Banks Collateral, Asymmetric Information and Aggregate Fluctuations
PRELIMINARY

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Abstract

We model liquidity shocks in debt markets as random variations in the riskiness of assets held by financial institutions and/or heightened uncertainty on the health of their balance sheets. We then characterize the effects of these shocks on both the interbank market and the overall supply of credit. We show that liquidity shocks can generate sizeable effects on real aggregate economic activity, especially when banks are highly leveraged.

Keywords: riskiness of assets, asymmetric information, interbank market, aggregate fluctuations

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1 Introduction

Macroeconomists have persistently downplayed the potential implications of the striking run-up of debt held by financial intitutions in the last decade. Figure 1 shows, for the US, the evolution of total credit market debt owed by alternative economic agents. Clearly, if we grant that a "debt problem" has somehow laid the ground for the financial crisis of 2008-09, this must have originated not only from the household sector but primarily from the banking sector, and to a much lesser extent from the corporate or the government sector.

It is well known that debt markets are sensitive to spirals.[1] Figure 2 plots the evolution of the TED spread (a measure of the banks' cost of finance) to illustrate how acute such amplification problems in debt markets might have been during the recent crisis.[2] Spirals involve feedback effects between the banks' cost of finance and liquidity. What we specifically mean by liquidity is the perceived safety of the assets traded in the market. As emphasized by Holmstrom (2008), in fact, a necessary condition for an asset to be liquid is that market participants know its value. When market participants become suddenly uncertain about the value of an asset, however, liquidity can dry up quickly. In those instances, and well beyond the equilibrium response of prices, elements of asymmetric information become increasingly more acute and drive the market behavior. Thus, sudden shifts in confidence about the riskiness of the assets held by financial institutions, and/or variations in the degree of opacity of the balance sheets of the same institutions, can be potentially lethal for the ability of financial markets to insure an adequate flow of credit to the real economy. Hence, a key lesson researchers have learnt from the crisis is actually an old one: namely, that amplification effects originating in debt markets can have sizeable implications for aggregate economic activity.

This old lesson notwithstanding, virtually all existing equilibrium models suited for business cycle analysis have been silent on both the qualitative and the quantitative implications of these channels for aggregate economic activity. In this paper, we place a

[1] Brunnermeier (2009), Krishnamurthy (2010)

[2] The TED spread is the difference between the three-month LIBOR rate and the Treasury bill rate of corresponding maturity.

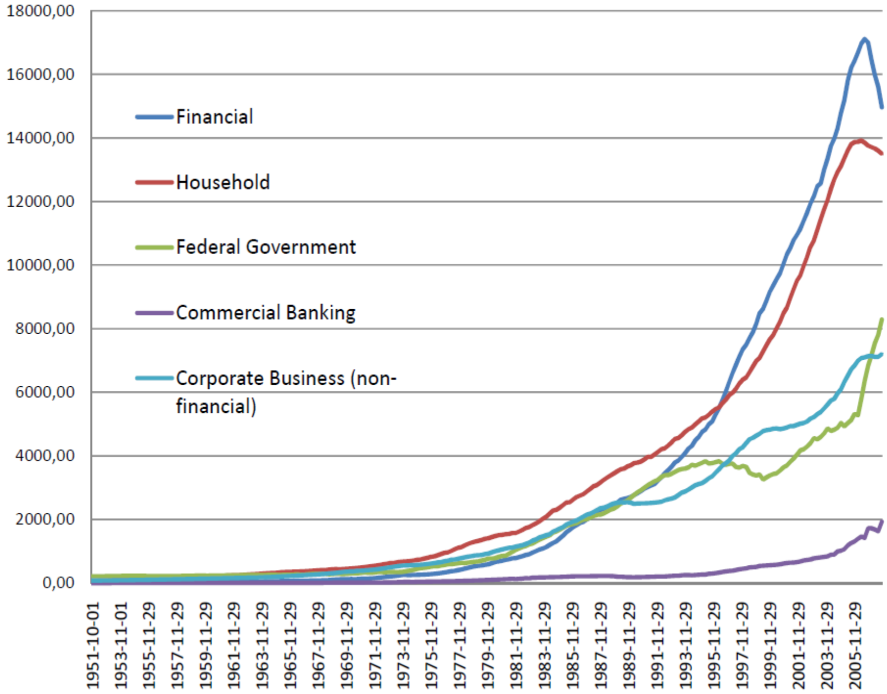

Figure 1: Total Credit Market Debt Owed by Different Sectors (bills. $, source St. Louis Fed)

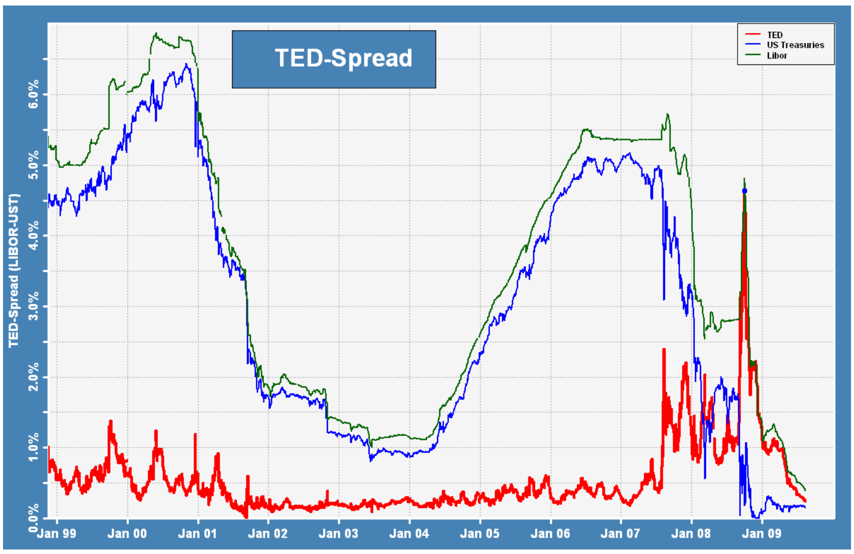

Figure 2: TED spread and its components (source Bloomberg).

problem of asymmetric information and moral hazard at the center-stage of the interbank market. Investment banks participate in the market and use assets (such as loans extended to the private sector) as collateral to finance short-term borrowing. The central element of our analysis is that the return on these assets is subject to random *idiosyncratic* disturbances. As a result, only the asset holders observe the true ex-post return from those assets. This is the asymmetric information problem at the heart of our analysis. The situation we have in mind is one in which, although the realization of an aggregate state affects the return on *all* assets simultaneously, only the individual bank (i.e., the asset holder) knows the true state of its own balance sheet.

In this setting, we study how an exogenous increase in the *riskiness* of assets held on the balance sheets of banks is transmitted to aggregate activity via the interbank market. This shock can be representative, for instance, of a state of the world in which market participants suddenly become more uncertain about the true health of the banks' balance sheet. In light of our previous discussion, however, probably the best way to think of this shock is in terms of a *liquidity* shock: namely, a shock that affects the perceived market uncertainty on the return of the assets and that therefore exasperates the underlying asymmetric information problems.

