Assessing the Linkages Between Financial Stress and Business Cycles**

Adrian Pagan[†] and Tim Robinson[‡]

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**The views expressed herein are those of the authors and are not necessarily those of the Reserve Bank of Australia. Mardi Dungey gave us useful comments on an earlier version.

[†]University of Technology, Sydney

[‡]Reserve Bank of Australia

1 Introduction

The global financial crisis (GFC) has led to a consideration of how one models the connections between financial stress and the business cycle. Quite a few models and analyses have emerged that aim to elucidate these relationships e.g. Gilchrist et al (2009), Christensen and Dib (2010), Zheng et al (2009), Gertler and Kiyotaki (2010), Greenlaw et al (2008), Liu et al (2009) and Benes at al (2009). Broadly, these papers deal with the issues of credit availability and collateral as well as the role of "animal spirits" in initializing and propagating cycles. Questions which naturally arise are what financial conditions contribute to the generation of a business cycle, whether this information can be used to predict recessions, and what type of features need to be accounted for in models of the cycle. In section 2 of the paper we review some of the major contributions to the literature, outlining the various strategies that have been employed, discussing them according to whether they influence the supply or demand for credit (debt) by agents.

In section 3 we subsequently select one of these models - Gilchrist et al (2009), subsequently termed GOZ - that seems to have enjoyed some success, and ask what the model contributes to the analysis of recessions. GOZ do not look at the latter directly but instead focus upon the decomposition of the transitory component of GDP into contributions from some identified shocks. In doing this assessment it is necessary to make reference to some "stylized facts" relating to recessions and credit. These are drawn from a number of sources, but principally from the work of the IMF reported in a number of issues of the *World Economic Outlook*. Many of the measures the IMF use are unfamiliar to the bulk of the applied macroeconomics literature, since they require a method for determining turning points in economic activity. We find that the model replicates some of the features but it is clear that it is unable to fully capture the effects of financial fragility upon recessions. A number of generalizations of the GOZ framework might be pursued. One is to allow for a stochastic permanent component in GDP, as GOZ only permit a deterministic permanent component to be present. Section 4 outlines that extension. We derive the VECM representation of the model and ask what it would tell us about the ability to quantitatively assess financial linkages.

2 Model Designs

Generally analysis has proceeded by first setting out a base macroeconomic model which is then adapted to incorporate financial stress. The latter is typically done by introducing a financial intermediary (FI) which responds to the demand for credit and which supplies this credit. Sometimes the FI is introduced explicitly and a detailed description is given of its operation. At other times there is a simple summary of what governs the demand for and supply of credit. Throughout we will use the terms "debt" and credit interchangeably, except when it is better to focus upon one term rather than the other

2.1 The Base Macro-economic Model

In order to explore financial-real linkages from a macroeconomic perspective it is necessary to begin with a basic macroeconomic model and then ask how it needs to be modified to provide an active role for credit. In most macroeconomic models where expenditure is not constrained by current income there is a passive role for credit, as some borrowing and lending will be occurring. Implicitly one might think there is an FI present. The macroeconomic model to be chosen as a base-line is contentious. Some e.g. Muellbauer (2010) seem to have a preference for what has been termed second generation (2G) models in Fukacs and Pagan (2009). These provide a set of equations describing macroeconomic outcomes, but are not designed to be fully consistent with actions. This may not be a bad thing, as data issues and the need to choose functional forms may mean that a fully consistent model is trying to describe outcomes that we don't observe. However, ultimately it seems to be more an argument for modifying the structure of some "ideal representation" than completely discarding it.

A popular base model is that in Smets and Wouters (2007). This distinguishes consumption, investment, wages, inflation, monetary policy and the supply side. There are clearly missing items in the model that are likely to be important to macro-economic outcomes e.g. although there is strictly exogenous government expenditure the fiscal side is largely absent since the model lacks taxes. Essentially government expenditure appears in the model only to satisfy the national income identity. Each of the structural equations for consumption, investment, the price of capital, inflation and wages have effects from expectations about the future as well as past events and these are driven by some variable constructed out of the model variables (w_t) i.e. they have the structure

$$z_t = \phi_1 z_{t-1} + \phi_2 E_t z_{t+1} + \phi_3 w_t.$$

In some cases $\phi_1 + \phi_2 = 1$ and, in others, the sum is the discount factor for consumers. Identities are also present and supply is constrained by a production function. It is this model that will be modified to produce an active role for financial factors.

2.2 The Demand For Credit

There are four broad areas of demand that credit may affect

1. Fixed Investment by firms
2. Residential investment by households
3. Consumer durable expenditures by households
4. Consumption of goods and services by households

Clearly the base model does not distinguish these categories, dealing only with an aggregate of types of investment and consumption. Moreover, the four types of expenditures given above does not exhaust the potential list. For example there is an extensive use of credit for financing inventories and this showed up during the GFC in the automobile market, where dealers were unable to get credit to hold the inventories on display in their saleyards. Even in more normal times inventories need to be financed for the short period of time between delivery and sale. Trade credit is also needed in order to pay for raw materials and even labour. But the analytical work on these latter elements has been much less than that on the four areas listed above.

2.2.1 Fixed Investment

This is by far the best developed and involves the financial accelerator. It comes in two versions and implicitly involves a financial intermediary. The financial intermediary can be thought of as taking deposits from the household sector and then lending to the business sector that is in need of credit to finance fixed investment. There has to be some way to distinguish the loaning and credit-using sectors. This is typically done by having the agents associated with each of these possessing different discount rates. The credit-using agents are taken to be less patient than the lending agents. This serves to produce two interest rates - one that is connected to the preferences of the lending agents (and which is typically taken to be the policy interest rate) and another that is the rate charged on loans by a financial intermediary. In many models there is no precise description of the operations of the intermediary, and its presence is simply summarized by an *external finance premium* charged over the policy rate. This strategy is common in the financial accelerator literature wherein credit comes at a cost that is a premium over internal financing resulting from the fact that there is asymmetric information between the borrower and lender. The external finance premium therefore governs the amount of credit that can be obtained, and so it is necessary to model its determinants. Mostly these are taken to be increasing in the degree of leverage. Ultimately, therefore increasing amounts of credit are costly, and this impacts on real and nominal quantities. Because the emphasis in this extension is on the demand for credit the external premium equations are often augmented with a shock that is intended to capture variations in supply i.e. the equation is more of a reduced form one than a structural equation.

