

# Termination rates and fixed mobile substitution\*

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*Abstract:*

*In this paper we consider fixed mobile substitution in a model of mobile network competition. We demonstrate that the profit neutrality result from the standard model of network competition (Laffont Rey Tirole 1998a) holds if the number of mobile subscribers is given. Thus the mobile termination rate does not have an impact on profits in the mobile sector in a mature market where all consumers are hooked up to a mobile network and fixed mobile substitution results in subscribers disconnecting from the fixed network. However, if fixed mobile substitution results in an increased number of subscribers in mobile networks, then the mobile termination rate will have an impact on profits in the mobile sector. The implication of the analysis is that there is a case for regulating mobile termination rates in the growth phase, whereas there is less need for regulation in mature markets characterized by a stable size of the mobile sector. This seems to be the opposite of the approach taken by regulators in Europe, where mobile firms were free to set termination rates in the growth phase and regulation of termination rates is introduced once markets mature.*

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## 1. Introduction

The demand for telephony services is derived from the need for communication. This communication will take place either from a fixed or a mobile phone. Intuitively one would accordingly expect fixed and mobile telephony to be substitutes. The implications of fixed mobile substitution on the regulation of mobile termination rates should thus be taken into account. To do this we extend the model of network competition due to Laffont Rey and Tirole (1998a) by adding a fixed network. We show that a necessary condition for the profit neutrality of mobile termination rates is that the size of the mobile sector is given. If, on the other hand, the size of the mobile sector is elastic, i.e. the growth phase of mobile telephony, then the mobile firms can increase their profits by raising the termination rate.

There is empirical evidence supporting the notion that fixed and mobile services are substitutes, see Cadima and Barros (2000) and Gruber and Verboven (2001b). There are however also examples of studies finding that fixed and mobile services are complements, e.g. Gruber and Verboven (2001a). According to a review article by Gans King and Wright (2005), the measured complementarity may be explained by network effects in the early phases with relatively few mobile subscribers. They argue that fixed and mobile services are likely to be substitutes in the mature phases of the life cycle of mobile telephony.

In order to obtain telephony connectivity, networks have to be interconnected, i.e. two-way access is required. Under the widespread principle of calling party pays, the interconnecting networks both buy and sell termination services. The termination service is accordingly an input to other phone companies, both fixed and mobile. The literature on interconnection of symmetric networks, i.e. mobile to mobile termination, starting with the papers by Laffont Rey and Tirole (1998a) as well as the paper by Armstrong (1998) is inconclusive with respect to whether interconnecting, competing firms have incentives to set termination rates above the welfare maximizing level. The results depend upon the pricing structure in the downstream market. The result that interconnected networks do not necessarily have incentives to set termination rates above

the welfare maximizing level is in contrast to the results in the literature on fixed to mobile termination. Fixed companies are typically former monopolies and they are subject to regulation. The argument is that fixed companies do not have any bargaining power when negotiating termination rates due to regulation. Thus mobile firms have incentives to raise termination rates above the welfare maximizing level in order to extract profits from the fixed sector (see e.g. Armstrong 2002 p. 339 and Wright 2002). A review of results on fixed to mobile termination can be found in de Bijl et al. (2004). None of these papers consider the implications of fixed mobile substitution.

The results cited above for mobile to mobile termination and fixed to mobile termination respectively are not derived within the same modelling frameworks. From the outset it is accordingly not evident whether the differences in conclusions with respect to the incentives to set termination rates survive within a model studying both issues simultaneously.

We consider a model where competing mobile firms set a termination rate that also applies to the fixed network. In the paper we demonstrate that the mobile firms pass termination revenues on to their subscribers due to competition in the mobile sector. On the one hand, the passing on of revenues implies that the profit neutrality result from the network competition literature (first derived in Laffont Rey and Tirole 1998a) still is valid given that the number of subscribers on mobile networks is given. On the other hand, the passing on of revenues implies that if the demand for mobile subscriptions is elastic, i.e. consumers substitute mobile for fixed, then mobile firms have incentives to raise their termination rate above the welfare maximizing level in order to attract more subscribers. In a mobile market in growth (“emerging market”) mobile firms can accordingly increase profits by raising the termination rate, whereas in a saturated mobile market (“mature market”) the profit neutrality of termination rates holds. To our knowledge, our analysis of fixed mobile substitution as well as multihoming is novel.

In the present paper we allow for multihoming, we analyze the effect of fixed and mobile services being substitutes in consumption and we assume non discriminatory termination rates, i.e. that mobile firms charge the same termination fee regardless of whether the traffic is originated in a fixed network or a mobile network. To fix ideas we can think of

two competing mobile firms negotiating a reciprocal termination fee,<sup>1</sup> and then due to the non discrimination assumption apply the same termination rate towards the fixed network. Interconnection is illustrated below:

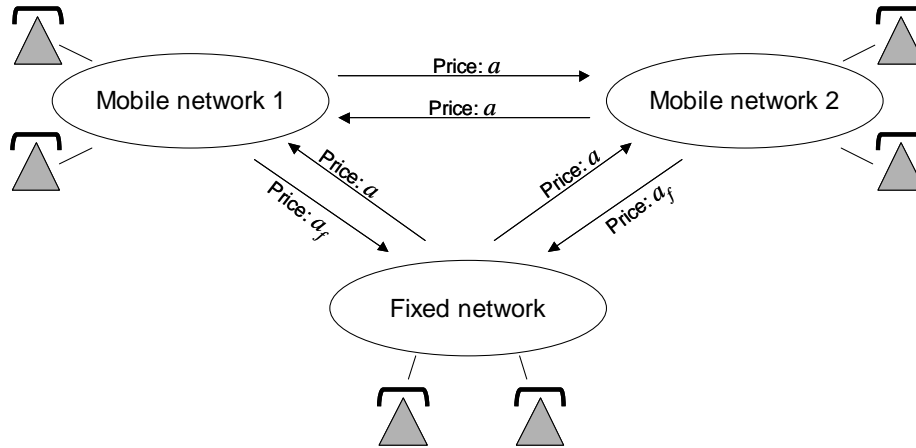


Figure 1 Termination and non discrimination

In the illustration above the reciprocal mobile to mobile termination fee is denoted  $a$ . The fixed network has to pay the same termination fee to both mobile networks. Furthermore,  $a_f$  denotes the termination fee charged by the fixed network. In European markets the mobile to mobile termination fees are typically 5 to 10 times higher than the termination fee charged by fixed networks. This is contrary to the US where reciprocal fixed to mobile termination rates are observed.

The assumed non discrimination is common among mobile operators (see de Bijl et al. 2004 p 108). This assumption is critical for our results and can be motivated in two ways; due to regulation and/or arbitrage. In many jurisdictions (e.g. most of the EU) non discrimination is mandatory on the termination market; i.e. mobile firms are not allowed to price discriminate based on whether the calls are originated in a fixed or a mobile network. Furthermore, suppose discrimination is allowed, then calls from the network

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<sup>1</sup> We assume that the reciprocal mobile termination fees are determined in negotiations between mobile firms, or by the regulator. The outcome of (symmetric) mobile firms setting termination rates non-cooperatively is also reciprocity as demonstrated by Gans and King (2001). However, when termination rates are determined non-cooperatively, Gans and King demonstrate that equilibrium level is high relative to the outcome under cooperative determination of termination rates. In many jurisdictions, regulators set the mobile termination rates. Regulated rates are reciprocal in some countries and non reciprocal in others.

facing the high termination charge can be routed via a network facing a low termination fee and thus the price discrimination is bypassed.<sup>2</sup>

The current paper is organized as follows. In section 2 of the paper we reproduce the reference model, i.e. a model of network competition with two part tariffs in the downstream market. Within the reference model we demonstrate that the profit neutrality result also holds if we add a fixed network of exogenous size. In section 3 of the paper we present our model of network competition and fixed mobile substitution. Then we proceed in section 4 by analyzing two types of market equilibriums, full multihoming and full singlehoming respectively. Finally in section 6 we conclude the paper. In appendix B we characterize two more possible equilibriums.

## **2. Adding an exogenous fixed network to the standard model of network competition**

Laffont Rey and Tirole (1998a) presented a model with Hotelling type differentiation between two mobile firms where the networks charge two part tariffs. A striking result from this model is the profit neutrality of reciprocal termination charges. In this section of the paper we will add a fixed network of exogenous size to a model of the Laffont Rey Tirole type. This model will serve as a benchmark and a motivation for the models where we take fixed mobile substitution into account.

The mobile market is assumed to have a given size normalized to unity. Subscribers are assumed to single home, calling patterns are assumed to be uniform and the two competing mobile networks charge two part tariffs. The networks are differentiated à la Hotelling. Network preferences are assumed to be uniformly distributed on the unit line, and the differentiation is assumed exogenous. The utility for a subscriber of type  $x$  connected to network  $i$  is given by:

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<sup>2</sup> *In the industry this type of bypass is called refiling. In Norway we had a case of refiling in 1999 – 2000, because the mobile firms had differentiated “domestic” and “international” termination fees. The international termination fees were below the domestic and calls were routed via Sweden in order to be subject to the lower international termination fee. Due to this bypass the differentiation of termination fees was abandoned.*

$$V_i + v_0 - \frac{1}{2\sigma} |x - x_i|$$

Where  $V_i = \omega(p_i) - T_i$  and  $\omega(p_i) = \max[u(q_i) - pq_i]$ , is the net utility from network subscription,  $q$  is the number of calls being made,  $u(q)$  is the utility from making calls and  $T_i$  is the fixed part of the two part tariff (subscription fee). Furthermore,  $v_0$  is the stand alone value of subscription to a mobile network,  $x_1 (=0,1)$  is the locus of the two networks. The disutility from not consuming an offering of the preferred type is  $\frac{1}{2\sigma}$ . Market shares,  $\alpha_i$ , are determined by the subscriber being indifferent as to the two offerings, thus:

$$\alpha_i = \frac{1}{2} + \sigma(V_i - V_j)$$

In this section we simplify the modelling by assuming that fixed subscribers make calls to the mobile subscribers, but mobile subscribers do not make calls to the fixed network.<sup>3</sup>

We assume that the volume of incoming calls per mobile subscriber from the fixed network is a decreasing (non increasing) function of the termination rate  $a$ :  $q_0 = q_0(a)$ .

