341 – 358.

Peitz, M., T. Valletti and J. Wright, 2004, Competition in telecommunications: an introduction, Information Economics and Policy, 16, 315–321.

PTS, 2004, LRIC prismetod för terminering av röstsamtal i mobilnät, 5th of July, available at: http://www.pts.se/Archive/Documents/SE/Prismetod_MLRIC.pdf.