We first study the implications of liquidity shocks for the interbank market by looking at the effects on the banks' cost of finance (the spread between the market rate of return and the opportunity cost of finance) and their leverage. We then discuss how these effects translate onto overall credit markets, where firms obtain resources to finance the purchase of physical capital, and in turn on real aggregate activity.

We obtain two main set of results. First, shocks to the riskiness of assets held by banks can have sizeable negative effects on real aggregate output, especially when financial institutions are highly leveraged. Second, the model delivers a series of interesting business cycle implications. Among other things, the model generates: (i) a positive co-movement between different asset prices, i.e., the price of assets held by banks and the price of physical capital; (ii) a positive co-movement between finance premia in the interbank market and in the credit market; and (iii) a positive co-movement betweeen deleveraging in the banking sector and contraction in the supply of business credit.

2 Asymmetric information and moral hazard in the interbank market

The interbank market is populated by two types of banks: commercial banks (the *lenders*), and (a continuum of) investment banks (the *borrowers*). Commercial banks derive funds from households' deposits (see more below), and are assumed to be perfectly competitive. Hence they perfectly pool the risk across investment banks. Our main goal is to characterize a problem of asymmetric information and moral hazard directly in the interbank market.

Financial technology At the end of period t each investment bank borrows an amount b_t to purchase a portfolio of real one-period riskless assets $a_t^{(+)}$. Alternatively, one can think of each bank using these assets as collateral for short-term borrowing. We assume that, besides internal resources, each bank finances itself via one-period loans.[3]

At the end of time t, the asset portfolio owned by the bank receives an idiosyncratic shock η drawn from a c.d.f $\Phi(\eta_t)$, with density function $\phi(\eta)$, mean μ_η, and standard deviation σ_η. We assume that $\Phi(\mu_\eta, \sigma_{\eta,t})$ is log-normal, with $\mathbb{E}_t\eta_{t+1} = \exp(\mu_{\eta,t+1} + (1/2)\sigma_{\eta,t+1}) = 1$. The shock is i.i.d. across banks/assets and time. The realization of η is private information to the bank. Thus, in this setup, even though the realizations of the aggregate states affect the return of *all* asset portfolios (i.e., banks) simultaneously, only the owner of each individual portfolio knows its true value, and therefore the true health of its own balance sheet.

We will assume that the standard deviation of the idiosyncratic shock η follows the stochastic process:

$$\log \sigma_{\eta,t} = (1 - \rho_\sigma) \log \sigma_\eta + \rho_\sigma \log \sigma_{\eta,t-1} + \varepsilon_{\sigma,t},$$

where $\varepsilon_{\sigma,t} \sim \mathbb{N}(0,1)$. Hence shocks to $\sigma_{\eta,t}$ represent exogenous variations in the *riski-*

[3]Hence we abstract here from the problem of portfolio *composition.* This dimension of the problem can be added to this framework in order to analyze related, and crucial issues, such as the influence of monetary policy on the yield curve and therefore on the banks' endogenous decision of engaging in maturity transformation (Tirole 2009). We defer this analysis to future work. Several authors have blamed the low interest rate policy of the years 2000s as critical in providing an incentive for investment banks to borrow in the interbank market at very short-term maturities.

ness of assets held by the investment banks. The assumption of a randomly time-varying standard deviation of the distribution of the idiosyncratic shock is analog to the one in Christiano et al. (2009). In Christiano et al. (2009), however, an asymmetric information problem characterizes directly the relationship between lenders and firms, whose productivity is subject to idiosyncratic shocks, as in the original seminal paper of Bernanke et al. (1999).

Let's define the *gross return* of an asset purchased at time t as:

$$\Delta_{t+1} \equiv \mathbb{E}_t \{q_{t+1}/q_t\}$$

We can then summarize each investment bank's financial technology as follows:

$$\underbrace{\text{purchase portfolio } q_t a_t^{(+)}}_{\text{end of t}} \quad \rightarrow \quad \underbrace{\eta_{t+1}\Delta_{t+1}\ q_t a_t^{(+)}}_{\substack{\text{end of t+1} \\ \text{total gross return}}}$$

Default Let R^b_{t+1} be the gross interest rate on the interbank loan acquired at t (and due at time $t+1$). The bank's default occurs when the return from the investment activity, $\eta_{t+1}\Delta_{t+1}\ q_t a_t^{(+)}$, falls short of the amount that needs to be repaid $R^b_{t+1}\ b_t$. This allows to define a *cutoff* value $\overline{\eta}_{t+1}$ for η which solves the condition:

$$\overline{\eta}_{t+1}\Delta_{t+1}\ q_t a_t^{(+)} = R^b_{t+1}\ b_t \tag{1}$$

Equation (1) states that the return on the asset is *exactly* enough to cover the gross service cost of the debt.

Lender's participation constraint In case of default, the lender recovers a fraction $1-\xi$ of the realized asset return. Each lender has an (ex-ante) incentive to participate in the financial contract if the following inequality holds:

$$\underbrace{\left[1-\Phi(\overline{\eta}_{t+1})\right] R^b_{t+1}\ b_t}_{\text{if no default}} + \underbrace{(1-\xi)\, M\left(\overline{\eta}_{t+1}\right) \Delta_{t+1}\ q_t a_t^{(+)}}_{\text{if default}} \geq R^d_t\ b_t, \tag{2}$$

where $\left[1-\Phi(\overline{\eta}_{t+1})\right]$ is the probability of non-default, $M\left(\overline{\eta}_{t+1}\right) \equiv \int_0^{\overline{\eta}_{t+1}} \eta d\Phi(\eta)$, and R^d_t is the return paid on households' riskless deposits.

The left-hand side of (2) is the lender's *expected* income (net of bankruptcy costs). The right hand side is the opportunity cost of funds. The lender, in fact, faces the alternative of hoarding the funds b_t in terms of (riskless) deposits, that need to be remunerated to the households at the interest rate R_t^d.

Let's define the fraction of gross return received by the lender as

$$\Gamma(\overline{\eta}_{t+1}) \equiv M\left(\overline{\eta}_{t+1}\right) + \overline{\eta}_{t+1}\left[1 - \Phi(\overline{\eta}_{t+1})\right] \tag{3}$$

Notice that this share depends only on $\overline{\eta}_{t+1}$, a distinguishing feature of the optimal financial contract under costly state verification (CSV).[4]

Substituting (1) into (2), and using (3), we can re-write the lender's participation constraint as:

$$\left[\Gamma(\overline{\eta}_{t+1}) - \xi M\left(\overline{\eta}_{t+1}\right)\right] \frac{\Delta_{t+1} q_t a_t^{(+)}}{R_t^d} \geq q_t a_t^{(+)} - nw_t \tag{4}$$

where

$$nw_t = q_t a_t^{(+)} - b_t$$

is the bank's net worth.

Net worth and leverage A key variable in our context will be the bank's debt-to-asset ratio V_t. The latter is related to banks' net worth and to the bank's leverage via the expression:

$$V_t \equiv \frac{b_t}{q_t a_t^{(+)}} = 1 - \frac{nw_t}{q_t a_t^{(+)}} = 1 - LV_t^{-1}$$

where $LV_t \equiv q_t a_t^{(+)}/nw_t$ is banks' leverage. Hence the debt-to-asset ratio is increasing in leverage in a non-linear fashion.