There are no direct series on the external finance premium, so either a proxy needs to be constructed or it needs to be left unobservable. GOZ use data on the spreads between medium risk long-maturity U.S. corporate bonds and the 10-year Treasury yield. They also utilize data on the leverage ratio of US firms to provide an estimate of the elasticity of the external finance premium to the leverage ratio.

2.2.2 Residential Investment

The events preceding the GFC led to an interest in the role of housing investment in the business cycle. Indeed some see it as the key to the latter e.g. Leamer (2005). But inspection of the cycle data has to cast doubt on such positions. Looking at the turning points in the series on real gross residential investment by quarter one finds that the duration of the residential investment cycle is quite short, on average around 12 quarters, which is almost half what the business cycle length is. This outcome is easily explained by an examination of the data. The growth rate in residential investment is around half that of GDP while the volatility is about five times. Thus, getting negative growth in residential investment is relatively easy, and such growth often results in a turning point in the series. These differences mean that, even if one had the knowledge that residential investment was in a recession, the probability of predicting an NBER-defined recession would just rise to .25 from its unconditional probability of .15. Thus it is hard to subscribe to Leamer's viewpoint that *housing is the business cycle*. It should be observed that Leamer carried out a good deal of massaging of the data before reaching his conclusion. This included smoothing the residential investment data to eliminate some of its peaks and troughs, so that these more closely resembled those of GDP, and eliminating the difference in the growth rates of the two series. Indeed the smoothing was done by the use of a kernel regression where the regressor was a time trend. This meant that, at time t, one would need to know future data on residential investment in order to compute what the value of the smoothed quantity would be at t i.e. one wouldn't even know at that time whether there was an investment slump.

A number of papers have appeared that augment the base model described above. Davis and Heathcote (2005) have 3 production sectors while Iacoviello and Neri (2009) disaggregate the standard macro model by having two production sectors. In Iacoviello and Neri (2009)'s case the first sector produces consumption and investment goods with capital and labour, while the other creates new houses using capital, labour and land. Some of the calibrated parameters they choose look odd e.g. they would " imply a ratio of non-residential investment to GDP around 27 per cent", which is far larger than in the data, unless one is augmenting private non-residential investment with structures and government investment. The very large rises in housing prices in the late 1970s and in the 2000s are not well explained by Iacoviello and Neri's model, indicating that some extra features would be needed. Apart from the sectorial disaggregation the model features the idea that housing could serve as a collateral asset to finance either investment or consumption, something introduced in Iacoviello(2005) and which is dealt with later in the sub-section on the supply of credit.

2.2.3 Consumption

In the base macro-economic model the consumption Euler equation (with habit persistence) takes the form (after log-linearization)

$$c_t = \alpha E_t c_{t+1} + (1-\alpha)c_{t-1} + \theta r_t \tag{1}$$

where r_t is the real rate of interest and small letters denote log departures from a steady state position. Preference shocks may also appear in the structural equation. (1) can be written as

$$\Delta c_t = \frac{\alpha}{1-\alpha} E_t \Delta c_{t+1} + \frac{\theta}{1-\alpha} r_t.$$

The term $E_t \Delta c_{t+1}$ in the base model varies with its nature. In general there will be a large number of influences on expected future consumption growth. When the model is extended to incorporate financial influences the number of factors would grow.

Aron et al (2010) take a more empirical approach to assessing how credit might affect consumption. In their work the $E_t \Delta c_{t+1}$ term is replaced by a number of factors involving liquid assets, housing wealth etc. The coefficients on these terms are made functions of a credit conditions index that is constructed differently for the different countries they are examining, but which essentially extracts a common factor from many series that aim to reflect the tightness of credit. The difference between the two approaches often revolves around the weights assigned to the variables used in constructing $E_t \Delta c_{t+1}$. These are essentially unrestricted in the Aron et al (2010) approach, but set by the augmented base model in the standard approach. Of course the credit conditions index used by Aron et al is generally constructed from information that is not in the model, but these could be employed in the model by adding them to the observation equations relating to the unknown external finance premium. If a number of series representing credit conditions are added a common factor among them would then be extracted by using the model. Another difference is that the base model described above is linear in logs, and so there would be no interaction terms with whatever is used to represent financial stress in the model. Again this might be emulated by performing a second order approximation for the base model as that will produce interaction terms involving covariances.

Benes et al (2009) augment the base model in a number of ways. Housing is introduced and credit is required by the household sector with the financial intermediary raising funds in a foreign market which are then loaned them out to the domestic market.

2.2.4 Supply

Credit could be rationed. There is no doubt that this was a primary financial mechanism in the models of the 1960s and 1970s, as it reflected the regulated financial markets then in operation. Since that time however the amount of credit supplied by FIs has been more endogenously determined, although some constraints still operate as a reflection of asymmetric information. In particular it is often assumed that credit is only supplied if there is an adequate amount of collateral put up by the borrower. This serves to make credit an endogenous variable and not just a rigid constraint. Collateral could be any asset which serves that purpose, such as the capital stock, but mostly a new asset is introduced that is demanded by both entrepreneurs and households. Entrepreneurs use the asset in production, and so it has a role in producing output as well as facilitating the acquisition of credit. Sometimes this asset is referred to as "housing" or "land", since the main component of the value of a house is generally the land value. Households consume housing services and it is often the case that small businesses use mortgages on their house as a way of arranging credit. As the price of this collateral asset rises greater quantities of credit can be raised. So it is not an external finance premium that regulates the amount of credit available but rather the price of the collateral asset (given the rate of return to capital). Thus a key informational variable will be something like a loan to value ratio. This need not be fixed and is generally allowed to vary in a stochastic way. Broadly, this is the mechanism at work in Iacovello (2005). It might be desirable to incorporate both the external finance premium and collateral into a single model and this might be done by having the external finance premium depend on the extent of collateral[1].

Iacovello (2005) has some quantitative work on this. Liu et al (2009) also deal with it, although in their case the empirical work might be regarded as more problematic because their model has unconventional elements e.g the degree of impatience of the entrepreneurial sector is allowed to follow a unit root process. As well, the level of general technology and investment-specific technology follow unit root processes. So there are three permanent

[1] In a sense it does that already since net worth affects the external premium, but this is essentially treating capital as the collateral asset.

components in their model. There is much to be said for the unit root in general technology. Moreover, for certain types of investment their relative prices leading to the need for a unit root in technology for that sector so as to generate a permanent component in relative prices. However, it is not clear whether this is a good solution for aggregate investment.