Given a market share of  $\alpha_i$ , mobile firm  $i$  will then receive incoming F2M traffic:

$\alpha_i q_0(a)$ . Then we can write profit of mobile firm  $i$ :

$$\pi_i = \alpha_i (T_i + q_0(a)(a - c_0) + q(p_i)(p_i - c - \alpha_j(a - c_0))) + \alpha_i \alpha_j q(p_j)(a - c_0).$$

Where  $c$  is the unit price of producing a call, and  $c_0$  is the cost of terminating a call.<sup>4</sup> We

use net utility as the strategic variable. Thus we substitute:  $T_i = \omega(p_i) - V_i$ . Furthermore

we define  $R(a) \equiv q_0(a)(a - c_0)$  as the per subscriber net revenue from incoming calls

from the fixed network. Then the profit function can be written:

$$\pi_i = \alpha_i (\omega(p_i) - V_i + R(a) + q(p_i)(p_i - c - \alpha_j(a - c_0))) + \alpha_i \alpha_j q(p_j)(a - c_0)$$

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<sup>3</sup> This assumption simplifies the modelling without changing the main insights, later on in the paper we will consider a richer model.

<sup>4</sup> In contrast to the Laffont Rey Tirole model, we do not include fixed cost per subscriber nor volume dependent costs in the transmission network. These parameters are not the focus of the current paper.

This function is to be maximized with respect to marginal price  $p$  and net utility  $V$ . For given net utilities market shares are given and we obtain the by now familiar result:

$$(1.) \quad \frac{\partial \pi_i}{\partial p_i} = 0 \Leftrightarrow p_i = c + \alpha_j(a - c_0)$$

Net profit from carrying traffic is accordingly zero. Consider next:  $\frac{\partial \pi_i}{\partial V_i} = 0$ , substituting

for usage pricing at marginal cost and then solving with respect to  $V_i$  yields:

$$V_i = -\frac{\alpha_i}{\sigma} + \omega(p_i) + R(a) + \alpha_i q(p_i)(a - c_0) + q(p_j)(1 - 2\alpha_i)(a - c_0)$$

In a symmetric equilibrium ( $\alpha_i = 0.5$ ,  $p_i = p_j$ ), the expression above simplifies to:

$$(2.) \quad V_i = -\frac{1}{2\sigma} + \omega(p_i) + R(a) + \frac{1}{2}q(p_i)(a - c_0)$$

Optimal pricing and optimal net utility can be substituted back into the profit function and then we obtain:

$$\pi_i = \frac{1}{4\sigma}$$

Hence profits are unaffected by the termination charge. Under Hotelling competition, any revenue (loss) from termination is passed on to consumers (see 2.).

Prior to the market game analyzed above, the mobile firms may negotiate a reciprocal termination charge. Since profits are unaffected, mobile firms are indifferent with respect to the level of this termination charge and they will (weakly) prefer to set it at the welfare maximizing level. A positive margin on termination to mobile networks will thus result in transfers from fixed to mobile subscribers.

Wright (2002) as well as Armstrong (2002) analyze the effects of mobile termination rates in a model with an exogenous fixed network of the type considered in above. In contrast to our result, both conclude that the mobile sector can increase profits by raising the termination rate. Wright (2002) is however also deriving a similar result to ours, but he argues that it is a special case. He focuses on cases where mobile firms set termination rates individually towards the fixed sector and/or situations with a less competitive

mobile sector. Armstrong (2002), (p. 337 and onwards) also analyzes the implications of mobile firms setting fixed to mobile termination rates individually, but he assumes perfect competition in the mobile sector.

### **3. A model of network competition and fixed mobile substitution**

In this section we will describe the extensions made to the model in order to analyze the effects of fixed mobile substitution.

#### **3.1. Preferences in the fixed mobile dimension**

As argued in the introduction to this paper, there is reason to believe that fixed and mobile services are substitutes.

Furthermore, in the market one can observe some consumers singlehoming in mobile, others singlehoming in fixed and some consumers multihoming in the sense that they subscribe to both fixed and mobile services. Taking Norway as an example, the number of mobile subscriptions exceeds the number of inhabitants<sup>5</sup>, and 83% of all households are hooked up to the fixed network. Most people in Norway are accordingly multihoming.<sup>6</sup> Furthermore, there seems to be a trend that consumers disconnect from the fixed network and become singlehomers in mobile. This phenomenon is called fixed mobile substitution.<sup>7</sup> Some predicts that this development will accelerate.

Our modelling of preferences in the fixed mobile dimension takes as its starting point that consumers differ in the degree that they are on the move. Some consumes are at fixed locations almost all the time and thus close to a fixed phone. Such consumers are assumed to have relatively low willingness to pay for mobile services. This is in contrast to people being mostly on the move. Such consumers have to rely on the mobile phone to

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<sup>5</sup> According to the Norwegian Post and Telecommunications Authority there were 104 mobile subscribers per 100 inhabitants in Norway in 2005.

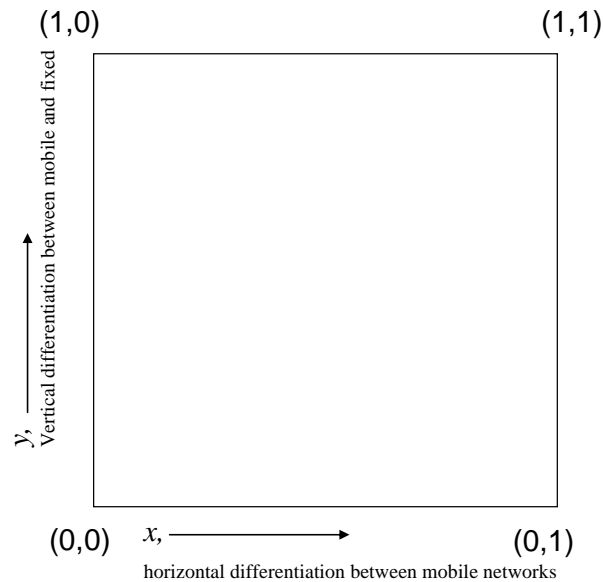
<sup>6</sup> June 2005, source: NPT 2005

<sup>7</sup> See also ITU 2003



be able to communicate, thus they have relatively high willingness to pay for being connected to a mobile service. Since fixed services typically are considerably cheaper than mobile services some consumers may even find that they are best off by multi-homing, i.e. by placing calls in the mobile network only when they are away from a fixed phone.

In our model the total number of customers is normalized to unity. We let every consumer be characterized by two parameters,  $(x, y)$  uniformly and independently distributed on the unit square.  $x$  measures preferences in the mobile dimension, i.e. the locus of preferences on the Hotelling line in the same way as in the model reviewed above.  $y$  is a measure of preferences in the fixed mobile dimension. This parameter can be given a straightforward interpretation, a consumer of type  $(x, y)$  is on the move and thus away from a fixed phone a fraction of time equal to  $y$ . The unit square is illustrated below:



Consumers with taste parameters in the upper left corner are likely to connect to mobile network 1, consumers with taste parameters in the lower half of the square are likely to connect to the fixed network, etc.

Note that differentiation between the two mobile services is assumed to be purely horizontal whereas the differentiation in the fixed to mobile dimension is purely vertical. Vertically differentiated mobile networks were analyzed by Carter and Wright 2003, as

well as by Peitz 2005. As for the fixed mobile dimension, it seems reasonable to assume that fixed and mobile services are vertically differentiated since a mobile phone gives the opportunity of communication in fixed locations as well as the opportunity to communicate while being on the move. Some may however argue that there is an element of horizontal differentiation since mobile services are characterized by radiation, poorer sound quality and hassle related to charging batteries. Thus, alternatively one could model horizontal differentiation in the fixed to mobile dimension as well. Altering the modelling in the present paper by assuming Hotelling type horizontal differentiation in the fixed to mobile dimension yields qualitatively identical results.

A consumer of type  $(x, y)$  single homing on mobile network  $i$  is assumed to receive utility:

$$(3.) \quad \omega(p_i) - \frac{1}{2\sigma}|x - x_i| + g(y) - T_i$$

The only difference from the utility function we considered in section 2 of the paper is that we have substituted the fixed term  $v_0$  for a type dependent term,  $g(y)$  capturing the preferences for mobile services.<sup>8</sup> We assume that  $g'(y) > 0$ .

The fixed network is assumed to be regulated in both the up- and downstream market, and the usage price on fixed is assumed to be an increasing function of the termination rate that the fixed network has to pay to mobile networks. The fixed network charges a single two-part tariff without discriminating between fixed to fixed and fixed to mobile traffic. Thus the indirect utility of a subscriber singlehoming in the fixed network (notation related to the fixed network has subscript  $f$  throughout the paper) can be written:

$$(4.) \quad V_f(a) = \omega_f(a) - T_f, \quad V'_f \leq 0$$

Finally, multihoming subscribers will place calls from the fixed network as well as calls from one of the mobile networks. These calls are terminated in fixed and mobile networks proportionally to the respective market shares in the same way as assumed in

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<sup>8</sup> Recall that the parameter  $y$  is measuring the fraction of time being away from a fixed phone. Thus, instead of adding a mobility premium to a mobile service one, an equivalent approach is to instead add a cost depending on  $y$  to the utility from fixed subscriptions.

the reference model considered in section 2. A multihoming subscriber is assumed to derive gross utility from making calls;  $U(\hat{q}, \hat{q}_f)$ , where  $\hat{q}$  is the quantity of calls originated in the mobile network and  $\hat{q}_f$  is the quantity of calls originated in the fixed network.<sup>9</sup> The multihoming consumer will optimize call consumption resulting in an indirect utility function;  $\hat{\omega}(p, a) = \max_{\hat{q}_i, \hat{q}_f} [U(\hat{q}, \hat{q}_f) - p\hat{q} - p_f\hat{q}_f]$ . Thus the utility of a subscriber of type

$(x, y)$  connected to mobile network  $i$  and to the fixed network is given by:

$$(5.) \quad \hat{\omega}(p_i, a) - \frac{1}{2\sigma} |x - x_i| + \hat{g}(y) - T_i - T_f$$

i.e. the sum of the following terms: indirect utility from making calls, the disutility from not consuming the most preferred mobile brand, the type dependent utility from being a multihomer, and finally the fixed fees on the fixed network as well as a mobile network. Note that the type dependent utility from subscribing to mobile services for multihoming consumers  $\hat{g}(y)$  may differ from the benefit of singlehoming in a mobile network. In the same way as for singlehomers, the willingness to pay for mobility is an increasing function of consumer type;  $\hat{g}'(y) > 0$ .