[4] See Townsend (1980) and Gale and Hellwig (1983). See more below on this point.

Financial contract The financial contract specifies a pair $\left\{a_t^{(+)}, \overline{\eta}_{t+1}\right\}$ to maximize the bank's expected income (taking the asset price q_t as given):[5]

$$\max \quad \left[1 - \Gamma(\overline{\eta}_{t+1})\right] \Delta_{t+1} q_t a_t^{(+)} \tag{5}$$

subject to the lender's participation constraint (4). In the Appendix we characterize this problem in detail.

Optimal financial contract The financial contract specifies a pair $\left\{a_t^{(+)}, \overline{\eta}_{t+1}\right\}$ to maximize the bank's expected income (taking the asset price q_t as given)

$$\max \quad \left[1 - \Gamma(\overline{\eta}_{t+1})\right] \Delta_{t+1} q_t a_t^{(+)}$$

subject to the lender's participation constraint (4). Let χ_t be the Lagrange multiplier on (4). First order conditions with respect to $\overline{\eta}_{t+1}$ and $a_t^{(+)}$ read respectively

$$\Gamma'(\overline{\eta}_{t+1}) = \chi_t \left[\Gamma'(\overline{\eta}_{t+1}) - \xi M'(\overline{\eta}_{t+1})\right] \tag{6}$$

$$\Delta_{t+1} \left[1 - \Gamma(\overline{\eta}_{t+1})\right] q_t + \chi_t \left[\Gamma(\overline{\eta}_{t+1}) - \xi M\left(\overline{\eta}_{t+1}\right)\right] \Delta_{t+1} q_t = \chi_t q_t R_t^d \tag{7}$$

Combining (6) and (7) to eliminate the multiplier χ_t and using (4) yields the equilibrium relation:

$$\mathbb{E}_t \left\{\frac{\Delta_{t+1}}{R_t^d}\right\} \left[1 - \Gamma_{t+1}\right] = \mathbb{E}_t \left\{\frac{1 - \Phi_{t+1}}{\left[1 - \Phi_{t+1} - \xi \overline{\eta}_{t+1} \Phi_{\eta,t+1}\right]}\right\} \frac{nw_t}{q_t a_t} \tag{8}$$

Timing To summarize, we can write the timing of events in the interbank market as follows

- End of period t.

[5] By appealing to the results of the CSV literature one can show that such a contract - characterized by a constant repayment function (i.e., with $\overline{\eta}_{t+1}$ as an argument for *any* realization of $\eta_{t+1} \geq \overline{\eta}_{t+1}$) - is incentive compatible. If the parties involved are also risk-neutral (as are the commercial and investment banks in our case), the contract is also efficient. See Freixat and Rochet (2008) for a simple proof of this result.

1. Each investment bank holds net worth nw_t, borrows b_t in the interbank market to purchase new assets $q_t a_t^{(+)}$

2. Idiosyncratic shock η_{t+1} to the newly purchased asset realizes.

- Period $t+1$.

1. Aggregate shocks realize.

2. Banks supply entrepreneurs with loans for purchase of capital and market production

3. Banks pay off short-term loan to lender.

4. Current net worth realizes.

Aggregation and return premium The linearity of the optimal contract allows easy aggregation, hence all variables can be henceforth interpreted as averages.

Combining (6) and (7), and aggregating, yields a relation between the expected return on the asset portfolio and the riskless return paid on deposits:

$$\mathbb{E}_t \left\{ \frac{\Delta_{t+1}}{R_t^d} \right\} = \theta(\overline{\eta}_{t+1}) \tag{9}$$

where

$$\theta(\overline{\eta}_{t+1}) \equiv \left[\left(1 - \Gamma(\overline{\eta}_{t+1})\right) \left(1 - \xi \frac{M'(\overline{\eta}_{t+1})}{\Gamma'(\overline{\eta}_{t+1})}\right) + \Gamma(\overline{\eta}_{t+1}) - \xi M\left(\overline{\eta}_{t+1}\right) \right]^{-1} > 1.$$

Equation (14) establishes that the equilibrium gross return on the portfolio held by the bank must exceed the riskless return on deposits by a premium $\theta(\cdot)$, which depends on the endogenous cutoff value $\overline{\eta}_{t+1}$. One can show that $\theta(\overline{\eta}_{t+1})$ is increasing in $\overline{\eta}_{t+1}$, and therefore in the probability of default.[6]

[6]This is a standard result in the CSV literature. See, for instance, Bernanke et al. (1999), where this type of contract is applied to the relationship between lenders and capital producers.

Asset demand and net worth By combining (4) with (14) one can further write a relationship between asset demand and banks' net worth whose proportionality factor depends endogenously on the return premium $\theta(\cdot)$:

$$q_t a_t^{(+)} = \frac{nw_t}{1 - \mathbb{E}_t \left\{ \theta(\overline{\eta}_{t+1}) \left[\Gamma(\overline{\eta}_{t+1}) - \xi M \left(\overline{\eta}_{t+1} \right) \right] \right\}} \tag{10}$$

Equation (10) is a key relationship in this context, for it explicitly shows the link between the demand for financial assets, the return premium $\theta(\cdot)$ and the banks' balance sheet (summarized by aggregate net worth).

On the other hand, one can also write the return premium as:

$$\begin{aligned} \mathbb{E}_t \left\{ \theta(\overline{\eta}_{t+1}) \right\} &= h(\overline{\eta}_{t+1}) \left(1 - \frac{nw_t}{q_t a_t^{(+)}} \right) \\ &= h(\overline{\eta}_{t+1}) \left(1 - \frac{1}{LV_t} \right) \end{aligned} \tag{11}$$

where $h(\overline{\eta}_{t+1}) \equiv \left[\Gamma(\overline{\eta}_{t+1}) - \xi M \left(\overline{\eta}_{t+1} \right) \right]^{-1}$. It is easy to show that $h'(\bullet) > 0$. This expression suggests that the interbank reurn premium is an equilibrium direct function of the bank's leverage LV_t.

Aggregate net worth and finance premium To derive the evolution of aggregate banks' net worth, we postulate that at the end of each period, a fraction ζ of banks exit the market. Aggregate net worth at the end of period t is proportional to the realization of the return on holding the assets:

$$nw_t = \zeta \left[1 - \Gamma(\overline{\eta}_t) \right] \Delta_t q_{t-1} a_{t-1}^{(+)} \tag{12}$$

By lagging (4) one period and combining with (12) one can describe the evolution between period $t-1$ and t of investment banks' aggregate net worth as:[7]

$$nw_t = \zeta \left[\underbrace{\Delta_t q_{t-1} a_{t-1}^{(+)}}_{\text{ex-post asset return}} - \underbrace{\left(R_{t-1}^d + \varphi_t \right) b_{t-1}}_{\text{effective cost of borrowing}} \right] \tag{13}$$

[7]See the Appendix for a detailed derivation.

where

$$\varphi_t \equiv \frac{\xi M(\overline{\eta}_t)\Delta_t q_{t-1} a_{t-1}^{(+)}}{q_{t-1}a_{t-1} - nw_{t-1}} \tag{14}$$

The above expression states that net worth at the end of period t (and before each banks enters the market to sign new financial contract) is the difference between the ex-post realized return on holding the assets and the effective cost of borrowing. The latter depends on two terms: first, the opportunity cost of finance,R_{t-1}^d, for lenders participating in the interbank market; second, a *finance premium* φ_t which reflects the presence of asymmetric information and monitoring costs in the interbank market. Notice that in the case in which monitoring is costless, i.e., $\xi = 0$, then the finance premium φ_t is always zero.