Two other ways of modelling the supply of credit by FIs should be mentioned. Gertler and Kiyotaki (2010) have many financial intermediaries which are aggregated. This serves to provide both a retail market for funds and a wholesale (inter-bank) market. Because one can observe data on the inter-bank market this extension looks promising for empirical work. Another important feature that might need to be captured in models was pointed out by Greenlaw et al (2008). They effectively observed that the credit supplied by financial institutions would likely vary with the VaR of their portfolio, as that was the standard way of determining the aggregate of loans that they could supply. Because the VaR is based on the probability of returns being less than a given value this will rise in a recession and so the "credit multiplier" would be smaller.

3 Financial Linkages in Model with Deterministic Permanent Components

3.1 Evidence

What features relating to cycles and credit might we be interested in examining? Some have emerged in the literature and five of these will be our focus here. Our source is International Monetary Fund (2009).

1. In the first two years of an expansion credit grows quite weakly, much more weakly than output does.

2. Restrictions on the supply of credit have a significant impact on the strength of the recovery. Here strength is measured as the cumulative output growth one year after the expansion begins.

3. The probability that an economy will stay in a recession beyond a certain number of quarters is higher when the recession was accompanied by a financial crisis. A crude interpretation of this would be that recessions with a financial crisis are of longer duration.

4. Annual output growth can be predicted by utilizing a measure of financial stress. Real investment growth shows even greater predictability.

The financial stress measure aims to measure the extra costs that firms have to encounter if they are required to borrow. One measure of this would be the external finance premium that arises in the financial accelerator literature.

5. The probability of a recession should increase markedly once the external finance premium exceeds some "crisis level"

The features noted above require that one locate turning points in the level of economic activity before they can be computed. These are found with the BBQ program that is a quarterly version of the method for locating turning points set out in Bry and Boschan (1971). The program is described in Harding and Pagan (2002).

3.2 The GOZ Model

The model we will work with is that in Gilchrist et al (2010). This uses the Smets-Wouters model as the base model and then augments it with four equations

$$E_t r_{t+1}^K = \frac{1-\delta}{\bar{R}_K + (1-\delta)} E_t q_{t+1} + \frac{\bar{R}_K}{\bar{R}_K + (1-\delta)} E_t mpk_{t+1} - q_t \tag{2}$$

$$s_t = E_t r_{t+1}^K - (r_t - E_t \pi_{t+1}) \tag{3}$$

$$s_t = \chi(q_t + k_t - n_t) + \varepsilon_t^{fd} \tag{4}$$

$$n_t = \frac{\bar{K}}{\bar{N}}(r_t^K - E_{t-1} r_t^K) + E_{t-1} r_t^K + \theta n_{t-1} + \varepsilon_t^{NW} \tag{5}$$

where the over-bars indicates a steady state value, r_t^K is the rate of return to capital, q_t is Tobin's Q, mpk_t is the marginal product of capital, s_t is the external finance premium, K_t is the capital stock, N_t is entrepreneurs' net worth and lower case values indicate log deviations from steady state. Of the coefficients δ is the depreciation rate of capital and θ is the survival rate of entrepreneurs. The first term in (5) is the leveraged return to entrepreneurs and the second is the cost of debt to them. The shocks ε_t^{fd} and ε_t^{NW} are meant to capture credit supply disruptions and matching of the model variable with net worth data. (4) shows how the external finance premium varies with the degree of leverage.

Using the GOZ parameter values we simulate their variant of the Smets-Wouters(SW) model and study the resulting business cycle. Table 1 contains the cycle output along with what we would get when BBQ is applied to quarterly U.S. GDP data over the period 1973:1-2009:4[2].

Table 1: Cycle Characteristics: Data and SW Model				
		Data		SW
Expan Dur		13.6		15.4
Contract Dur		4.8		4.5
Expan Amp		9.2		9.5
Contract Amp		-2.8		-1.7
Expan Cum Amp		132.4		125.5
Contact Cum Amp		-8.1		-5.98

The basic macro model shows quite a good match to the business cycle characteristics, although expansions are longer and recessions less severe than they should be. This suggests that there could be a role for credit within the cycle. Table 2 produces the same statistics as for Table 1 but now for the GOZ model. In general however the effects of credit upon the average cycle is relatively small, with only about a two quarter reduction in its duration. This does not mean that it is unimportant in particular cycles, and we turn to that later.

[2] Alberto Ortiz kindly provided us with a Dynare program that simulated the model they use. The parameter values set in that code are different to those reported in their paper, but this is most likely due to the fact that a longer period of data, 1973:1-2009:4 was now available.

Table 2: Cycle Characteristics: GOZ and SW Models

		GOZ		SW
Expan Dur		14.2		15.4
Contract Dur		4.3		4.5
Expan Amp		8.9		9.5
Contract Amp		-1.6		-1.7
Expan Cum Amp		107.9		125.5
Contact Cum Amp		-5.6		-5.9

Some experiments can be conducted here. Doubling the standard deviation of the credit supply shocks has a very small effect upon the cycle. It is necessary to make much bigger changes in order to have an impact, well outside the range of values of the external finance premium than has been observed. Thus, quadrupling the standard deviation reduces expansion length to 12.8 quarters and raises the amplitude of recessions, although only to -1.9%. But it does this by producing premia that can go to 1000 basis points. At those levels the probability of a recession is .72, but one might think that this is rather low for such an extreme case. Doubling the coefficient χ in (4) also has relatively small effects, but does move the durations and amplitudes closer towards what is in the data.