Call demand functions for a subscriber multihoming in mobile network  $i$  and the fixed network are given by:

$$\hat{q}_i = q_i(p_i, a) = -\frac{\partial \hat{\omega}(p_i, a)}{\partial p_i}, \quad \frac{\partial \hat{q}_i}{\partial p_i} < 0$$

$$\hat{q}_f = q_f(a, p_i) \quad , \quad \frac{\partial \hat{q}_f}{\partial a} \leq 0$$

Finally, we assume that traffic originated in fixed and traffic originated in mobile are substitutes, i.e.:

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<sup>9</sup> Throughout the paper, variables and functions with a hat are related to multihoming consumers, i.e.  $\hat{q}$  is the quantity of mobile to mobile calls for multihoming subscribers and  $q$  is the quantity of mobile to mobile calls made by singlehoming consumers.

$$\frac{\partial q_f}{\partial p_i} \geq 0$$

Note that this assumption *per se* not is contradictory to fixed and mobile services being complements at an aggregate level. Consider the following example, in an uncovered market, a reduction in the fixed usage price will have two opposing effects; a direct substitution effect and an indirect network effect. The indirect network effect is due to some unsubscribing consumers joining the fixed network. This will again result in more potential communication partners, resulting in increased mobile usage. The aggregate effect may be that the network effect dominates the substitution effect such that fixed and mobile services appear to be complements.

### **3.2. Timing of the games**

In this paper we endogenize the homing decisions made by subscribers, i.e. the choice between:

- a. Singlehoming in mobile
- b. Multihoming in mobile and fixed
- c. Singlehoming in fixed

In order to simplify the modelling we will assume that this homing decision is made prior to consumers learning their preferences over mobile services (the  $x$  parameter, location on the Hotelling line). By doing so the strategic interaction between the two mobile firms will be directly comparable to the reference model.<sup>10</sup>

This timing structure is introduced in order to simplify the modelling, but it can be motivated by assuming that there is a search cost related to learning the characteristics of the mobile services. Suppose consumers only are willing to incur the cost of learning characteristics of the mobile services after the homing decisions are made; i.e. consumers first make their homing decision, and if the decision is to join a mobile network they start searching for the preferable offering.

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<sup>10</sup> Without this assumption consumers located in the middle of the Hotelling line would be more likely to choose the fixed network. Then the strategic interaction in the standard Hotelling model changes. In particular, the change in market share as a result of changing prices takes a different (and more complicated) form.

We assume that mobile termination rates are determined prior to the game we are analyzing. The termination rates are either a result of negotiations between the mobile firms or from regulation. Thus we consider the following multistage game:

1. Consumers make their homing decision
2. Consumers homing in mobile learn their preferences in the mobile dimension, i.e. the location of their preferences on the Hotelling line
3. Mobile firms compete in two-part tariffs

The outcome of stage 1 of the game may be a corner-solution where either all subscribers singlehome in fixed or mobile; alternatively we obtain a corner solution where all consumers multihome. The stage 1 outcome may also be an interior solution where some subscribers choose singlehoming in fixed or mobile, others choose multihoming. The game is solved by backward induction. Thus, in principle, one has to consider all possible stage 1 outcomes. If all subscribers singlehome in mobile we are however back to the reference model. Furthermore, if all subscribers singlehome in fixed, the effect of termination rates vanishes since all traffic will be internal in the fixed network. These outcomes are not interesting in our context. Our focus is the implications of fixed mobile substitution and it turns out that it is sufficient to analyze two outcomes from stage 1 of the game: 1) The outcome where all consumers multihome in fixed and mobile, and 2) the outcome characterized by singlehoming, where subscribers are either on fixed or on mobile. In appendix B we also look into two other stage 1 outcomes, namely: B.1.) “An emerging market” where all subscribers are on fixed and some subscribers multihome and B.2.) “A mature market” where all subscribers are on mobile, and some multihome in fixed and mobile.

### **3.3.     *The homing decision***

The offered mobile services are located on the extremes of the unit line. Consumer preferences are uniformly distributed; thus expected traveling distance is 0.25. The expected disutility from not consuming the most preferred variety is accordingly

$$\frac{1}{2\sigma} \cdot \frac{1}{4} = \frac{1}{8\sigma}.$$

Thus at stage 1 of the game a subscriber will choose to singlehome in fixed if this homing decision is preferred over both singlehoming in mobile (*i*) and multihoming (*ii*):

$$i) \quad \omega(p_f) - T_f \geq \omega(p_i) - \frac{1}{8\sigma} + g(y) - T_i$$

$$ii) \quad \omega(p_f) \geq \hat{\omega}(p_i, a) - \frac{1}{8\sigma} + \hat{g}(y) - T_i$$

Similarly, a consumer will, at stage 1, prefer to singlehome in mobile over both singlehoming in fixed (*i*) and multihoming (*ii*):

$$i) \quad \omega(p_i) - \frac{1}{8\sigma} + g(y) - T_i \geq \omega(p_f) - T_f$$

$$ii) \quad \omega(p_i) + g(y) \geq \hat{\omega}(p_i, a) + \hat{g}(y) - T_f$$

Finally, a consumer will prefer to multihome if:

$$i) \quad \hat{\omega}(p_i, a) - \frac{1}{8\sigma} + \hat{g}(y) - T_i \geq \omega(p_f)$$

$$ii) \quad \hat{\omega}(p_i, a) + \hat{g}(y) - T_f \geq \omega(p_i) + g(y)$$

Note that depending upon prices, the shape and locus of the indirect utility functions for singlehoming and multihoming consumers  $\omega, \hat{\omega}$  as well as the shape and locus of the additional utility from mobility  $g, \hat{g}$ , we may end up in scenarios where all subscribers make the same homing decisions or we may end up in mixed situations. As indicated above we will focus on two outcomes: 1) The outcome where all subscribers multihome, and 2) the outcome where some subscribers singlehome in mobile and others singlehome in fixed. In the appendix we briefly look into other outcomes as well.

#### 4. All subscribers multihome

In this section of the paper we will make the extreme assumption that the outcome of stage 1 of the game is that all subscribers choose to multihome. This is the case if at stage 1 of the game, all subscribers prefer multihoming over singlehoming in mobile; i.e. for all  $y \in [0,1]$ ,  $\hat{\omega}(p_i, a) + \hat{g}(y) - T_f \geq \omega(p_i) + g(y)$  and they also prefer multihoming over singlehoming in fixed; i.e. for all  $y \in [0,1]$ ,  $\hat{\omega}(p_i, a) - \frac{1}{8\sigma} + \hat{g}(y) - T_i \geq \omega(p_f)$ .

#### 4.1. Market shares

Market shares of the two mobile firms are determined in the standard Hotelling way:

$$\alpha_i = \frac{1}{2} + \sigma(\hat{w}(p_i, a) - \hat{w}(p_j, a) - T_i + T_j)$$

#### 4.2. Stage 3

In the following we will without loss of generality focus on mobile firm 1, and to save notation we write  $\alpha = \alpha_1$ . Retail profits are:

$$\pi_R = \alpha[T_1 + \hat{q}(p_1, a)(p_1 - c - (a - c_0)(1 - \alpha))]$$

i.e. market share multiplied with the fixed fee plus profits on traffic. Note that we have made one important simplification in this section; when consumers originate a call in a mobile network, their call will also be terminated in a mobile network.<sup>11</sup>

Consider then profits in the wholesale market. It consists of three elements:

1. Calls from mobile network 2 terminated in network 1:

$$(a - c_0)\alpha(1 - \alpha)\hat{q}(p_2, a)$$

2. Calls from multihoming subscribers in network 1 originated in fixed, terminating in mobile network 1:

$$(a - c_0)\alpha_1^2 \hat{q}_f(p_1, a)$$

3. Finally calls from multihoming subscribers in network 2 originated in fixed, terminating in mobile network 1

$$(a - c_0)\alpha_1(1 - \alpha_1)\hat{q}_f(p_2, a)$$

We substitute  $T_1 = \hat{w}(p_1, a) - V_1$ , collect terms and obtain the following profit function:

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<sup>11</sup> Since all subscribers multihome, all subscribers have a mobile phone, and since we assume that there is no price discrimination between traffic terminated in fixed and mobile, subscribers will (weakly) prefer to terminate calls on mobile phones.

$$\pi_1 = \max_{p_1, V_1} \left[ \alpha(\hat{w}(p_1, a) - V_1 + \hat{q}(p_1, a)(p_1 - c - (a - c_0)(1 - \alpha))) \right. \\ \left. + (a - c_0)(\alpha(1 - \alpha)\hat{q}(p_2, a) + \alpha^2\hat{q}_f(p_1, a) + \alpha(1 - \alpha)\hat{q}_f(p_2, a)) \right]$$