2.1 Households

The households purchase physical capital $k_{h,t}$ (which is in fixed supply) to be used in a home production technology. They also save in the form of real one-period deposits that earn a rate of return R_t^d. The households' problem can be written:

$$\max \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta_h^t \mathbb{U}(c_{h,t}) \right\}$$

subject to the flow budget constraint:

$$c_{h,t} + z_t \left(k_{h,t} - k_{h,t-1}\right) + d_t = R_{t-1}^d d_{t-1} + G(k_{h,t}) \tag{15}$$

where $c_{h,t}$ is households' consumption, z_t is the price of capital, d_{t-1} are one-period riskless deposits held at the beginning of time t and yielding a gross rate of interest R_{t-1}^d, and $G(k_{h,t})$ is a home production function, with $G_k(\cdot) > 0$ and $G_{kk}(\cdot) < 0$.

Efficiency conditions of the household's problem read:

$$\lambda_{h,t} = \mathbb{U}_{c_h,t} \tag{16}$$

$$\lambda_{h,t} = \beta_h \mathbb{E}_t \left\{ \lambda_{h,t+1} R_t^d \right\} \tag{17}$$

$$\lambda_{h,t} = \beta_h \mathbb{E}_t \left\{ \lambda_{h,t+1} \left[z_{t+1} + G^{'} \left(k_{h,t+1} \right) \right] \right\} \tag{18}$$

2.2 Entrepreneurs

Entrepreneurs issue bonds in order to finance the purchase of consumption and physical capital. The entrepreneurs are relatively more impatient than the households. This assumption justifies why, in equilibrium, they act as borrowers, i.e., as *issuers* of bonds. The entrepreneurs use physical capital for the production of a homogenous final good. Due to the presence of an enforcement problem, they can issue bonds only up to the expected market value of their capital stock at the date of maturity. In other words, lenders cannot secure their loans, and therefore require those loans to be collateralized. A key assumption is that the bonds issued by the entrepreneurs are purchased directly by the investment banks. To a first approximation, this assumption captures a crucial ingredient of a process of asset-backed securitization, thereby (investment) banks use underlying assets (such as "re-packaged" loans) as collateral for short-term borrowing in the interbank market:

Formally, the problem of a representative entrepreneur can be written:

$$\max \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta_e^t \mathbb{U}(c_{e,t}) \right\} \quad \beta_e < \beta_h$$

subject to the flow budget constraint

$$c_{e,t} + z_t \left(k_{e,t} - k_{e,t-1} \right) + q_{t-1}^{-1} a_{t-1}^{(-)} = a_t^{(-)} + y_t, \tag{19}$$

and to the collateral constraint on the issue of bonds

$$a_t^{(-)} \leq (1 - \gamma_t) \mathbb{E}_t \left\{ k_{e,t} \ z_{t+1} \right\}, \tag{20}$$

where $c_{e,t}$ is entrepreneurs' consumption, and

$$y_t = F(k_{e,t-1})$$

is the market production function of the homogeneous good, with $F_k(\cdot) > 0$ and $F_{kk}(\cdot) < 0$.

In equation (20) γ_t is the inverse loan-to-value ratio, which evolves exogenously according to:

$$\log \gamma_t = (1-\rho_\gamma)\log \gamma + \rho_\gamma \log \gamma_{t-1} + \varepsilon_{\gamma,t},$$

where $\varepsilon_{\gamma,t} \sim \mathbb{N}(0,1)$. Random variations in the LTV ratio are an additional source of "financial shocks" in the economy. Unlike shocks to the riskiness of assets, that specifically originate in the interbank market, shocks to the LTV ratio characterize directly the financial relationship between the entrepreneurs and the lenders.

Define by $\lambda_{e,t}$ and $\lambda_{e,t}\psi_t$ the multipliers on the above constraints respectively. First order conditions of the entrepreneur's problem read:

$$\lambda_{e,t} = \mathbb{U}_{c_e,t}, \tag{21}$$

$$\lambda_{e,t} z_t = \beta_e \mathbb{E}_t \left\{ \lambda_{e,t+1} \left[z_{t+1} + mpk_{t+1} \right] \right\} \; + s_{k,t}, \tag{22}$$

$$\lambda_{e,t} = \beta_e \mathbb{E}_t \left\{ \lambda_{e,t+1} R_t^a \right\} + \lambda_{e,t}\psi_t, \tag{23}$$

where

$$s_{k,t} \equiv (1-\gamma_t)\psi_t \lambda_{e,t} \mathbb{E}_t \left\{ z_{t+1} \right\} \tag{24}$$

is the shadow value of one unit of capital, and

$$R_t^a = q_t^{-1}$$

is the gross interest rate on a one-period bond issued in time t-1 (and therefore maturing at t).[8]

[8] Let $q_{m,t}$ be the price of a riskless asset of maturity m. The corresponding (compounded) yield to maturity of this asset is

$$r_{m,t} = -(1/m)\log q_{m,t}$$

The yield to maturity can alternatively be written:

$$\exp\left\{ r_{m,t} \right\} \simeq \exp\left\{ \log(1+r_{m,t}) \right\} \simeq R_{m,t}$$

where $R_{m,t} \equiv 1 + r_{m,t}$ is the gross yield. Combining the above equations one can write

Equation (22) equates the marginal cost of one unit of capital $\lambda_{e,t}z_t$ to its (expected discounted) marginal benefit. The latter has three components: (i) the expected future price of capital (capital acquired today can be resold to morrow); (ii) the marginal product (capital purchased today can be used in production tomorrow); (iii) the shadow value of borrowing. The last component captures the idea that, at the margin, a new unit of capital acquired at time t can be used as collateral in borrowing. When the collateral constraint (20) is not binding ($\psi_t = 0$ all t), then we have $s_{k,t} = 0$ for all t. Notice that shocks to the LTV ratio, via random variations in γ_t, hit directly the shadow value $s_{k,t}$, and therefore the entrepreneurs' ability to borrow. This type of shocks will be the prototype "credit supply shocks" in our context.

Equation (23) is the entrepreneurs' bond Euler condition. When the collateral constraint is binding, that condition states that the marginal utility of consumption, $\lambda_{e,t}$, exceeds the (expected) marginal utility of saving, $\beta_e \mathbb{E}_t \{\lambda_{e,t+1} R_t^a\}$.