It is also necessary that the credit growth rates implied by the model be determined. To do the latter we proceed as follows. Let leverage be $r_t = \frac{Q_t K_t}{N_t}$. Then

$$r_t = \frac{Q_t K_t}{Q_t K_t - D_t} = \frac{1}{1 - d_t}$$

where $d_t = D_t / Q_t K_t$, and

$$\begin{aligned} d_t &= 1 - r_t^{-1} \\ &\implies D_t = Q_t K_t (1 - r^{-1}). \end{aligned}$$

It immediately follows that

$$
\begin{aligned}
\Delta \ln D_t &= \Delta \ln(Q_t K_t) + \Delta \ln(1 - r_t^{-1}) \\
&= \Delta q_t + \Delta k_t + \gamma + \Delta \ln(1 - r_t^{-1}) \qquad (6)
\end{aligned}
$$

where $Q_t = \ln(Q_t/\bar{Q}_K), k = \ln(K/\bar{K}\gamma^t)$, since the steady state growth rate of capital will be the same as output (γ). An expression for r_t is available from

$$
\begin{aligned}
\ln r_t &= \ln Q_t + \ln R_{KN,t} \\
&= \ln Q_t - \ln \bar{Q}_K + \ln R_{KN} - \ln \bar{R}_{KN} + \ln \bar{P}_K + \ln \bar{R}_{KN} \\
&= q_t + k_t - n_t + \ln \bar{Q} + \ln \bar{R}_{KN} \\
\therefore &\quad r_t = \exp(q_t + k_t - n_t + \ln \bar{Q} + \ln \bar{R}_{KN})
\end{aligned}
$$

Now Gilchrist et al use percentage changes for their variables. Designating these by $*$ we get

$$
r_t = \exp((q_t^* + k_t^* - n_t^*)/100 + \ln \bar{Q} + \ln \bar{R}_{KN}).
$$

$\ln \bar{Q} + \ln \bar{R}_{KN}$ is available from the GOZ code as $\ln(1.3634)$. Hence we have

$$
\begin{aligned}
\Delta \ln D_t^* &= \Delta q_t^* + \Delta k_t^* + \gamma_t^* + 100\Delta \ln(1 - r_t^{-1}) \\
r_t &= \exp((q_t^* + k^* - n^*/100) + \ln(1.3634))
\end{aligned}
$$

We now seek to examine some of the characteristics listed in the preceding sub-section. *Prima facie*, the first characteristic seems satisfied as the growth of credit over the first eight quarters of an expansion is on average -2.6 versus the 4.8 in output. But this hides an enormous variation. There are many simulations in which the growth in credit over the first two years of an expansion exceeds that of output. In fact the growth rates in credit are extremely volatile with a standard deviation of quarterly growth being 5.86 versus only .69 for output. The extreme volatility comes from the factor $100\Delta \ln(1 - r_t^{-1})$ in the growth of credit. If one computes the standard deviation of $100\Delta \ln(1 - r_t^{-1})$ from the data and the model we find these to be 5.52 and 8.83 respectively, so that the volatility in the implied credit raising is also in the data used by GOZ[3].

We can investigate the dependence of recessions upon the external finance premium by taking advantage of the binary nature of the recession indicator R_t ($R_t = 1$ if the economy is in recession but zero otherwise). Thus we wish to compute $\Pr(R_t|s_t)$. This might be done non-parametrically as in Harding and Pagan (2010a) but here we compute this probability using a Probit specification $\Pr(R_t|s_t) = \Phi(s_t)$, where $\Phi(\cdot)$ is the cumulative standard normal distribution function. In fact the non-parametric estimate tended to follow the Probit values except at the boundaries where it ceased to be monotonic. Harding (2010) has suggested a way to resolve this but, at this point, we simply summarize the outcomes using the Probit model. Fig 1 presents this probability, while Fig 2 gives what it is if one used the BAA bond spread in the US data as a measure of the external finance premium. Note that the unconditional probability of a recession over 1953:2-2009:3 was .12 so that the rise in the external premium does increase the probability quite substantially, although it never gets to a standard critical value often used in predicting recessions of .5.

The final question we seek to examine is whether once one is in a recession the duration of the recession depends upon the magnitude of the external finance premium. Although it is not exactly what one wants, a simple way to get an appreciation of what the model says about this is to compute $\Pr(R_{t+m} = 1|R_t = 1, s_t)$ i.e. the probability that in m periods time the economy will be in a recession when a recession was in progress at time t. Table 3 shows what this probability is for three levels of s_t and for $K = 1, 2, 3$. It should be noted that, since BBQ has a restriction that recessions and expansions must last at least two quarters, the only reason that $\Pr(R_{t+1} = 1|R_t = 1, s_t) \neq 1$ is that there will be an $R_t = 0$ at the end of a recession and $R_{t+1} = 0$[4]. Table 3 shows this probability for various levels of the external risk premium and m.

[3] Credit data is often readily available but difficult to work with due to its volatility and persistence. In Catao and Pagan (2010) it is introduced into a model like that used as the base one for Chile and Brazil and there were some noticeable effects.

[4] This fact also means that for $m \leq 3$ the value of m must be the duration of a recession since, if $R_t = 1$, then $R_{t+1} = 1$, owing to the fact that recessions last two quarters. If $R_{t+3} = 1$ then $R_{t+2} = 1$, otherwise we would have a one period expansion.