This function is to be maximized with respect to net utility  $V_1$  and usage price  $p_1$ . The first order conditions are:

$$\frac{\partial \pi_1}{\partial p_1} = \alpha \frac{\partial \hat{q}(p_1, a)}{\partial p_1} (p_1 - c - (a - c_0)(1 - \alpha)) + \alpha^2 (a - c_0) \frac{\partial \hat{q}_f(p_1, a)}{\partial p_1} = 0$$

$$\frac{\partial \pi_1}{\partial V_1} = -\alpha + \sigma(\hat{w}(p_1, a) - V_1 + \hat{q}(p_1, a)(p_1 - c - (a - c_0)(1 - 2\alpha))) \\ + (a - c_0)\sigma((1 - 2\alpha)\hat{q}(p_2, a) + 2\alpha\hat{q}_f(p_1, a) + (1 - 2\alpha)\hat{q}_f(p_2, a)) = 0$$

Consider first optimal usage price:

$$\frac{\partial \pi_1}{\partial p_1} = 0 \Leftrightarrow p_1 = c + (a - c_0)(1 - \alpha) - \alpha(a - c_0) \frac{\frac{\partial \hat{q}_f(p_1, a)}{\partial p_1}}{\frac{\partial \hat{q}(p_1, a)}{\partial p_1}}$$

**Proposition 1:**

As compared to a model without fixed mobile substitution, multihoming and fixed mobile substitution results in:

- An upward adjustment of usage prices if the termination margin is positive
- A downward adjustment of usage prices if the termination margin is negative

**Proof:**

The result follows directly from fixed and mobile being substitutes,  $\frac{\partial \hat{q}_f(p_1, a)}{\partial p_1} \geq 0$ ;

$$\text{sign} \left[ -\alpha(a - c_0) \frac{\frac{\partial \hat{q}_f(p_1, a)}{\partial p_1}}{\frac{\partial \hat{q}(p_1, a)}{\partial p_1}} \right] = \text{sign}[(a - c_0)] \text{ QED}$$



This result is in contrast to the result on usage pricing in the benchmark model in section 2 of this paper. Mobile firms deviate from pricing at perceived marginal cost when traffic originated in the fixed network is a substitute for traffic in the mobile network. This adjustment is increasing in the cross price effect and decreasing in the own price effect.

Assume there is a positive termination margin, and take pricing at perceived marginal cost as a starting point, then a marginal increase in the usage price will result in two effects: 1) An increase in wholesale profits since consumers will increase the number of calls originated in the fixed network resulting in increased termination revenues. 2) A loss in retail profits since the subscription fee will have to be reduced in order to compensate for the loss in consumer surplus due to the increased usage price. The wholesale effect is a first order effect, whereas the effect on retail profits is a second order effect. When there is a termination margin the mobile firms will accordingly increase their profits by raising usage prices above the perceived marginal cost. A negative termination margin will result in the opposite adjustment in usage prices.

Define:

$$\delta \equiv -\frac{\frac{\partial \hat{q}_f(p_1, a)}{\partial p_1}}{\frac{\partial \hat{q}(p_1, a)}{\partial p_1}} > 0$$

In order to simplify the modelling, we will assume that  $\delta$  is a constant.<sup>12</sup> The condition for optimal usage price can then be written:

$$p_1 = c + (1 - \alpha)(a - c_0) + \delta\alpha(a - c_0)$$

**Proposition 2:**

Reciprocal termination rates are profit neutral under full multihoming.

**Proof:**

Inserting optimal usage price in the condition for optimal net utility yields:

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<sup>12</sup> *Fulfilled for linear demand functions*

$$\begin{aligned}
0 &= -\alpha + \sigma(\hat{\omega}(p_1, a) - V_1 + \hat{q}(p_1, a)\alpha(1 + \delta)(a - c_0)) \\
&\quad + (a - c_0)\sigma((1 - 2\alpha)\hat{q}(p_2, a) + 2\alpha\hat{q}_f(p_1, a) + (1 - 2\alpha)\hat{q}_f(p_2, a)) \\
V_1 &= -\frac{\alpha}{\sigma} + \hat{\omega}(p_1, a) + (a - c_0)(\alpha(1 + \delta)\hat{q}(p_1, a) + 2\alpha\hat{q}_f(p_1, a) + (1 - 2\alpha)(\hat{q}(p_2, a) + \hat{q}_f(p_2, a)))
\end{aligned}$$

In any symmetric equilibrium market shares = 0.5, thus we obtain equilibrium net utility:

$$V_1 = -\frac{1}{2\sigma} + \hat{\omega}(p_1, a) + (a - c_0)\left(\frac{1}{2}(1 + \delta)\hat{q}(p_1, a) + \hat{q}_f(p_1, a)\right)$$

Finally, inserting equilibrium-, market shares, net utility and usage prices (where  $p_1 = p_2$ ) into the profit function yields:

$$\begin{aligned}
\pi_1 &= \frac{1}{2} \left( \hat{\omega}(p_1, a) - \left\{ -\frac{1}{2\sigma} + \hat{\omega}(p_1, a) + (a - c_0)\left(\frac{1}{2}(1 + \delta)\hat{q}(p_1, a) + \hat{q}_f(p_1, a)\right) \right\} \right) \\
&\quad + \frac{1}{2} \hat{q}(p_1, a) \left( \{c + (1 - \alpha)(a - c_0) + \delta\alpha(a - c_0)\} - c - (a - c_0)\frac{1}{2} \right) \\
&\quad + (a - c_0)\frac{1}{4}(\hat{q}(p_2, a) + \hat{q}_f(p_1, a) + \hat{q}_f(p_2, a)) = \frac{1}{4\sigma}
\end{aligned}$$

QED

Under full multihoming mobile firms cannot increase their profits by using the termination rate as a collusive device. The mechanism driving this result is the same as in the benchmark model considered in section 2. Termination revenues are passed on to consumers. Thus a margin on mobile termination will result in a reduction of the mobile fixed fees. This reduction is exactly equal to the generated profits on termination. These profits are partly from fixed to mobile traffic and partly from incoming mobile to mobile traffic.

## 5. Singlehoming

In this section we will assume that all subscribers, at stage 1 of the game, have chosen to singlehome; i.e. all subscribers  $\{x, y\}$  are either characterized by preferring singlehoming in mobile over multihoming,  $\hat{\omega}(p_i, a) + \hat{g}(y) - T_f \leq \omega(p_i) + g(y)$  or they are

characterized by  $\hat{\omega}(p_i, a) - \frac{1}{8\sigma} + \hat{g}(y) - T_i \leq \omega(p_f)$ , i.e. preferring singlehoming in fixed over multihoming.

Thus in this section we assume that at stage 1 of the game a fraction of subscribers  $m$ , where  $m \in (0,1)$  has chosen to singlehome in a mobile network, and a fraction  $(1 - m)$  has chosen to singlehome in the fixed network.

### 5.1. Stage 3 of the game

A consumer singlehoming in the fixed network has utility  $V_f = \omega_f(p_f) - T_f$ , whereas a subscriber singlehoming in mobile network  $i$  has net utility  $V_i = \omega(p_i) - T_i$ . The two mobile networks are competing over the  $m$  customers in the mobile segment. The market share of mobile firm 1 of the mobile segment is accordingly:

$$\alpha = \frac{1}{2} + \sigma(\omega(p_1) - \omega(p_2) - T_1 + T_2)$$

Retail profit of firm 1 is now:

$$\pi_R = m\alpha[T_1 + q(p_1)(p_1 - c - (a - c_0)(1 - \alpha)m - (a_f - c_0)(1 - m))]$$

The difference from the formulation in the previous section is that all market shares are scaled by  $m$ , and we have included a term capturing the cost of traffic terminated in the fixed network,  $(a_f - c_0)$  where  $a_f$  denotes the regulated termination fee in the fixed network.<sup>13</sup> We assume  $a_f \leq c_0$ , i.e. that the regulated termination fee in the fixed network is no larger than the cost of terminating calls in the mobile network. Consider then profits in the wholesale market. It consists of two elements:

1. Calls from mobile network 2 terminated in network 1:

$$(a - c_0)m^2\alpha(1 - \alpha)q(p_2)$$

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<sup>13</sup> In the previous section we considered only multihoming consumers and then we simplified the modelling by assuming that all mobile originated traffic also terminated in mobile phones. Under singlehoming some traffic has to be terminated in fixed in order to allow for calls to the group of customers singlehoming in fixed.

2. Calls originated in the fixed network terminated in mobile network 1:

$$(a - c_0)m(1 - m)\alpha_1 q_0(a)$$

As in the previous section we substitute  $T_1 = \omega(p_1) - V_1$ , collect terms and obtain the following profit function:

$$\pi = \max_{p_1, V_1} \left[ m\alpha(\omega(p_1) - V_1 + q(p_1)(p_1 - c - (a - c_0)(1 - \alpha)m - (a_f - c_0)(1 - m))) \right. \\ \left. + m\alpha(a - c_0)((1 - \alpha)m q(p_2) + (1 - m)q_0(a)) \right]$$

Maximization with respect to usage price and net utility yields:

$$\frac{\partial \pi}{\partial p_1} = m\alpha q'(p_1)(p_1 - c - (a - c_0)(1 - \alpha)m - (a_f - c_0)(1 - m)) = 0$$

$$\frac{\partial \pi}{\partial V_1} = -m\alpha + m\sigma(\omega(p_1) - V_1 + q(p_1)(p_1 - c - (a - c_0)(1 - 2\alpha)m - (a_f - c_0)(1 - m))) \\ + m\sigma(a - c_0)((1 - 2\alpha)m q(p_2) + (1 - m)q_0(a)) = 0$$

Optimal usage price is accordingly:

$$p_1 = c + (1 - \alpha)m(a - c_0) + (a_f - c_0)(1 - m)$$

As compared to the reference model the usage price is adjusted to reflect the termination rate on fixed, but the result is similar in the sense that usage is priced at perceived marginal cost. Consider next optimal net utility, where we insert optimal usage price and obtain:

$$V_1 = -\frac{\alpha}{\sigma} + \omega(p_1) + (a - c_0)(\alpha m q(p_1) + (1 - 2\alpha)m q(p_2) + (1 - m)q_0(a))$$

In a symmetric equilibrium we have  $p_1 = p_2 = p_i$  and  $\alpha = \frac{1}{2}$ , thus equilibrium net utility is:

$$V_1 = -\frac{1}{2\sigma} + \omega(p_1) + (a - c_0)\left(\frac{1}{2}m q(p_1) + (1 - m)q_0(a)\right)$$

**Proposition 3:**

Under singlehoming the profit in the mobile sector is proportional to the number of consumers in the mobile segment.