Let's define the *effective* real interest rate faced by the entrepreneurs, $m_{e,t}$, as:

$$m_{e,t} \equiv \frac{\lambda_{e,t}}{\beta_e \lambda_{e,t+1}} \tag{25}$$

Then the entrepreneurs' external finance premium turns out to be proportional to the shadow value of borrowing ψ_t (the multiplier on the collateral constraint (20)). From (23) and(25), in fact, we can write:

$$\begin{aligned} f_t &\equiv \mathbb{E}_t \{m_{e,t} - R_t^a\} \\ &= R_t^a \frac{\psi_t}{1 - \psi_t} \end{aligned} \tag{26}$$

In turn, the above expression can be re-written, using (24), as:

$$R_{m,t} = (q_{m,t})^{-1/m}$$

This establishes a standard negative relationship between the price of an asset and its gross yield to maturity. In our context, $m = 1$, and therefore

$$R_{1,t} = q_{1,t}^{-1} = R_{a,t} = R_{d,t}$$

$$f_t = R_t^a \mathbb{E}_t \left\{ \frac{s_{k,t}}{(1-\gamma_t)\lambda_{e,t} z_{t+1} - s_{k,t}} \right\} \tag{27}$$

Equation (27) shows that the finance premium depends positively on both the shadow value of capital, and on the inverse LTV ratio γ_t. Hence shocks to the latter have the alternative interpretation of credit supply shocks, expressed in the form of risk premia shocks.

2.3 Commercial banks

Commercial banks collect deposits from the households and lend in the interbank market. As in Curdia and Woodford (2010), this intermediation activity is subject to a resource cost captured by a function $\Xi(b_t)$ which is increasing andt convex in the amount b_t lent. The balance sheet of the commercial banks therefore reads (in real units):

$$b_t + \Xi(b_t) = d_t \tag{28}$$

The presence of intermediation costs generates a spread between the deposit rate and the interbank interest rate. Hence in equilibrium it will hold

$$R_t^b = (1 + \Xi_b(b_t)) R_t^d \tag{29}$$

3 Equilibrium

In the following, to simplify the notation, we adopt the convention $\Gamma_t \equiv \Gamma(\overline{\eta}_t, \sigma_{\eta,t})$, $M_t \equiv M(\overline{\eta}_t, \sigma_{\eta,t})$, $\Phi_t \equiv \Phi(\overline{\eta}_t, \sigma_{\eta,t})$, $\Phi_{\eta,t} \equiv \partial\Phi(\overline{\eta}_t, \sigma_{\eta,t})/\partial\overline{\eta}_t$. For any exgoenous process $\{\log \sigma_{\eta,t},\ \log \gamma_t\}$ an equilibrium in this economy is a set of allocations for $\{d_t, b_t,\ a_t, nw_t,\ c_{e,t}, c_{h,t}, \psi_t, k_{e,t},$ that solve the following set of conditions:

- First-order condition of the optimal financial contract.

$$\mathbb{E}_t \left\{ \frac{\Delta_{t+1}}{R_t^d} \right\} [1 - \Gamma_{t+1}] = \mathbb{E}_t \left\{ \frac{1 - \Phi_{t+1}}{\left[1 - \Phi_{t+1} - \xi \overline{\eta}_{t+1} \Phi_{\eta,t+1}\right]} \right\} \frac{nw_t}{q_t a_t} \tag{30}$$

- Lender's zero-profit condition, holding for any time t and state.

$$[\Gamma_t - \xi M_t] \frac{\Delta_t}{R^d_{t-1}} = 1 - \frac{nw_{t-1}}{q_{t-1}a_{t-1}} \tag{31}$$

- Definition of banks' net worth

$$q_t a_t = nw_t + b_t \tag{32}$$

- Evolution of banks' net worth,

$$nw_t = \zeta \left[\Delta_t q_{t-1} a^{(+)}_{t-1} - \left(R^d_{t-1} + \frac{\xi M_t \Delta_t q_{t-1} a^{(+)}_{t-1}}{q_{t-1}a_{t-1} - nw_{t-1}} \right) b_{t-1} \right] \tag{33}$$

- Asset return

$$\mathbb{E}_t \left\{ \Delta_{t+1} \right\} = \mathbb{E}_t \left\{ q_{t+1}/q_t \right\} \tag{34}$$

- Commercial banks' conditions

$$b_t + \Xi(b_t) = d_t \tag{35}$$

$$R^b_t = (1 + \Xi^{'}(b_t))R^d_t \tag{36}$$

- Households' conditions

$$c_{h,t} + z_t \left(k_{h,t} - k_{h,t-1} \right) + d_t = R^d_{t-1} d_{t-1} + G(k_{h,t}) \tag{37}$$

$$\lambda_{h,t} = \frac{1}{c_{h,t}} \tag{38}$$

$$\lambda_{h,t} = \beta_h \mathbb{E}_t \left\{ \lambda_{h,t+1} R_t^d \right\} \tag{39}$$

$$\lambda_{h,t} = \beta_h \mathbb{E}_t \left\{ \lambda_{h,t+1} \left[z_{t+1} + G^{'}\left(k_{h,t+1}\right) \right] \right\} \tag{40}$$

- Entrepreneurs' conditions

$$c_{e,t} + z_t \left(k_{e,t} - k_{e,t-1}\right) + q_{t-1}^{-1} a_{t-1}^{(-)} = a_t^{(-)} + F(k_{e,t-1}) \tag{41}$$

$$a_t^{(-)} = (1-\gamma_t) \mathbb{E}_t \left\{ k_{e,t} \; z_{t+1} \right\} \tag{42}$$

$$\lambda_{e,t} = \frac{1}{c_{e,t}} \tag{43}$$

$$\lambda_{e,t} z_t = \beta_e \mathbb{E}_t \left\{ \lambda_{e,t+1} \left[z_{t+1} + mpk_{t+1} \right] \right\} \quad + (1-\gamma_t) \psi_t \lambda_{e,t} \mathbb{E}_t \left\{ z_{t+1} \right\} \tag{44}$$

$$\lambda_{e,t} = \beta_e \mathbb{E}_t \left\{ \lambda_{e,t+1} R_t^a \right\} + \lambda_{e,t} \psi_t \tag{45}$$

- Equilibrium in the asset market

$$a_t^{(+)} + a_t^{(-)} = 0 \tag{46}$$

- Equilibrium in the market for physical capital

$$k_{e,t} + k_{h,t} = \overline{k} \tag{47}$$

4 Steady state

Using equations (6), (7), and (4) that characterize the optimal financial contract, setting the risk-free rate $R^d = 1/\beta_h$, we can write the system:

$$\frac{\Delta}{1/\beta_h}\left[1-\Gamma(\overline{\eta},\sigma_\eta)\right] = \frac{1-\widetilde{\Phi}(\overline{\eta},\sigma_\eta)}{\left[1-\widetilde{\Phi}(\overline{\eta},\sigma_\eta)-\xi\overline{\eta}\Phi_\eta(\overline{\eta},\sigma_\eta)\right]}(1-V) \tag{48}$$

$$\left[\Gamma(\overline{\eta},\sigma_\eta)-\xi M(\overline{\eta},\sigma_\eta)\right]\frac{\Delta}{1/\beta_h} = V \tag{49}$$

$$\Phi(\overline{\eta},\sigma_\eta) = \widetilde{\Phi}(\overline{\eta},\sigma_\eta) \tag{50}$$

By setting values for $\widetilde{\Phi}(\cdot)$, the probability of banks' (or asset) default, ξ (the share of income going to the lender after bankruptcy), and V (the bank's debt-to-asset ratio), the above system can be solved for Δ, the steady-state asset return, $\overline{\eta}$, the steady-state threshold value for the idiosyncratic shock (such that $\eta < \overline{\eta}$ implies asset default), and σ_η, the steady-state standard deviation of $\overline{\eta}$. Using the assumption $E(\eta) = 1$, recall that (as an implication of η being log-normally distributed), any given value of σ_η implies also $\mu_\eta = -(1/2)\sigma_\eta^2$.