Fig 1 : Prob of Recession as Function of External Finance Premium

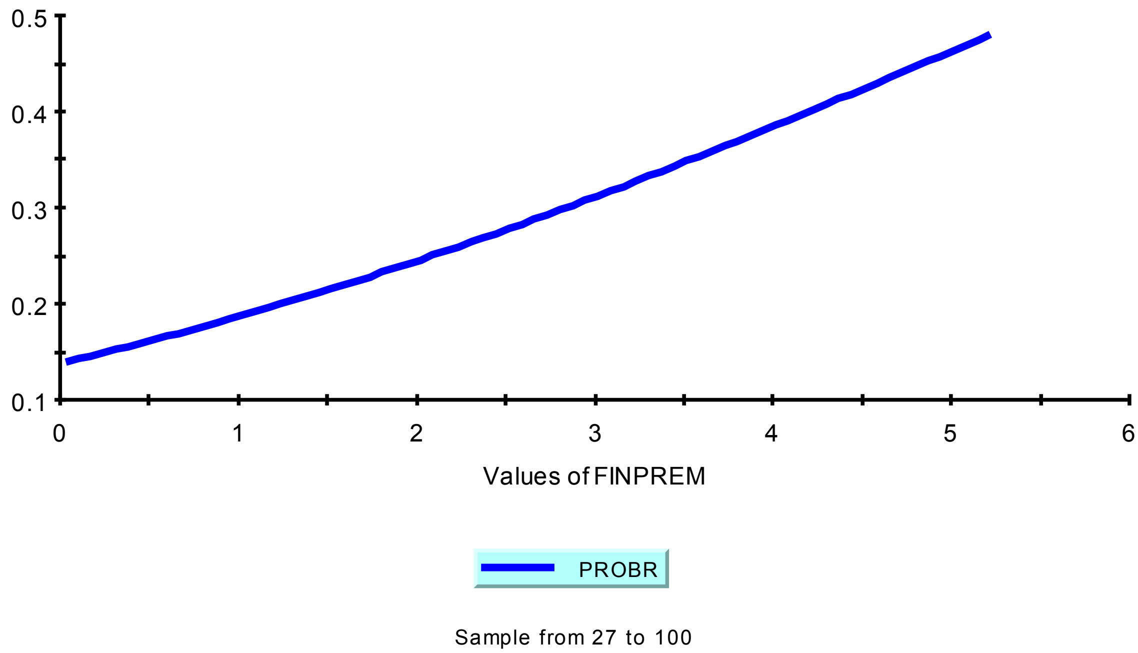


Fig 2 Probability of Recession Given External Premium in Data

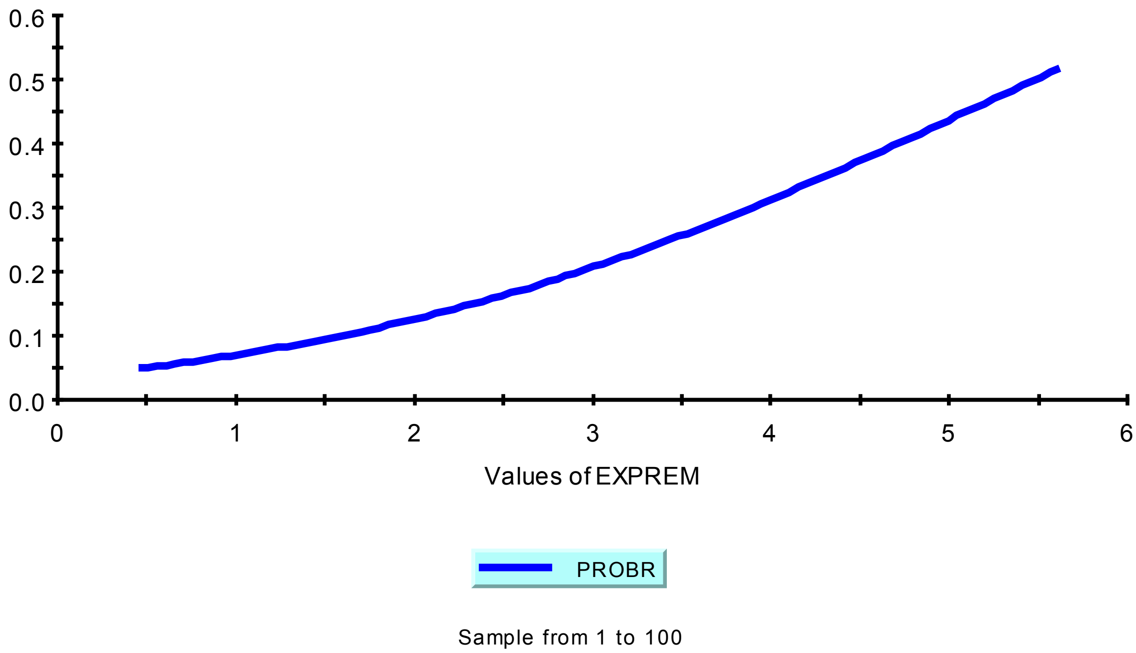


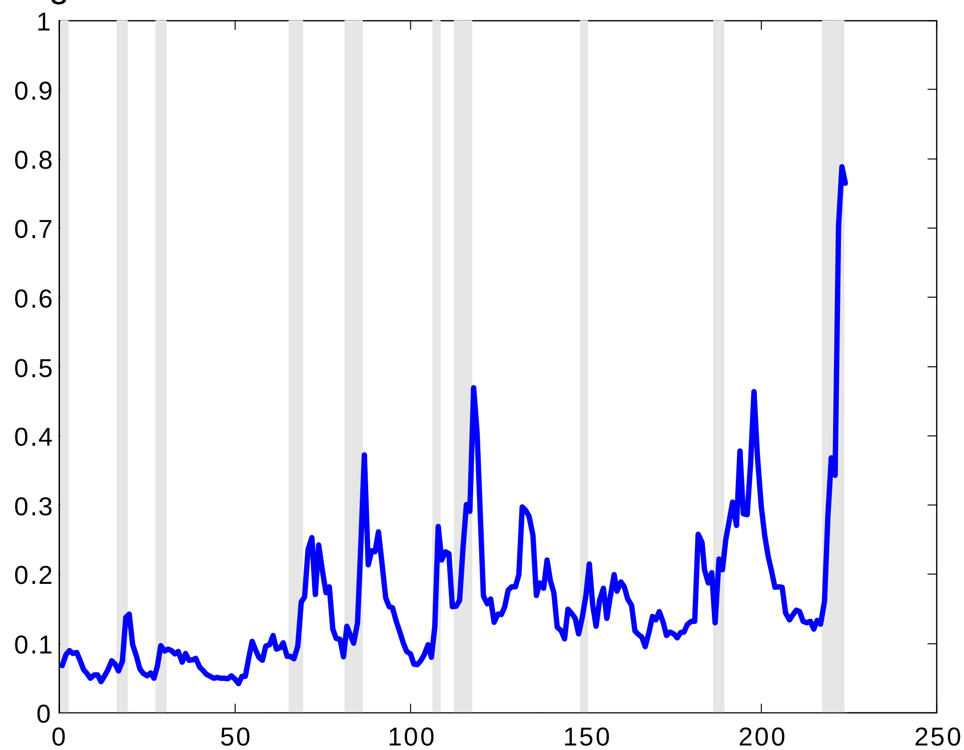
Table 3 Probability of Recession for m Periods as Ext Premium Varies				
Ext premium (basis points)		$m = 1$	$m = 2$	$m = 3$
25		.70	.38	.16
300		.72	.42	.20
485		.74	.46	.23

It is clear that there is an increase in the probability of the duration of a recession as the external finance premium rises, but what is striking is how small the rise in this probability is. Essentially the computation addresses the often-quoted result that a recession associated with a financial crisis is around twice as long as one that does not have one. As we would think that a crisis would involve a high external interest rate premium, given that there would be little credit available, the GOZ model would fail to deliver such a prediction. One would certainly associate a crisis with a high probability of recession, as seen in Table 1, but its duration does not seem to depend much on that. Indeed one gets the impression that the external premium itself would lead to short recessions. Perhaps this is a consequence of some persistence problems in the growth in credit. It is interesting to note that the persistence in $\{\ln(1-(1/r_t)) - \ln(1-(1/r_{t-1}))\}$ used to form credit growth in (6) is quite different in the data than the model.