**Proof:**

Equilibrium profits are:

$$\begin{aligned}\pi &= m \frac{1}{2} \left( \omega(p_1) - \left\{ -\frac{1}{2\sigma} + \omega(p_1) + (a - c_0) \left( \frac{1}{2} m q(p) + (1 - m) q_0(a) \right) \right\} \right) \\ &\quad + m \frac{1}{2} (a - c_0) \left( \frac{1}{2} m q(p) + (1 - m) q_0(a) \right) \\ &= m \frac{1}{4\sigma}\end{aligned}$$

QED

Since profits increase with the size of the mobile segment it is interesting to analyze stage 1 of the game in order to study whether the termination rate has an impact on the homing decisions.

## 5.2. Stage 1, homing decisions

If joining a mobile network, a consumer of type  $(x, y)$  receives expected utility:

$$V_i - \frac{1}{8\sigma} + g(y)$$

Where  $\frac{1}{8\sigma}$  is the expected disutility from not consuming the most preferred mobile variety. The utility if joining the fixed network is given by  $V_f(a)$ . The size of the mobile segment is accordingly determined by finding the taste parameter  $y^*$  so that subscribers are indifferent as to singlehoming in fixed or singlehoming in mobile:

$$V_i - \frac{1}{8\sigma} + g(y^*) - V_f = 0 \Leftrightarrow g(y^*) = V_f - V_i + \frac{1}{8\sigma}$$

It is the consumers with high willingness to pay for mobility that join the mobile segment. Thus the size of the mobile sector is  $m = 1 - y^*$ . The function  $g(\cdot)$  is everywhere increasing, thus we can write the number of customers, at stage 2 of the game, choosing to join the mobile segment, as a function of the difference in offered net utilities:

$$m = m(V_i - V_f) = 1 - g^{-1}\left(V_f - V_i + \frac{1}{8\sigma}\right), \quad m' > 0$$

Recall that the net utility from joining the fixed network is a non increasing function of the termination fee, i.e.  $V_0'(a) \leq 0$ . The size of the mobile segment is accordingly given by the solution of the following system of equations:

- i.  $m = m(V_i - V_f), \quad m' > 0$
- ii.  $V_i = -\frac{1}{2\sigma} + \omega(p) + (a - c_0)\left(\frac{1}{2}mq(p) + (1 - m)q_f(a)\right)$
- iii.  $p_i = c + \frac{1}{2}m(a - c_0) + (a_f - c_0)(1 - m)$

**Proposition 4**

Under singlehoming the termination rate is not profit neutral. Profit in the mobile sector is a function of the size of the mobile sector. The size of the mobile sector is a function of the termination rate. Furthermore:

- a) The profit of the mobile firms is increasing in the termination rate in the point where the termination rate is cost based.
- b) If mobile firms are free to raise the termination rate, the fixed network may be driven out of the market or there may exist an interior solution.

The proof is in the appendix.

In this scenario, the mobile firms have incentives to raise the termination rate above the welfare maximizing level, the reason being that the profits in the mobile sector are proportional to the size of the mobile sector. Since termination revenues are passed on to mobile consumers, the utility of mobile subscribers increases in the termination rate. Thus an increase in the termination rate will result in a larger mobile sector. If the mobile firms are free to set the termination rate they may drive the fixed network out of the market or they may end up in an interior solution.

The fixed network is not necessarily driven out of the market. This result deserves a comment. Starting from cost based termination rates there is a first order effect when increasing the termination rate resulting in increased termination revenues. At stage 3 of the game, these revenues are passed on to consumers. Thus the size of the mobile sector increases and so do the profits of the mobile firms. As the termination rate increases

further there is however some effects that come into play and some of these effects constrain the mobile firm's ability to increase the difference in offered utility:

- The usage price increase which result in a deadweight loss,
- There is a positive price effect and a negative volume effect; the revenues on a given volume of F2M traffic increase, but the volume decreases
- As the termination rate increases, the number of mobile customers sharing the (possibly) decreasing termination revenues increases

Note that, on the one hand, if there is a strong positive link between the termination rate on the mobile network and the regulated downstream prices charged by the fixed network, then it is more likely that the mobile firms are able to drive the fixed network out of the market. On the other hand, even if downstream prices in the fixed sector are unaffected by the mobile termination rate, the mobile firms will gain from increasing the termination rate above the cost based level.

Dessein (2003) considers a case with heterogeneous consumers, non linear pricing and elastic subscription. Similarly to the results presented here, Dessein finds that the termination rate is not profit neutral. In his model mobile firms prefer a termination rate below costs. This result is in contrast to the result above that the mobile firms prefer a high termination rate. The difference is due to network effects. Consumers joining the mobile networks in Dessein's model are genuinely new network members. Thus, increasing the number of subscribers in one of the competing mobile networks results in increased utility for all consumers in both networks. This is in contrast to the result from our model of fixed mobile substitution. In our model, an expansion of the mobile sector results in a reduction of the fixed sector. Thus the number of communication partners is constant.

According to proposition 4 above, mobile firms prefer a termination rate above marginal costs. Furthermore, proposition 4 seems to indicate that the mobile firms may set a relatively high termination rate such that the fixed network is driven out of the market. Such a high termination rate may however violate conditions for a shared market equilibrium in the mobile sector. In Laffont Rey and Tirole (1998a) appendix B it is

demonstrated that if the termination margins are large and/or there is high substitutability between the networks no equilibrium exists. This result is derived in a model with the same structure as our stage 3 game. If termination rates at stage 1 of the game are determined at such a high level that stage 3 equilibrium breaks down, then our modelling is no longer valid since stage 3 results are derived by assuming the existence of a symmetric equilibrium. It is outside the scope of the current paper to analyze such a game.

## **6. Conclusions**

The implication of the analysis in the current paper is that there is a case for regulating mobile termination rates in the growth phases of mobile telephony, whereas there is less need for regulation in mature markets characterized by a stable size of the mobile sector. This seems to be the opposite of the approach taken by regulators in Europe, where mobile firms were free to set termination rates in the growth phase and where regulation is introduced once markets mature.

These results have been derived by considering fixed mobile substitution in a model of mobile network competition. We have demonstrated that the termination rates are profit neutral if the size of the mobile sector is given. An implication of this result is that the mobile termination rate does not have an impact on profits in the mobile sector if all subscribers multihome. Furthermore, the termination rate is also profit neutral if there is fixed mobile substitution of a type where consumers change status from multihoming in fixed and mobile to a status where they singlehome in mobile. In situations where consumers multihome and there is a positive termination margin, mobile firms will set usage prices above perceived marginal cost.

Furthermore, if fixed mobile substitution results in an increased number of mobile subscribers, then the mobile termination rate will have an impact on profits in the mobile sector. The mechanism behind this result is that profits in the mobile sector are proportional to the size of the mobile sector. The size of the mobile sector is an increasing function of the net utility offered to mobile subscribers. This net utility is increasing in the termination rate because termination revenues are being passed on to consumers due to competition in the mobile sector. Thus the mobile termination rate will



have an impact on profitability in the mobile sector if the size of the mobile sector is affected.

In a mixed market situation where the size of the mobile sector is not given and there are some subscribers multihoming, the two effects described above will in combination result in two kinds of market distortions. At stage 1 of the game mobile firms will set termination rates above cost in order to induce more subscribers to join the mobile networks, then at stage 3, due to the termination margin and the existence of multihoming subscribers, mobile firms will have an incentive to raise usage prices above perceived marginal cost in order to make multihomers substitute traffic originated in mobile for traffic originated in fixed because it results in increased termination revenues.

## Appendix A, Proof of proposition 4

We have the following system of equations:

$$i) \quad m = m(V_i - V_f) = 1 - g^{-1}\left(V_f - V_i + \frac{1}{8\sigma}\right), \quad m' > 0$$

$$ii) \quad V_i = -\frac{1}{2\sigma} + \omega(p) + (a - c_0)\left(\frac{1}{2}mq(p) + (1 - m)q_f(a)\right)$$

$$iii) \quad p_i = c + \frac{1}{2}m(a - c_0) + (a_f - c_0)(1 - m)$$

Total differentiation yields:

$$\frac{dm}{da} = m' \left( \frac{dV_i}{da} - \frac{\partial V_f}{\partial a} \right)$$

$$\begin{aligned} \frac{dV_i}{da} &= -q(p_i) \frac{dp_i}{da} + \frac{1}{2}mq(p) + (1 - m)q_f(a) \\ &\quad + (a - c_0) \left( \frac{1}{2}q(p) \frac{dm}{da} + \frac{1}{2}mq'(p) \frac{dp_i}{da} - \frac{dm}{da} q_f(a) + (1 - m) \frac{\partial q_f(a)}{\partial a} \right) \end{aligned}$$

$$\frac{dp_i}{da} = \frac{1}{2}m + \frac{1}{2}(a - c_0) \frac{dm}{da} - (a_f - c_0) \frac{dm}{da}$$

Combining these expressions yields:

$$(A1) \quad \frac{dm}{da} = \frac{m' \left( (1 - m)q_f(a) + (a - c_0) \left( \frac{1}{4}m^2q'(p) + (1 - m) \frac{\partial q_f(a)}{\partial a} \right) - \frac{\partial V_f}{\partial a} \right)}{1 + m' \left[ -(a_f - c_0)q(p_i) - (a - c_0) \left( \frac{1}{2}mq'(p) \left( \frac{1}{2}(a + c_0) - a_f \right) - q_f(a) \right) \right]}$$

a)