Next, we can solve for the remaining part of the steady state recursively. The entrepreneurs' Euler condition (23) implies:

$$\psi = 1 - \frac{\beta_e}{\beta_h} > 0$$

which states that,as a result of heterogeneity in patience rates, the collateral constraint is binding in the steady state.

The households' capital Euler equation (18), the entrepreneurs' budget constraint (19) and the Euler condition on physical capital (22), respectively imply:

$$z = \frac{\beta_h G_k(k-k_e)}{1-\beta_h} \tag{51}$$

$$c_e = (1-\gamma)k_e z\left(\frac{1-\beta_h}{\beta_h}\right) + F(k_e) \tag{52}$$

$$z = \frac{\beta_e F_k(k_e)}{B} \tag{53}$$

where $B \equiv 1 - \beta_e(1-\gamma)\left[1 - (\beta_e/\beta_h)\right] > 0$.

For any given value of the (fixed) aggregate capital stock, k, and of the steady state inverse LTV ratio, γ, the system (51), (52), (53) can be solved for the three unknowns z, k_e, c_e. We proceed by normalizing the relative price of capital $z = 1$, and solving for the aggregate capital stock consistent with that value. Hence from (51) we obtain:

$$k_e = F_k^{-1}\left(\frac{B}{\beta_e}\right).$$

Using (**??**), and susbtituting into (51), one finally obtains:

$$k = \left[G_k^{-1}\left(\frac{1-\beta_h}{\beta_h}\right)\right] + \left[F_k^{-1}\left(\frac{B}{\beta_e}\right)\right]$$

5 Calibration

We compare two alternative steady states, with low and high banks leverage, respectively $V_{low} = 0.3$, and $V_{high} = 0.6$. We set the steady-state asset default rate $\widetilde{\Phi}(\cdot) = 0.04$ at the quarterly level. This is higher than the quarterly rate of business failure observed in the data, and calibrated accordingly by Bernanke et al. (1999) and Christiano et. al (2009). In general, by increasing the parameterized value of $\widetilde{\Phi}(\cdot)$, one can model alternative scenarios with higher (steady-state) asset return uncertainty. The remaining calibrated values are listed in Table 1 below.

Table 1. Calibration in Simulation Exercise		
Parameter	**Description**	**Value**
β_e	entrepreneurs' rate of time preference	0.98
β_h	households' rate of time preference	0.99
ζ	share of banks surviving each period	0.987
ξ	share of bankruptcy costs	0.2
σ_η	match $\Phi(\overline{\eta}, \sigma_\eta) = 0.04$ at quarterly level	-
ω	K share in home production	0.1
α	K share in market production	0.4
ρ_σ	persistence of σ shock	0.9
ρ_γ	persistence of γshock	0.9

6 Liquidity shocks and real activity

We are interested in studying the transmission to real economic activity of a specific type of "financial shock". This is a shock to the riskiness of assets, namely a sudden increase in uncertainty in the distribution of the idiosyncratic disturbance to asset returns. In the sense we want to think of this shock as a shock to liquidity in the interbank market. In practice, the shock is modelled as a random innovation in the stochastic process for $\sigma_{\eta,t}$.

Riskiness of assets and return premium In order to better understand the effects of variations in the riskiness of assets it is useful to log-linearize conditions (30) and (31), to obtain

$$\mathbb{E}_t \widehat{\Delta}_{t+1} = \widehat{R}^d_t - \mathcal{D}_{\eta,1} \mathbb{E}_t \left\{ \widehat{\overline{\eta}}_{t+1} \right\} - \mathcal{S}_{\sigma,1} \mathbb{E}_t \left\{ \widehat{\sigma}_{t+1} \right\} + nw_t - \widehat{q}_t - \widehat{a}_t \tag{54}$$

$$\Delta_t = \widehat{R}^d_{t-1} - \mathcal{D}_{\eta,2} \widehat{\overline{\eta}}_t - \mathcal{S}_{\sigma,2} \widehat{\sigma}_t + nw_{t-1} - \widehat{q}_{t-1} - a_{t-1} \tag{55}$$

where variables with a hat denote percentage deviations from respective steady state values. The impact of a rise in the time-varying standard deviation $\widehat{\sigma}_t$ depends on the sign of the coefficients $\mathcal{S}_{\sigma,j}$ and $\mathcal{D}_{\sigma,j}$, $j = 1, 2$.

In the Appendix, as in Christiano et al. (2010) and Fernandez-Villaverde (2010), we show in detail how to derive the coefficients $\mathcal{S}_{\sigma,j}$ and $\mathcal{D}_{\sigma,j}$. We also show that for a large class of parameter values we have

$$\mathcal{S}_{\sigma,j} < 0 \quad j = 1, 2$$

Hence a rise in the riskiness of assets $\widehat{\sigma}_t$ produces a *rise* in both the actual and the expected future return premium Δ_t.

Figure (3) displays the effects of a rise of 5% in the standard deviation $\sigma_{\eta,t}$ of the idiosyncratic disturbance η. In this scenario we set the steady state banks' debt-to-asset ratio $V = 0.5$. This corresponds to a value of leverage of $LV = 1/(1 - V) = 2$.

The rise in the return premium produces two competing effects on banks' end-of-period net worth: on the one hand, it raises net worth directly by raising the asset ex-post return; but on the other, it raises the effective cost of borrowing (the finance premium), thereby - ceteris paribus - lowering net worth.

The downward pressure on net worth is reinforced by an additional effect: namely, a rise in the probability of asset default, induced by a rise in $\overline{\eta}_t$. In order to understand this point, consider equation (9). Recall that the return premium is increasing in $\overline{\eta}_t$, hence the rise in the return premium must be accompanied by an equilibrium rise in the default threshold $\overline{\eta}_t$. Thus, intuitively, an initial rise in the riskiness of asset returns produces a higher rate of default in the same assets. In turn, this raises $M(\overline{\eta}_t)$ and therefore the finance premium φ_t, contributing to a further reduction in net worth.

The overall equilibrium effect is therefore a *fall* in banks' net worth, which depresses the demand for assets, and drives asset prices down. In turn, the fall in asset prices lowers banks' net worth even further, inducing a downward acceleration effect.