One might ask if there is any evidence that a recession can be predicted with the external finance spread. Using the Baa spread over the 10 year government bond rate as a measure of this one can evaluate the predictability of a recession from 1973[5]. Here the Baa spread is that available at the beginning of the quarter a prediction is to be made about. Although the spread is highly significant in a Probit model fitted to the recession indicator (t ratio of 4), it is clear from the graph in Figure 3 that it adds little to the predictive power. Even in the 2008 recession it was not indicating one until the recession was well under way (the predicted probability in the first quarter of 2008 was just .27). This finding is something which concurs with that found by Harding and Pagan (2010b) for many series recommended as useful for predicting recessions. Of course it may be that there are advantages to using the spread that GOZ constructed rather than the Baa spread above,

[5]Because there are only two interest rates in the model and one of these is the policy rate it might be better to use the spread over a three month T-Bill rate. But doing this does not change the results very much.

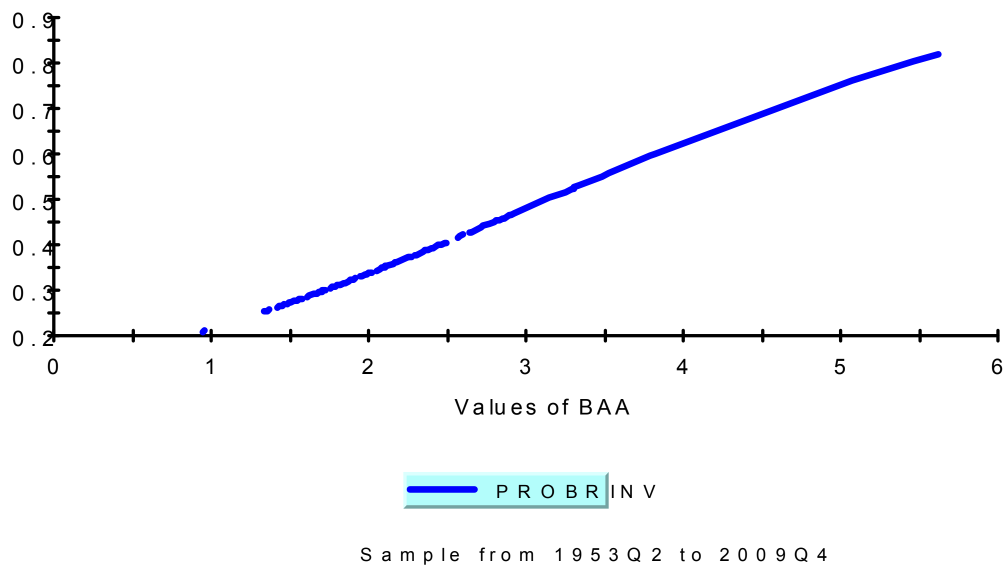
Fig 3 Prob GDP Recession and Recession Periods for US



but that data was not available to us.

One of the observed cycle characteristics listed above was that there would be a stronger response by investment than output. We therefore studied the investment cycle present in the data on U.S. non-residential investment. Here expansions on average were 12 quarters long and recessions were 6.5. So, while the investment cycle length is not far from that of GDP, the recessions are longer and the expansions shorter. Figure 4 presents the probability of an investment recession as a function of the Baa spread and it is apparent that it rises very quickly with the spread. This is also true of the model, where the probability of an investment recession is .45 when there is an (annualized) spread of 200 basis points, .58 when the spread is 300 points, and .83 when it is 520 basis points. Indeed, when viewed in conjunction with Figure 4, it is clear that the model predicts an even stronger effect than seen in the data (of course the Baa spread is not the external finance premium used by GOZ

Fig 4 Probability of an Investment Recession as a Function of the BAA Spread- Data

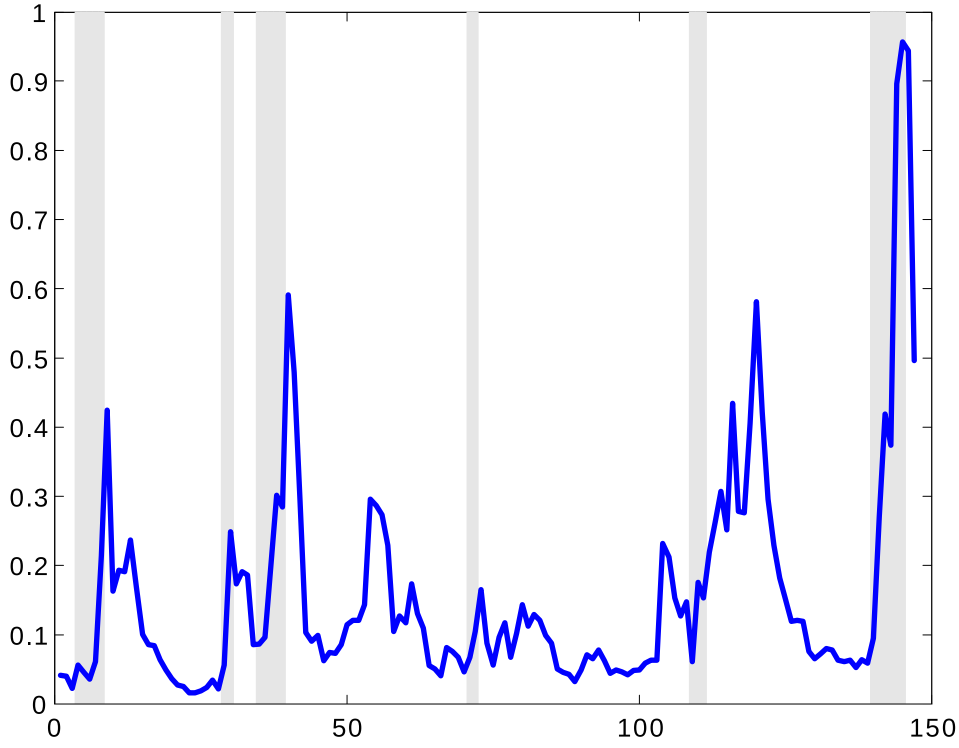


and so it might be different if that was used).

The equivalent of figure 3 is shown in figure 5.

Why is it that the probability of an output recession for a given spread is so much lower than that for investment? One reason is that investment in the GOZ model is only 10% of GDP, so a very large negative growth rate in investment is need to cause a negative growth in output. This suggests that one needs to work with a broader set of investment expenditures i.e. housing and consumer durables could be crucial to getting the financial linkages right. In turn this implies that collateral effects will be important. Integrating housing and consumer durables into the GOZ model would seem a useful extension of it.