Consider first the point of cost based termination rates,  $a = c_0$ , then the expression simplifies to:

$$\frac{dm}{da} = \frac{m' \left( (1-m)q_f(a) - \frac{\partial V_f}{\partial a} \right)}{1 + m' \left[ -\underbrace{(a_f - c_0)}_{-} q(p_i) \right]} > 0$$

This proves part a) of the proposition. Note that  $a_f < c_0$  is a sufficient, but not necessary condition.

b)

Consider next the denominator in the expression (A1):

$$1 + m' \left[ -\underbrace{(a_f - c_0)}_{-} q(p_i) - (a - c_0) \left( \frac{1}{2} m q'(p) \left( \frac{1}{2} (a + c_0) - a_f \right) - q_f(a) \right) \right]$$

When the termination margin is positive we have  $\frac{1}{2}(a + c_0) > c_0$ , furthermore, by assumption,  $a_f < c_0$ , thus  $\left( \frac{1}{2}(a + c_0) - a_f \right) > 0$ , the denominator is accordingly positive for positive termination margins. The sign of (A1) is accordingly determined by the numerator:

$$\begin{aligned} m' \left( (1-m)q_f(a) + (a - c_0) \left( \frac{1}{4} m^2 q'(p) + (1-m) \frac{\partial q_f(a)}{\partial a} \right) - \frac{\partial V_f}{\partial a} \right) &= 0 \\ (a - c_0) \underbrace{\left( \frac{1}{4} m^2 q'(p) + (1-m) \frac{\partial q_f(a)}{\partial a} \right)}_{-} &\geq \frac{\partial V_f}{\partial a} - (1-m)q_f(a) \\ (a - c_0) &\leq \frac{\overbrace{(1-m)q_f(a)}^{+} - \overbrace{\frac{\partial V_f}{\partial a}}^{+}}{\underbrace{-\frac{1}{4} m^2 q'(p) - (1-m) \frac{\partial q_f(a)}{\partial a}}_{+}} \end{aligned}$$

The right hand side of this expression is always positive, but the inequality may not hold for sufficiently high termination margins, thus there may exist an interior optimum where the mobile sector has its maximum size, and that this size is below 1.

## Appendix B

### ***B.1. An emerging market; all in fixed, some multihome F&M***

In this section we let  $m$  denote the size of the segment multihoming in fixed and mobile, and we let  $\alpha$  denote the market share of mobile firm 1 within the multihoming segment.

#### **B.1.1. Stage 3**

Retail profit of firm 1 is now:

$$\pi_R = m\alpha \left[ \hat{T}_1 + \hat{q}(\hat{p}_1) (\hat{p}_1 - c - (a - c_0)(1 - \alpha)m - (a_f - c_0)(1 - m)) \right]$$

Profits in the wholesale market consist of three elements:

1. Calls from multihomers in mobile network 2 terminated in network 1:

$$(a - c_0)m^2\alpha(1 - \alpha)(\hat{q}(\hat{p}_2) + q_0(\hat{p}_2))$$

2. Calls from multihomers in mobile network 1 originated in the fixed network terminating in mobile network 1:

$$(a - c_0)m^2\alpha^2q_f(\hat{p}_1)$$

3. Calls from singlehomers in fixed:

$$(a - c_0)m(1 - m)\alpha\tilde{q}(a)$$

Collecting terms and substituting for net utility yields the following profit function:

$$\begin{aligned} \pi = m\alpha & \left[ \omega(\hat{p}_1) - \hat{V}_1 + \hat{q}(\hat{p}_1) (\hat{p}_1 - c - (a - c_0)(1 - \alpha)m - (a_f - c_0)(1 - m)) \right] \\ & + m\alpha(a - c_0) \left[ m(1 - \alpha)(\hat{q}(\hat{p}_2) + q_f(\hat{p}_2)) + m\alpha q_f(\hat{p}_1) + (1 - m)\tilde{q}(a) \right] \end{aligned}$$

Then we can maximize profits:

$$\frac{\partial \pi}{\partial \hat{p}_1} = 0 \Leftrightarrow \hat{p}_1 = c + (a - c_0)(1 - \alpha)m + (a_f - c_0)(1 - m) + (a - c_0)m\alpha\delta$$

where :

$$\delta = -\frac{q_0'(\hat{p}_1)}{\hat{q}'(\hat{p}_1)}$$

This pricing rule is similar to the one we derived under full multihoming. Consider next the condition for optimal net utility where we insert the optimal pricing rule:

$$\frac{\partial \pi}{\partial \hat{V}_1} = 0 \Leftrightarrow$$

$$\hat{V}_1 = -\frac{\alpha}{\sigma} + \omega(\hat{p}_1) + \hat{q}(\hat{p}_1)((a - c_0)m\alpha\delta + (a - c_0)\alpha m)$$

$$+ (a - c_0)(m(1 - 2\alpha)(\hat{q}(\hat{p}_2) + q_0(\hat{p}_2)) + m2\alpha q_0(\hat{p}_1) + (1 - m)\tilde{q}(a))$$

Inserting equilibrium prices and market shares:

$$\hat{V}_1 = -\frac{1}{2\sigma} + \hat{\omega} + (a - c_0)(mq_0 + (1 - m)\tilde{q} + \hat{q}\frac{1}{2}m(1 + \delta))$$

Finally, inserting all equilibrium values back into the profit function:

$$\pi = m\frac{1}{2} \left[ \hat{\omega} - \left( -\frac{1}{2\sigma} + \hat{\omega} + (a - c_0)(mq_0 + (1 - m)\tilde{q} + \hat{q}\frac{1}{2}m(1 + \delta)) \right) \right.$$

$$\left. + \hat{q} \left( c + (a - c_0)\frac{1}{2}m + (a_f - c_0)(1 - m) + (a - c_0)m\frac{1}{2}\delta - c - (a - c_0)\frac{1}{2}m - (a_f - c_0)(1 - m) \right) \right]$$

$$+ m\frac{1}{2}(a - c_0) \left[ m\frac{1}{2}(\hat{q} + q_0) + m\frac{1}{2}q_0 + (1 - m)\tilde{q} \right]$$

$$= m\frac{1}{4\sigma}$$

Profits in the mobile sector are accordingly proportional to the size of the mobile sector.

Then we are back at the same structure as the one we considered in section 5 of the paper.

### B.1.2. Stage 1

At stage 1 of the game consumers choose between becoming singlehomers in fixed or multihomers in fixed and mobile. By similar reasoning as in section 5, the size of the multihoming segment is given by  $m = \hat{m}(\hat{V}_i - V_f)$ ,  $m' > 0$ .

Similarly to the case of singlehoming, the size of the mobile sector is then given by the solution of the following system of equations:

Similarly to the case of singlehoming, the size of the mobile sector is then given by the solution of the following system of equations:

$$(i) \quad m = \hat{m}(\hat{V}_i - V_f), \quad m' > 0$$

$$(ii) \quad \hat{V} = -\frac{1}{2\sigma} + \hat{\omega}(p_i) - C + (a - c_0)(mq_0(a) + (1 - m)\tilde{q}(a) + \frac{1}{2}m(1 + \delta)\hat{q}(p_i))$$

$$(iii) \quad p_i = c + (a - c_0)(1 - \alpha)m + (a_f - c_0)(1 - m) + (a - c_0)m\alpha\delta$$

### Proposition 5

In a market characterized by all consumers being in the fixed network and some consumers multihoming in fixed and mobile:

- The profit neutrality result does not hold.
- The profit of the mobile firms is increasing in the point of cost based termination rates.

### Proof

The solutions are given as the solution of the following system of equations (where we have inserted optimal usage price):

$$(i) \quad m = m(\hat{V}_i - V_f), \quad m' > 0$$

$$(ii) \quad \hat{V} = -\frac{1}{2\sigma} + \hat{w}\left(c + \frac{1}{2}(a - c_0)m(1 + \delta) + (a_f - c_0)(1 - m)\right) + \\ (a - c_0)\left(mq_0(a) + (1 - m)\tilde{q}(a) + \frac{1}{2}m(1 + \delta)\hat{q}\left(c + \frac{1}{2}(a - c_0)m(1 + \delta) + (a_f - c_0)(1 - m)\right)\right)$$

Total differentiation of this system yields:

$$\frac{dm}{da} = m' \left( \frac{d\hat{V}}{da} - \frac{\partial V_f}{\partial a} \right)$$

$$\frac{d\hat{V}}{da} = -\hat{q}(\hat{p}) \left( \frac{1}{2}m(1 + \delta) + \frac{1}{2}(a - c_0)(1 + \delta) \frac{dm}{da} - (a_f - c_0) \frac{dm}{da} \right) \\ + mq_0(a) + (1 - m)\tilde{q}(a) + \frac{1}{2}m(1 + \delta)\hat{q}\left(c + \frac{1}{2}(a - c_0)m(1 + \delta) + (a_f - c_0)(1 - m)\right) \\ + (a - c_0) \left( m \frac{\partial q_0(a)}{\partial a} + q_0(a) \frac{dm}{da} + (1 - m) \frac{\partial \tilde{q}(a)}{\partial a} - \tilde{q}(a) \frac{dm}{da} \right) \\ + (a - c_0) \frac{1}{2}(1 + \delta)\hat{q}\left(c + \frac{1}{2}(a - c_0)m(1 + \delta) + (a_f - c_0)(1 - m)\right) \frac{dm}{da} \\ (a - c_0) \frac{1}{2}m(1 + \delta)\hat{q}'(\hat{p}) \left( \frac{1}{2}m(1 + \delta) + \frac{1}{2}(a - c_0)(1 + \delta) \frac{dm}{da} - (a_f - c_0) \frac{dm}{da} \right)$$