Both the fall in asset prices and in the demand for assets induce a process of deleveraging in the investment banking sector. The key to the transmission of the shock to aggregate economic activity is that the process of banks' deleveraging induces a contraction in credit supply. This effect, coupled with the fall in the price of capital, forces the entrepreneurs to reduce their borrowing and therefore their capital stock. In turn, this induces a contraction of output, with this effect being particularly severe in the scenario with high banks' leverage.

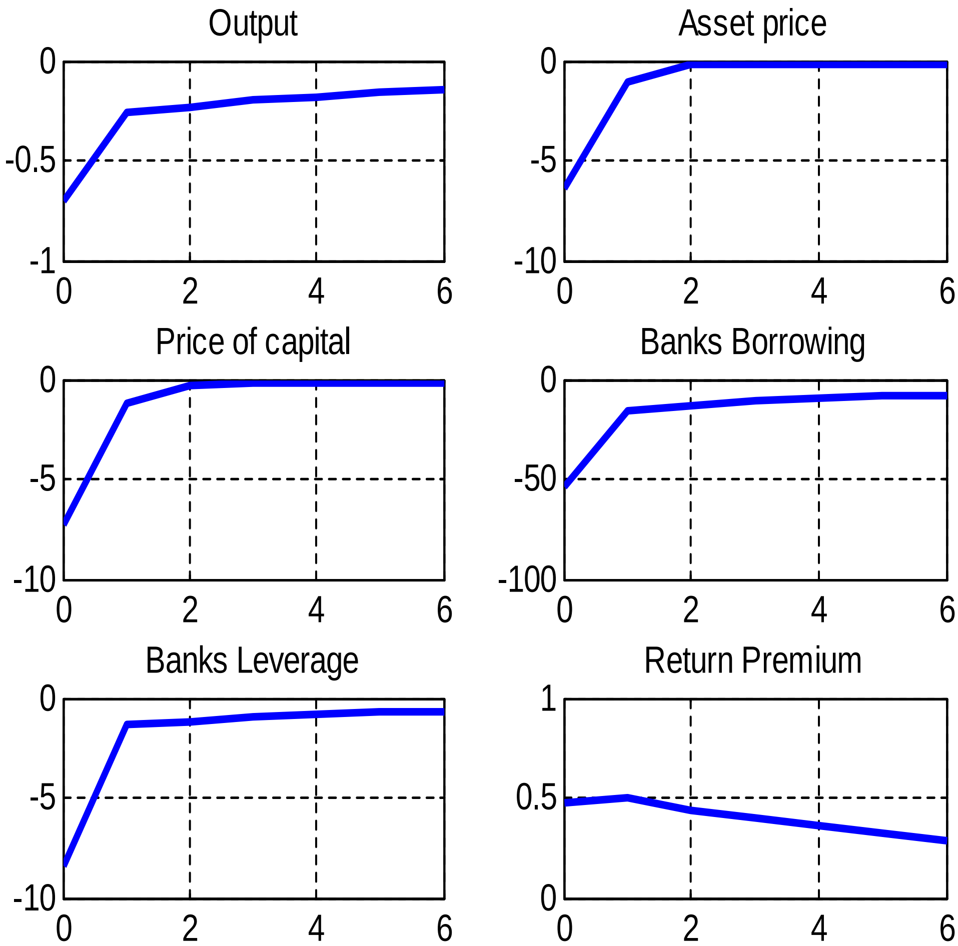

Figure 3: Effect of a Negative Liquidty Shock: Impulse Responses to a Rise in the Riskiness of Banks' Assets. (% deviations from steady state)

Sensitivity analysis (to be added)

References

(to be added)

Appendix

Evolution of aggregate net worth Aggregate (average) net worth at the end of period t is proportional to the banks' ex-post asset return:

$$nw_t = \zeta \left[1 - \Gamma(\overline{\eta}_t)\right] \Delta_t q_{t-1} a_{t-1}^{(+)}$$

From the lenders' zero profit condition (4) holding with equality (i.e., under the assumption $\chi_t > 0$), after lagging one period, one can write:

$$\Gamma(\overline{\eta}_t) = \left(q_{t-1} a_{t-1}^{(+)} - nw_{t-1}\right) \left[\frac{R_{t-1}^d}{\Delta_t q_{t-1} a_{t-1}^{(+)}} + \frac{\xi M\left(\overline{\eta}_t\right)}{\left(q_{t-1} a_{t-1}^{(+)} - nw_{t-1}\right)} \right]$$

Substituting (**??**) into (**??**) and rearranging one obtains:

$$\begin{aligned} nw_t &= \zeta \Delta_t q_{t-1} a_{t-1}^{(+)} - \\ &\quad \zeta \left(q_{t-1} a_{t-1}^{(+)} - nw_{t-1}\right) \left[\frac{R_{t-1}^d}{\Delta_t q_{t-1} a_{t-1}^{(+)}} + \frac{\xi M\left(\overline{\eta}_t\right)}{\left(q_{t-1} a_{t-1}^{(+)} - nw_{t-1}\right)} \right] \Delta_t q_{t-1} a_{t-1}^{(+)} \\ &= \zeta \Delta_t q_{t-1} a_{t-1}^{(+)} - \zeta \left[R_{t-1}^d + \frac{\xi M\left(\overline{\eta}_t\right) \Delta_t q_{t-1} a_{t-1}^{(+)}}{\left(q_{t-1} a_{t-1}^{(+)} - nw_{t-1}\right)} \right] \left(q_{t-1} a_{t-1}^{(+)} - nw_{t-1}\right) \end{aligned}$$

Deriving $\Gamma\left(\overline{\eta}, \sigma_\eta\right)$ and $M\left(\overline{\eta}, \sigma_\eta\right)$ Recall, first, that steady state values for $\overline{\eta}$ and σ_η are derived from the solution of the system (48)-(50). Also, we already know that:

$$\Gamma\left(\overline{\eta}, \sigma_\eta\right) = \overline{\eta}\left(1 - \Phi\left(\overline{\eta}, \sigma_\eta\right)\right) + M\left(\overline{\eta}, \sigma_\eta\right) \tag{56}$$

Notice that we can write:

$$
\begin{aligned}
M(\overline{\eta},\sigma_\eta) &\equiv \int_0^{\overline{\eta}} \eta d\Phi(\overline{\eta},\sigma_\eta) \\
&= \int_0^{\infty} \eta\phi(\overline{\eta},\sigma_\eta)\,d\eta - \int_{\overline{\eta}}^{\infty} \eta\phi(\overline{\eta},\sigma_\eta)\,d\eta \\
&= \mathbb{E}\{\eta\} - \underbrace{\exp\left(\mu_\eta + \frac{\sigma_\eta^2}{2}\right)}_{=1} \Phi_N\left(\frac{(1/2)\sigma_\eta^2 - \log\overline{\eta}}{\sigma_\eta^2}\right)
\end{aligned} \tag{57}
$$

where Φ_N is the *normal* cdf. Both $M(\overline{\eta},\sigma_\eta)$ and $\Gamma(\overline{\eta},\sigma_\eta)$ can be easily computed using the *normcdf* the *logncdf* routines in Matlab.