Fig 5 Prob Invest Recession and Recession Periods for US



4 Financial Linkages in Model with Stochastic Permanent Components

The estimated autoregressive coefficient for technology in the GOZ model is close to unity. This suggests that we might wish to impose the coefficient at unity. To do so one needs to generalize the models above in order to allow for a permanent stochastic component to output. A second reason for doing this is that one sometimes wants to perform a comparative analysis with papers that have a unit root in technology, such as Liu et al (2009). Such an extension is not always straightforward and the next sub-section details the difficulties. Broadly, the model needs to be designed to enable balanced growth, and some functional forms used when series are stationary do not have this property. We look at the options. In fact the GOZ model is relatively simple to adapt because of the utility function that was employed. The following sub-section then looks at the quantitative effects of financial stress upon output in the GOZ model extended to allow for permanent components. Finally, we consider what a model set up to measure the impact of financial stress upon activity would imply for a VECM representation of the data. Since papers such as Liu et al (2009) summarize the data with a VECM model, it is interesting to see how this relates to the models being used for interpretation. We explain some of the difficulties in interpreting empirical VECM models by working with a simple RBC model that incorporates habit persistence and a unit root in technology, as this model underlies many of the base macro models used in the literature.

4.1 Model Design Issues with Permanent Components

In many models used when assuming stationary series the utility function of households has the power form of $\frac{(C_t)^{1-\sigma}}{1-\sigma} - \psi N_t$. In many early versions of RBC models the utility function had this form. The consumer maximizes $E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \psi N_t\right]$ with respect to consumption (C_t) and hours (N_t) subject to the budget constraint

$$P_t C_t + B_{t+1} = W_t N_t + R_{t-1} B_t,$$

where B_t is the real level of bonds, R_t the rate of return on them and W_t is the nominal wage rate. The standard first order conditions for inter-temporal optimization will be

$$C_t^{-\sigma} = \lambda_t P_t \quad (7)$$
$$\psi = \lambda_t W_t, \quad (8)$$

where λ_t is the Lagrange Multiplier attached to the budget constraint. These equations imply that the real wage will be $\frac{W_t}{P_t} = \psi C_t^{\sigma}$. Now, if there is a permanent component in the log of technology, the log of the real wage would co-integrate with the log of consumption, and the cointegrating vector will be $\begin{pmatrix} 1 & -\sigma \end{pmatrix}$. However, if the production function is Cobb-Douglas, and hours are an $I(0)$ process, the log of the real wage must cointegrate with output, and the co-integrating vector is $\begin{pmatrix} 1 & -1 \end{pmatrix}$. Hence this raises a conflict as $\ln C_t$ and the log level of output cannot co-integrate with vector $\begin{pmatrix} 1 & -1 \end{pmatrix}$, meaning that one does not have balanced growth. If $\sigma = 1$ there will be balanced growth, explaining why it is common to see log utility being used in many macro models where a non-separable utility function is prescribed.

There are a number of ways to avoid this difficulty. As noted above log utility- $\ln C_t \ -\psi N_t -$ would suffice. Another is to have nonseparability between consumption and hours i.e. to specify utility as having the form $\frac{C_t^{1-\sigma}}{1-\sigma} \times g(N_t)$, where $g(\cdot)$ is some smooth function. In this instance the two equations above would become

$$C_t^{-\sigma} g(N_t) = \lambda_t P_t \quad (9)$$
$$\frac{C_t^{1-\sigma}}{1-\sigma} g'(N_t) = \lambda_t W_t, \quad (10)$$

which gives $\frac{1}{1-\sigma} C_t g'(N_t) = \frac{W_t}{P_t}$, and this is consistent with balanced growth.

If one adopted with log utility of non-separable utility the equations can be simply re-expressed in terms of re-scaled consumption, $\tilde{C}_t = \frac{C_t}{T_t}$, where T_t is some $I(1)$ variable such that $\ln T_t$ co-integrates with the $\ln C_t$. Thus we would have $\frac{1}{1-\sigma} \tilde{C}_t g'(N_t) = (\frac{W_t}{P_t}/T_t)$ and the model would be solved for $\tilde{C}_t$ and the adjusted real wage. In fact such a transformation has also been used as a way of handling the balanced growth problems if the utility function is separable by del Negro and Schorfheide (2008). In their work the objective function to be optimized is taken to be $E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{(\frac{C_t}{T_t})^{1-\sigma}}{1-\sigma} - N_t \right]$, and this yields first order conditions of

$$\tilde{C}_t^{-\sigma} = \lambda_t P_t A_t \quad (11)$$
$$\psi = \lambda_t W_t. \quad (12)$$

Solving these gives $\frac{W_t}{P_t} = \psi A_t \tilde{C}_t^{\sigma}$. which results in co-integration between $\ln \frac{W_t}{P_t}$ and $\ln T_t$ with co-integrating vector $\begin{pmatrix} 1 & -1 \end{pmatrix}$. There is also cointegration between $\ln C_t$ and $\ln T_t$, as well as between the log of income and $\ln T_t$. Thus one gets balanced growth.

What should one use for T_t? If there is a single $I(1)$ factor driving variables, and in most macro models this is technology A_t, then it makes sense to use the permanent value of this. In the case that $\ln A_t$ is a pure random walk then the permanent component of $\ln A_t$ is just $\ln A_t$, meaning that one would work with $\frac{C_t}{A_t}$. But, if technology followed $\ln \Delta A_t = \rho_a \ln \Delta A_{t-1} + \varepsilon_{at}$, then we would use $\frac{C_t}{A_t^p}$ where $\ln A_t^p = \ln A_t + \frac{\rho_z}{1-\rho_z} \Delta \ln A_t$.

Of course these different specifications will produce a different set of Euler equations than if one normalized by A_t. So why would one prefer A_t^p? If it is simply a re-scaling factor then the only advantage would come from the fact that $E_t \Delta \ln A_{t+1}^P = 0$. But if one adopts the del Negro and Schorfheide solution it would be preferable to set $T_t = A_t^P$. In the long run consumption will settle to A_t^p and not A_t (except in the case when there is no serial correlation in technology growth). It makes sense then to formulate the households problem as one that aims to target the long run value of consumption, that is it is the *transitory* part of consumption which appears in the utility function rather than the level of consumption as in the traditional formulation.