Consider now, as a reference point, cost based termination rates, i.e.  $a = c_0$ , then the second equation simplifies to:

$$\frac{d\hat{V}}{da} = -\hat{q}(\hat{p}) \left( \frac{1}{2} m(1+\delta) - (a_f - c_0) \frac{dm}{da} \right) + mq_0(a) + (1-m)\tilde{q}(a) + \frac{1}{2} m(1+\delta)\hat{q}(c + (a_f - c_0)(1-m))$$

and we can combine the two expressions to obtain:

$$\begin{aligned} \frac{dm}{da} &= m' \left( \begin{array}{l} -\hat{q}(\hat{p}) \left( \frac{1}{2} m(1+\delta) - (a_f - c_0) \frac{dm}{da} \right) \\ + mq_0(a) + (1-m)\tilde{q}(a) + \frac{1}{2} m(1+\delta)\hat{q} \end{array} \right) \\ &\quad - m' \frac{\partial V_f}{\partial a} \\ \frac{dm}{da} (1 - m'(a_f - c_0)\hat{q}(\hat{p})) &= m' \left( -\hat{q}(\hat{p}) \left( \frac{1}{2} m(1+\delta) \right) + mq_0(a) + (1-m)\tilde{q}(a) + \frac{1}{2} m(1+\delta)\hat{q} - \frac{\partial V_f}{\partial a} \right) \\ \frac{dm}{da} &= \frac{m' \left( \frac{1}{2} (1+\delta) m \hat{q}(\hat{p}) + mq_0(a) + (1-m)\tilde{q}(a) + \frac{1}{2} m(1+\delta)\hat{q} - \frac{\partial V_f}{\partial a} \right)}{1 - \underbrace{m'(a_f - c_0)\hat{q}(\hat{p})}_{+}} > 0 \end{aligned}$$

This is similar to the expression under singlehoming.

QED

The result is similar to what we found under singlehoming. Thus, if mobile firms are free to set termination rates they can increase profits by increasing the termination rate above costs. Furthermore, if the resulting termination rate is sufficiently high, stage 3 equilibrium will break down.

## **B.2. A mature market, All in mobile, some multihome FM**

In this section we assume that consumers, at stage 3 of the game, are divided into two groups, a segment of singlehomers in mobile and a segment of multihomers. The size of the singlehoming segment is  $m_s$  and the multihoming segment is  $1 - m_s$ . Firm  $i$  offers tariffs targeted at the single- and multihoming segments respectively:  $\{(T_i, p_i), (\hat{T}_i, \hat{p}_i)\}$ , thus market shares of firm 1 within the two segments become:

$$\alpha_s = \frac{1}{2} + \sigma(\omega(p_i) - \omega(\hat{p}_i) - T_i + \hat{T}_i)$$

$$\alpha_m = \frac{1}{2} + \sigma(\hat{\omega}(\hat{p}_1, a) - \hat{\omega}(\hat{p}_2, a) - \hat{T}_1 + \hat{T}_2)$$

We assume that firms are able (and allowed) to condition the offered mobile tariff on whether the subscribers are within the single- or multihoming segment, thus they can do third degree price discrimination.<sup>14</sup>

Retail profit is now:

$$\begin{aligned} \pi_R = & m_s \alpha_s [T + q(p_1)(p_1 - c - (1 - \alpha)(a - c_0))] \\ & + (1 - m_s) \alpha_m [\hat{T} - \hat{q}(\hat{p}_1, a)(\hat{p}_1 - c - (1 - \alpha)(a - c_0))] \end{aligned}$$

Recall that net utility is given by  $V_i = \omega(p_i) - T_i$ , and  $\hat{V}_i = \omega(\hat{p}_i, a) - \hat{T}_i$ , substituting  $V$  for  $T$  yields:

$$\begin{aligned} \pi_R = & m_s \alpha_s [\omega(p_1) - V_1 + q(p_1)(p_1 - c - (1 - \alpha)(a - c_0))] \\ & + (1 - m_s) \alpha_m [\hat{\omega}(\hat{p}_1, a) - \hat{V}_1 - \hat{q}(\hat{p}_1, a)(\hat{p}_1 - c - (1 - \alpha)(a - c_0))] \end{aligned}$$

Wholesale profits consist of four elements, incoming traffic from the other mobile network originated by single- and multihomers respectively, and incoming traffic originated in the fixed network by customers multihoming in the two mobile networks:

$$\begin{aligned} \pi_w = & m_s \alpha (a - c_0) [(1 - \alpha_s) q(p_2)] \\ & + (1 - m_s) \alpha (a - c_0) [(1 - \alpha_m) \hat{q}(p_2, a) + \alpha_m q_0(\hat{p}_1, a) + (1 - \alpha_m) q_0(p_2, a)] \end{aligned}$$

In order to simplify calculations we carry out profit maximization in two steps by first maximizing profits,  $\pi_R + \pi_w$ , subject to the constraint that  $\alpha = m_s \alpha_s + (1 - m_s) \alpha_m$ , and then finding optimal total market share.<sup>15</sup> Thus we maximize the Lagrangian (where  $\lambda$  is the Lagrange multiplier):

$$L = \pi_R + \pi_w + \lambda(\alpha - m_s \alpha_s + (1 - m_s) \alpha_m)$$

And then we maximize the Lagrangian with respect to total market share  $\alpha$  by applying the envelope theorem.

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<sup>14</sup> Note that the homing decisions made at stage 1 of the game typically is observable.

<sup>15</sup> This approach is due to Hahn, J. H., 2004.



### Proposition 6

In a market characterized by all consumers being in one of the mobile networks and some consumers multihoming in fixed and mobile:

- The profit neutrality result holds
- The usage price charged from singlehoming consumers is at perceived marginal cost
- The usage price charged from multihoming consumers is adjusted upwards (for positive termination margins)
- The fixed fee charge from singlehoming consumers exceeds the fixed fee charged from multihoming subscribers

### Proof

Profits,  $\pi_R + \pi_W$ , are to be maximized subject to the constraint that

$\alpha = m_s \alpha_s + (1 - m_s) \alpha_m$ , thus we have the following Lagrangian: (where  $\lambda$  is the Lagrange multiplier):

$$\begin{aligned}
 L = & m_s \{ \alpha_s (\omega(p_1) - V_1 + q(p_1)(p_1 - c - (1 - \alpha)(a - c_0))) \} \\
 & + (1 - m_s) \{ \alpha_m (\hat{\omega}(\hat{p}_1, a) - \hat{V}_1 + \hat{q}(\hat{p}_1, a)(\hat{p}_1 - c - (1 - \alpha)(a - c_0))) \} \\
 & + \alpha(a - c_0)(m_s(1 - \alpha_s)q(p_2) + (1 - m_s)(1 - \alpha_m)\hat{q}(\hat{p}_2, a) + \alpha_m(1 - m_s)q_0(\hat{p}_1, a) + (1 - \alpha_m)(1 - m_s)q_0(\hat{p}_2, a)) \\
 & + \lambda(\alpha - m_s \alpha_s - (1 - m_s) \alpha_m)
 \end{aligned}$$

Consider:

$$\frac{\partial L}{\partial p_1} = m_s \alpha_s (-q(p_1) + q(p_1) + q'(p_1)(p_1 - c - (1 - \alpha)(a - c_0))) \Leftrightarrow p_1 = c + (1 - \alpha)(a - c_0)$$

and:

$$\frac{\partial L}{\partial \hat{p}_1} = (1 - m_s) \alpha_m \left( -\hat{q}(\hat{p}_1, a) + \hat{q}(\hat{p}_1, a) + \frac{\partial \hat{q}(\hat{p}_1, a)}{\partial \hat{p}_1} (\hat{p}_1 - c - (1 - \alpha)(a - c_0)) \right) + \alpha(a - c_0) \alpha_m (1 - m_s) \frac{\partial q_0(\hat{p}_1, a)}{\partial \hat{p}_1} = 0$$

$$(1 - m_s) \alpha_m \frac{\partial \hat{q}(\hat{p}_1, a)}{\partial \hat{p}_1} \left[ \hat{p}_1 - c - (1 - \alpha)(a - c_0) + \alpha(a - c_0) \frac{\frac{\partial q_0(\hat{p}_1, a)}{\partial \hat{p}_1}}{\frac{\partial \hat{q}(\hat{p}_1, a)}{\partial \hat{p}_1}} \right]$$

$$\hat{p}_1 = c + (1 - \alpha)(a - c_0) - \alpha(a - c_0) \frac{\frac{\partial q_0(\hat{p}_1, a)}{\partial \hat{p}_1}}{\frac{\partial \hat{q}(\hat{p}_1, a)}{\partial \hat{p}_1}}$$

Simplification: as in earlier sections, assume constant  $-\frac{\frac{\partial q_0(\hat{p}_1, a)}{\partial \hat{p}_1}}{\frac{\partial \hat{q}(\hat{p}_1, a)}{\partial \hat{p}_1}} = \delta$ . Then we can

write:  $\hat{p}_1 = c + (1 - \alpha)(a - c_0) + \alpha(a - c_0)\delta$

Consider next:

$$\frac{\partial L}{\partial V_1} = -m_s \alpha_s + \sigma m_s (\omega(p_1) - V_1 + q(p_1)(p_1 - c - (1 - \alpha)(a - c_0))) - \alpha \sigma (a - c_0) m_s q(p_2) - \lambda m_s \sigma = 0$$

Inserting optimal usage price and solving with respect to  $V_1$ :

$$\frac{\partial L}{\partial V_1} = -m_s \alpha_s + \sigma m_s (\omega(p_1) - V_1) - \alpha \sigma (a - c_0) m_s q(p_2) - \lambda m_s \sigma = 0$$

$$\sigma m_s V_1 = -m_s \alpha_s + \sigma m_s (\omega(p_1)) - \alpha \sigma (a - c_0) m_s q(p_2) - \lambda m_s \sigma$$

$$V_1 = -\frac{\alpha_s}{\sigma} + \omega(p_1) - \alpha(a - c_0)q(p_2) - \lambda$$