Deriving coefficient $\mathcal{S}_{\sigma,1}$ Differentiating (56) we can write:

$$
\Gamma_{\sigma_\eta} = -\overline{\eta}\Phi_{\sigma_\eta} + M_{\sigma_\eta}
$$

where $\Gamma_{\sigma_\eta} \equiv \partial\Gamma(\overline{\eta},\sigma_\eta)/\partial\sigma_\eta$ and $M_{\sigma_\eta} \equiv \partial M(\overline{\eta},\sigma_\eta)/\partial\sigma_\eta$, with all the expressions evaluated at the deterministic steady state.

The expressions for coefficient $\mathcal{S}_{\sigma,1}$ is :

$$
\begin{aligned}
\mathcal{S}_{\sigma,1} &= \frac{-\overline{\eta}\Gamma_{\sigma_\eta}}{1-\Gamma(\overline{\eta},\sigma_\eta)} - s_{\sigma,1} \\
&= \frac{\overline{\eta}^2\Phi_{\sigma_\eta} - \overline{\eta}M_{\sigma_\eta}}{1-\Gamma(\overline{\eta},\sigma_\eta)} - s_{\sigma,1},
\end{aligned}
$$

where

$$
s_{\sigma,1} = \frac{-\Phi_{\sigma_\eta}}{1-\Phi} + \frac{\Phi_{\sigma_\eta}(1+\xi) + \xi\overline{\eta}\Phi_{\sigma_\eta,\sigma_\eta}}{1-\Phi-\xi\overline{\eta}\Phi_{\sigma_\eta}},
$$

and $\Phi_{\sigma_\eta,\sigma_\eta} \equiv \partial^2\Phi/\partial\sigma_\eta$.

Hence, in order to compute $\mathcal{S}_{\sigma,1}$ one needs to evaluate three objects: Φ_{σ_η}, M_{σ_η}, and $\Phi_{\sigma_\eta,\sigma_\eta}$. First, we can write:

$$
\Phi_{\sigma_\eta} = \frac{\partial}{\partial\sigma_\eta}\left\{\int_0^{\overline{\eta}} \phi(\eta)d\eta\right\} = \int_0^{\overline{\eta}} \phi_{\sigma_\eta}d\eta \tag{58}
$$

where $\phi(\eta) \equiv \left(1/\eta\sigma_\eta\sqrt{2\pi}\right) \exp\left(-\frac{\left[\log(\overline{\eta})+(1/2)\sigma_\eta^2\right]^2}{2\sigma_\eta^2}\right)$ and $\phi_{\sigma_\eta} \equiv \partial\phi(\eta,\sigma_\eta)/\partial\sigma_\eta$.

Differentiating $\phi(\eta)$ with respect to σ_η yields:

$$\phi_{\sigma_\eta} = -\frac{\sigma_\eta^4 + 4\left(\sigma_\eta^2 - \log\left(\overline{\eta}\right)^2\right)}{4\sqrt{2\pi}\overline{\eta}\sigma_\eta^4 \exp\left(-\frac{\left[2\log(\overline{\eta})+\sigma_\eta^2\right]^2}{8\sigma_\eta^2}\right)} \tag{59}$$

Given the above expression for ϕ_{σ_η}, and given values for $\overline{\eta}$ and σ_η, one can then evaluate the integral in (58) numerically. We use the Matlab routine $quad(f,a,b)$ that approximates the integral of scalar-valued function f from a to b using the recursive adaptive Simpson quadrature.

We proceed similarly to compute M_{σ_η}. We have:

$$M_{\sigma_\eta} = \frac{\partial}{\partial\sigma_\eta}\int_0^{\overline{\eta}} \eta\phi_{\sigma_\eta} d\eta = \int_0^{\overline{\eta}} \eta\phi_{\sigma_\eta} d\eta \tag{60}$$

Using (59) to obtain an expression for $\eta\phi_{\sigma_\eta}$, one can then evaluate (60) numerically.

Finally, we have:

$$\Phi_{\sigma_\eta,\sigma_\eta} = \frac{\partial}{\partial\sigma_\eta}\left\{\int_0^{\overline{\eta}} \phi_{\sigma_\eta} d\eta\right\} = \int_0^{\overline{\eta}} \phi_{\sigma_\eta,\sigma_\eta} d\eta \tag{61}$$

where $\phi_{\sigma_\eta,\sigma_\eta} \equiv \partial^2\phi(\eta,\sigma_\eta)/\partial\sigma_\eta$. Differentiating (59) with respect to σ_η, and plugging it into (61), one can then evaluate the expression numerically.

Deriving coefficient $\mathcal{S}_{\sigma,2}$ To obtain coefficient $\mathcal{S}_{\sigma,2}$ we proceed similarly to above. The corresponding expression is:

$$\begin{aligned}
\mathcal{S}_{\sigma,2} &= \frac{\left(\Gamma_{\sigma_\eta} - \xi M_{\sigma_\eta}\right)\sigma_\eta}{\Gamma\left(\overline{\eta},\sigma_\eta\right) - \xi M\left(\overline{\eta},\sigma_\eta\right)} \\
&= \frac{\left[-\xi\Phi_{\sigma_\eta} + M_{\sigma_\eta}(1-\xi)\right]\sigma_\eta}{\Gamma\left(\overline{\eta},\sigma_\eta\right) - \xi M\left(\overline{\eta},\sigma_\eta\right)}
\end{aligned}$$

Deriving coefficient $\mathcal{D}_{\eta,1}$ and $\mathcal{D}_{\eta,2}$ The expressions for coefficients $\mathcal{D}_{\eta,1}$ and $\mathcal{D}_{\eta,2}$ are as follows:

$$\mathcal{D}_{\eta,1} = \frac{(1 - \Phi(\eta, \sigma_\eta))\,\overline{\eta}}{\Gamma\left(\overline{\eta}, \sigma_\eta\right) - 1} - d_{\eta,1} \tag{62}$$

where

$$d_{\eta,1} \equiv \overline{\eta}\left[-\frac{\phi(\eta, \sigma_\eta)}{1 - \Phi(\eta, \sigma_\eta)} + \frac{\phi(\eta, \sigma_\eta) + \xi\overline{\eta}\phi_\eta + \xi\phi(\eta, \sigma_\eta)}{1 - \Phi(\eta, \sigma_\eta) - \xi\overline{\eta}\phi(\eta, \sigma_\eta)}\right],$$

Also,

$$\mathcal{D}_{\eta,2} = \overline{\eta}\left(\frac{1 - \Phi(\eta, \sigma_\eta) - \xi\overline{\eta}\phi(\eta, \sigma_\eta)}{\Gamma\left(\overline{\eta}, \sigma_\eta\right) - \xi M\left(\overline{\eta}, \sigma_\eta\right)}\right) \tag{63}$$

Since both $\phi(\eta, \sigma_\eta)$ and ϕ_η have analytical expressions, all that is needed is to employ, as above, the *logcdf* routine in Matlab to compute $\Gamma\left(\overline{\eta}, \sigma_\eta\right)$, $M\left(\overline{\eta}, \sigma_\eta\right)$ and $\Phi(\eta, \sigma_\eta)$.