4.2 Financial Effects in the Extended GOZ Model

We utilize the same parameter values as in the previous variants of the GOZ and SW models but now assume that technology has both a stochastic and deterministic trend. Because the GOZ utility function is non-separable, it is a simple matter to adjust the model equations to reflect this changed emphasis. Table 4 gives the cycle characteristics from this modified model (along with the model without financial effects, designated SW). Perhaps the most interesting outcome is that the presence of a unit root means recessions are now longer and deeper than they were before. There is a small rise in the probability of a recession for large values of the external premium but only from .48 to .49.

Table 4: Cycle Characteristics: Unit Root GOZ and SW Models				
		GOZ	Data	SW
Expan Dur		14.2	13.6	14.5
Contract Dur		4.6	4.8	4.3
Expan Amp		9.4	9.2	8.8
Contract Amp		-2.0	-2.8	-1.4
Expan Cum Amp		105	132	101
Contact Cum Amp		-7.0	-8.1	-3.3

4.3 The VECM Representation of a DSGE Model

Having introduced a unit root in technology to the model we will sometimes want to compare the VECM representation of it with that in the data to determine if there are any particular features that are poorly matched. Moreover, as was mentioned earlier, the use of VECMs was important in Liu et al (2009). Prior to doing this for the GOZ model we look at the idea in the context of a simple RBC model, in order to highlight some of the principal issues about what can be learned from empirical VECMs.

In the models that we have dealt with previously (but not in Liu et al (2009)) there is a single $I(1)$ factor that is responsible for the permanent components in model variables, and it is a feature of such models that the permanent component of consumption, investment etc equals the permanent component of technology. This means that the variables $y_{t-1} = \ln Y_{t-1} - \ln A^p_{t-1}$ will be the error correction terms. Using the conversion equations in Christensen et al (2010) a DSGE model written in terms of any $I(1)$ variables scaled by A^p_t can be converted to a VECM form. To illustrate, consider an RBC model in which there is habit persistence and preference shocks. For simplicity we adopt log utility. Assuming that there is no serial correlation in the technology shocks we have $a^p_t = \ln A^P_t = \ln A_t = a_t$, and the equations to solve will be

$$
\begin{aligned}
l_t - y_t + c_t &= y_t - c_t \\
\bar{C}_y c_t + \bar{K}_y k_t &= y + \bar{K}_y(1-\delta)(k_{t-1} - \Delta a_t) \\
c_t &= hc_{t-1} + (1-h)E_t c_{t+1} + (1-h)\Delta a_t - r_{t+1} + \varepsilon_t^c \\
\bar{R} r_t &= \alpha(1/\bar{K}_y)(y - k_{t-1} - \Delta a_t) \\
y_t &= \alpha k_{t-1} + (1-\alpha)l - \alpha \Delta a_t.
\end{aligned}
$$

Here ε_t^c and Δa_t are white noise preference and technology shocks, l_t is the log deviation from steady state of hours, $\{c_t, k_t, y_t\}$ are log deviations from their permanent components, and r_t is the rate of return to capital. Quantities with over-bars are steady state values. Thus $\bar{R} = 1/\beta, \bar{K}_y = \frac{\alpha}{R^* - 1 + \delta}, \bar{C}_y = 1 - \delta \bar{K}_y$ are the steady state rate of return, capital-income and consumption-income ratios. The parameters are set to the following values - $\beta = .99$,$\alpha = .3, h = .4, \delta = .3$.

As mentioned earlier c_t, k_t, y_t will also be the ECM terms in a system composed of l_t,$\ln C_t, \ln K_t, \ln y_t$ and r_t. Converting the solution to a VECM produces

$$
\begin{aligned}
\Delta \ln Y_t &= -y_{t-1} - .04k_{t-1} - 1.26c_{t-1} + e_{1t} \\
l_t &= -.48k_{t-1} - 1.8c_{t-1} + e_{2t} \\
\Delta \ln K_t &= -.07k_{t-1} - .19c_{t-1} + e_{3t} \\
\Delta \ln C_t &= .44k_{t-1} - .46c_{t-1} + e_{4t} \\
r_t &= -.4k_{t-1} - .04c_{t-1}
\end{aligned}
$$

Now it is important to note that the ECM terms $c_{t-1}, k_{t-1}, y_{t-1}$ are latent, since a_{t-1}^p is not known. If we assume that $\ln Y_t, \ln C_t$ and $\ln K_t$ are observable, then we could write

$$
\begin{aligned}
k_{t-1} &= \ln K_{t-1} - a_{t-1}^p = \ln K_{t-1} - \ln Y_{t-1} + \ln Y_{t-1} - a_{t-1}^p \\
&= \ln K_{t-1} - \ln Y_{t-1} + y_{t-1},
\end{aligned}
$$

and thereby re-express the equation for $\Delta \ln C_t$ as

$$
\Delta \ln C_t = .44(\ln K_{t-1} - \ln Y_{t-1}) - .46(\ln C_{t-1} - Y_{t-1}) - .02y_{t-1} + e_{4t}.
$$

Here $y_t = \ln Y_t - a^p_{t-1} = \ln Y_t - \ln Y^p_t$ is the transitory component of output. Consequently, if one worked only with a system composed of the observable variables $\Delta \ln Y_t, \Delta \ln K_t$ and $\Delta \ln C_t$, there would be two ECM terms $(\ln K_{t-1} - \ln Y_{t-1})$ and $(\ln C_{t-1} - Y_{t-1})$, and a specification error would be committed, as the transitory component for output will have been omitted from the equations. How important this is depends on the context. One might expect that the coefficients of $(\ln K_{t-1} - \ln Y_{t-1})$ and $(\ln C_{t-1} - Y_{t-1})$ in the $\Delta \ln C_t$ equation might not have much bias, since the coefficient on y_{t-1} is small. Indeed simulations of data from the RBC model confirm this. However, for the output equation, there is a large coefficient on y_{t-1}, and a very large bias on the coefficients results. This issue becomes amplified when there are other unobservable variables e.g. if $\ln K_t$ was omitted from the VECM system only one ECM term, $\ln C_t - \ln Y_t$, would be present, and therefore there would be a second transitory component (for capital) missing. Of course this is just a consequence of the well-known fact that omission of variables from the solved solution for a DSGE model makes the solution a VARMA process rather than a VAR. However, it does suggest that one needs to be careful if trying to establish relations between credit and economic activity using VECMs in observable variables, and some attempt needs to be made to correct for the missing transitory variables.

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