Then consider:

$$\frac{\partial L}{\partial \hat{V}_1} = -(1 - m_s) \alpha_m + (1 - m_s) \sigma \left\{ \hat{\omega}(\hat{p}_1, a) - \hat{V}_1 + \hat{q}(\hat{p}_1, a) (\hat{p}_1 - c - (1 - \alpha)(a - c_0)) \right\} + \alpha(a - c_0) \sigma (1 - m_s) (-\hat{q}(\hat{p}_2, a) + q_0(\hat{p}_1, a) - q_0(\hat{p}_2, a)) - \lambda \sigma (1 - m_s) = 0$$

Inserting optimal usage price and solving with respect to  $\hat{V}_1$ :

$$-(1-m_s)\alpha_m + (1-m_s)\sigma\left(\hat{\omega}(\hat{p}_1, a) - \hat{V}_1 + \hat{q}(\hat{p}_1, a)\alpha(a-c_0)\delta\right) \\ + \alpha(a-c_0)\sigma(1-m_s)\left(-\hat{q}(\hat{p}_2, a) + q_0(\hat{p}_1, a) - q_0(\hat{p}_2, a)\right) - \lambda\sigma(1-m_s) = 0$$

$$\hat{V}_1 = -\frac{\alpha_m}{\sigma} + \hat{\omega}(\hat{p}_1, a) + \hat{q}(\hat{p}_1, a)\alpha(a-c_0)\delta + \alpha(a-c_0)\left(-\hat{q}(\hat{p}_2, a) + q_0(\hat{p}_1, a) - q_0(\hat{p}_2, a)\right) - \lambda$$

Consider then optimal target network size:

$$L = m_s\left\{\alpha_s(\omega(p_1) - V_1 + q(p_1)(p_1 - c - (1-\alpha)(a-c_0)))\right\} \\ + (1-m_s)\left\{\alpha_m(\hat{\omega}(\hat{p}_1, a) - \hat{V}_1 + \hat{q}(\hat{p}_1, a)(\hat{p}_1 - c - (1-\alpha)(a-c_0)))\right\} \\ + \alpha(a-c_0)\left(m_s(1-\alpha_s)q(p_2) + (1-m_s)(1-\alpha_m)\hat{q}(\hat{p}_2, a) + \alpha_m(1-m_s)q_0(\hat{p}_1, a) + (1-\alpha_m)(1-m_s)q_0(\hat{p}_2, a)\right) \\ + \lambda(\alpha - m_s\alpha_s - (1-m_s)\alpha_m)$$

$$\frac{\partial L}{\partial \alpha} = m_s\alpha_s q(p_1)(a-c_0) + (1-m_s)\alpha_m \hat{q}(\hat{p}_1, a)(a-c_0) \\ + (a-c_0)\left(m_s(1-\alpha_s)q(p_2) + (1-m_s)(1-\alpha_m)\hat{q}(\hat{p}_2, a) + \alpha_m(1-m_s)q_0(\hat{p}_1, a) + (1-\alpha_m)(1-m_s)q_0(\hat{p}_2, a)\right) \\ + \lambda = 0 \\ \lambda = -(a-c_0)\left[\frac{m_s\alpha_s q(p_1) + (1-m_s)\alpha_m \hat{q}(\hat{p}_1, a) + m_s(1-\alpha_s)q(p_2) + (1-m_s)(1-\alpha_m)\hat{q}(\hat{p}_2, a)}{\alpha_m(1-m_s)q_0(\hat{p}_1, a) + (1-\alpha_m)(1-m_s)q_0(\hat{p}_2, a)}\right]$$

We can now characterize equilibrium by combining optimal pricing, optimal net utilities, and optimal market shares, i.e.  $p = p_1 = p_2$ ,  $\hat{p} = \hat{p}_1 = \hat{p}_2$ ,  $\alpha = \alpha_s = \alpha_m = \frac{1}{2}$ . Then the condition for optimal market share simplifies to  $\lambda = -(a-c_0)[m_s q + (1-m_s)\hat{q} + (1-m_s)q_0]$ , and we obtain:

$$V_1 = -\frac{\alpha_s}{\sigma} + \omega(p_1) - \alpha(a-c_0)q(p_2) - \lambda \\ = -\frac{1}{2\sigma} + \omega + (a-c_0) - \frac{1}{2}(a-c_0)q - \lambda \\ V_1 = -\frac{1}{2\sigma} + \omega + (a-c_0)\left((m_s - \frac{1}{2})q + (1-m_s)\hat{q} + (1-m_s)q_0\right)$$

and:

$$\hat{V}_1 = -\frac{\alpha_m}{\sigma} + \hat{\omega}(\hat{p}_1, a) + \hat{q}(\hat{p}_1, a)\alpha(a-c_0)\delta + \alpha(a-c_0)\left(-\hat{q}(\hat{p}_2, a) + q_0(\hat{p}_1, a) - q_0(\hat{p}_2, a)\right) - \lambda \\ \hat{V}_1 = -\frac{1}{2\sigma} + \hat{\omega} + \frac{1}{2}\hat{q}(a-c_0)\delta + (a-c_0)\left(m_s q + \left(\frac{1}{2} - m_s\right)\hat{q} + (1-m_s)q_0\right)$$

Finally, equilibrium values can be inserted into the profit function:

$$\begin{aligned}
L &= m_s \frac{1}{2} (\omega - V_1 + q(p_1 - c - (1 - \alpha)(a - c_0))) + (1 - m_s) \frac{1}{2} (\hat{\omega} - \hat{V}_1 + \hat{q}(\hat{p}_1 - c - (1 - \alpha)(a - c_0))) \\
&\quad + \frac{1}{2} (a - c_0) (m_s \frac{1}{2} q + (1 - m_s) \frac{1}{2} \hat{q} + (1 - m_s) q_0) \\
&= m_s \frac{1}{2} (\omega - V_1) + (1 - m_s) \frac{1}{2} (\hat{\omega} - \hat{V}_1 + \frac{1}{2} (a - c_0) \delta \hat{q}) \\
&\quad + \frac{1}{2} (a - c_0) (m_s \frac{1}{2} q + (1 - m_s) \frac{1}{2} \hat{q} + (1 - m_s) q_0)
\end{aligned}$$

Then inserting net utilities:

$$\begin{aligned}
\pi &= m_s \frac{1}{2} \left( \omega - \left( -\frac{1}{2\sigma} + \omega + (a - c_0) \left( (m_s - \frac{1}{2}) q + (1 - m_s) \hat{q} + (1 - m_s) q_0 \right) \right) \right) \\
&\quad + (1 - m_s) \frac{1}{2} \left( \hat{\omega} - \left( -\frac{1}{2\sigma} + \hat{\omega} + \frac{1}{2} \hat{q} (a - c_0) \delta + (a - c_0) \left( m_s q + (\frac{1}{2} - m_s) \hat{q} + (1 - m_s) q_0 \right) \right) + \frac{1}{2} (a - c_0) \delta \hat{q} \right) \\
&\quad + \frac{1}{2} (a - c_0) (m_s \frac{1}{2} q + (1 - m_s) \frac{1}{2} \hat{q} + (1 - m_s) q_0) \\
&= \frac{1}{4\sigma}
\end{aligned}$$

Thus profit neutrality holds.

Consider next equilibrium fixed fees:

$$\begin{aligned}
T &= \omega - V_1 = \omega - \left( -\frac{1}{2\sigma} + \omega + (a - c_0) \left( (m_s - \frac{1}{2}) q + (1 - m_s) \hat{q} + (1 - m_s) q_0 \right) \right) \\
&= \frac{1}{2\sigma} - (a - c_0) \left( (m_s - \frac{1}{2}) q + (1 - m_s) \hat{q} + (1 - m_s) q_0 \right)
\end{aligned}$$

$$\begin{aligned}
\hat{T} &= \hat{\omega} - \hat{V}_1 = \hat{\omega} - \left( -\frac{1}{2\sigma} + \hat{\omega} + \frac{1}{2} \hat{q} (a - c_0) \delta + (a - c_0) \left( m_s q + (\frac{1}{2} - m_s) \hat{q} + (1 - m_s) q_0 \right) \right) \\
&= \frac{1}{2\sigma} - \frac{1}{2} \hat{q} (a - c_0) \delta - (a - c_0) \left( m_s q + (\frac{1}{2} - m_s) \hat{q} + (1 - m_s) q_0 \right)
\end{aligned}$$

and note that:

$$\begin{aligned}
T - \hat{T} &= \frac{1}{2\sigma} - (a - c_0) \left( (m_s - \frac{1}{2}) q + (1 - m_s) \hat{q} + (1 - m_s) q_0 \right) \\
&\quad - \left( \frac{1}{2\sigma} - \frac{1}{2} \hat{q} (a - c_0) \delta - (a - c_0) \left( m_s q + (\frac{1}{2} - m_s) \hat{q} + (1 - m_s) q_0 \right) \right) \\
&= (a - c_0) \left( (-m_s + \frac{1}{2} + m_s) q + (-1 + m_s + \frac{1}{2} - m_s) \hat{q} + (-1 + m_s + 1 - m_s) q_0 \right) + \frac{1}{2} \hat{q} (a - c_0) \delta \\
&= (a - c_0) \frac{1}{2} (q - \hat{q}) + \frac{1}{2} \hat{q} (a - c_0) \delta > 0
\end{aligned}$$

The sign indicated above holds if singlehomers in mobile originate more calls in the mobile network as compared to multihomers.

QED

The results above are derived by assuming third degree price discrimination, but the profit neutrality result is likely to hold under second degree price discrimination as well. Seen from a mobile network, singlehoming and multihoming consumers can be seen as high volume and low volume customers respectively. As demonstrated by Dessein 2003 (two type model) and Hahn 2004 (continuum of types) the profit neutrality holds in models with consumer heterogeneity, as long as the total number of mobile subscribers is given.

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