Changes in International Business Cycle Affiliations

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Abstract

We investigate changes in international business cycle affiliations using a procedure designed to separate breaks in dynamics and covariances. Further, we propose a procedure for decomposing the latter into volatility and (conditional) contemporaneous correlation breaks. We analyze GDP growth rates for leading developed economies through two VAR systems, one with the US, Euro area, UK and Canada and the other for the Euro area countries of France, Germany and Italy. No significant breaks are found in the dynamic interactions between these international economies since 1970, although there are breaks in both the volatilities of growth and cross-economy conditional correlations. Contemporaneous correlations between the Euro area countries increase in both 1984 and around 2002, with a large increase in correlations also evident across the international system at the latter date, which may be associated with the 2008/09 world recession.

Keywords: International business cycle, structural breaks, spillovers, business cycle synchronization

JEL codes: C32, E32, F43

1. **Introduction**

There is now a substantial body of empirical evidence relating to the nature of international business cycle linkages and whether these have changed over the recent so-called globalization era[1]. There is no doubt that some recessions are essentially global events, with those of the 1970s and the latest episode associated with the credit crunch being especially notable in this sense. However, the issue investigated in this paper is not the causes of these specific events, but the more general one of whether international business cycle affiliations, measured using cross-country linkages in output growth, have altered over the last 40 years. If cross-country affiliations have increased, as implicitly assumed in much of the general discussion of globalization, then purely domestic models become less relevant for explaining economic growth, even in the large G-7 countries.

Nevertheless, a somewhat surprising consensus appears to be emerging from many recent studies, namely that the era of globalization has not been associated with a general increase in the strength of international business cycle affiliations. For example, Heathcote and Perri (2004) find the business cycle correlation of US output with the rest of the world to be lower after 1986 compared with the earlier post-Bretton Woods period (1972-1986). Similar conclusions are drawn by Kose, Otrok and Whiteman (2008) in relation to output and consumption for the G-7 countries and by Del Negro and Otrok (2008) for output across 19 developed countries. Also, both Stock and Watson (2005) and Doyle and Faust (2005) find relatively little evidence of increased synchronization of business cycles across the G-7 countries since the 1960s.

However, other studies draw a different conclusion. Both Helbling and Bayoumi (2003) and Perez, Osborn and Artis (2006) find evidence of time-varying affiliations, with the early 1990s being distinctive as a period of relative disconnection between the US and major

[1] See Doyle and Faust (2005), Kose, Otrok and Prasad (2008) and de Haan, Inklaar and Jong-a-Pin (2008).

countries of mainland Europe and a restoration of strong trans-Atlantic links in the latter part of that decade. In contrast to other studies, Bordo and Helbling (2003) take a long-run perspective by using data over one and a quarter centuries, and find an increasing role for globalization over time. Kose, Otrok and Prasad (2008) find increased business cycle convergence within groups of industrial economies and groups of emerging market economies but between them a decoupling has been observed with divergence of these business cycle fluctuations and a decline in the importance of the global factor.

In the light of the huge changes in the formal structures linking European economies since the 1960s, which culminated in the establishment of the Euro area in January 1999, an important strand of analysis has focussed on these countries; see the review in de Haan, Inklaar and Jong-a-Pin (2008). There is a general finding of stronger business cycle linkages over time between countries that are now Euro area members, especially for previously 'peripheral countries', such as Italy and Spain becoming more closely integrated with Germany and the Euro area more generally (for example, Artis and Zhang, 1997, 1999, Koopman and Azevedo, 2008). Nevertheless there is also some evidence of regimes in affiliations rather than monotonic movement towards a common cycle (Inklaar and de Haan, 2001, Massmann and Mitchell, 2004). Interestingly, however, the literature in an international context often finds no evidence of changing affiliations for the three major economies of the Euro area, that is France, Germany and Italy (Canova *et al*., 2007, Del Negro and Otrok, 2008, Kose, Otrok and Whiteman, 2003).

Although this literature employs a variety of econometric techniques, relatively little use has been made of formal tests for structural change at unknown dates. Rather, most studies either use essentially descriptive methods (such as output growth correlations based on rolling windows) or assume known dates of change. Although major international events (including the breakdown of the Bretton Woods system in the early 1970s and the 'Maastricht Treaty' that firmly committed a number of European countries to the formation of the Euro area in

1992) may lead to changes in business cycle linkages, such changes are not necessarily synchronous with the legal dates of such events and, indeed, could pre-date changes to formal structures when the latter are pre-announced or otherwise anticipated. Further, events in individual countries may affect these affiliations. For example, Helbling and Bayoumi (2003) associate low trans-Altantic linkages in the early 1990s with a sequence of country-specific shocks over that period. Therefore, an appropriate econometric methodology for examining changes in international business cycle affiliations should allow for changes that are both unknown in number and occur at unknown dates. This is exactly what we do in this paper.

A further econometric complication is that many countries have experienced substantial changes in output volatility over the last four decades. This is best documented for the US (see Kim and Nelson, 1999, and McConnell and Perez-Quiros, 2000, Sensier and van Dijk, 2004, among others), but has also been established for other G-7 countries (van Dijk, Osborn and Sensier, 2002; Doyle and Faust, 2005), while Del Negro and Otrok (2008) refer to business cycle volatility as converging across countries. Consequently, results based on an explicit or implicit assumption of a constant cross-country disturbance covariance matrix may not be valid. To our knowledge, Doyle and Faust (2005) is the only previous study to undertake a similar examination of changes in both the co-movement and volatility of international business cycle linkages. However, they assume that the number of breaks is known, and hence do not apply any tests for the overall null of no breaks, and are able to consider only the possibility of concurrent coefficient and covariance breaks.

In common with many previous analyses of the international business cycle, this paper examines quarterly GDP growth for G-7 countries within a vector autoregressive (VAR) framework. Our sample period of 1970 to early 2010 focuses on changes in business cycle affiliations over the post-Bretton Woods period, which allows us to examine the impact of changes relevant to the international economy, including globalization and the establishment of the Euro area. Further, and unlike previous analyses, we are able to include information from the world recession of 2008/09, which was triggered by the financial crisis that commenced in 2007. Following Doyle and Faust (2005), and also based on evidence in many other recent studies (including Canova *et al*., 2007, Del Negro and Otrok, 2008, Kose, Otrok and Whiteman, 2008), Japan is excluded as it has not been closely linked to other G-7 economies since the 1970s.

Our analysis seeks to examine changes in the business cycle for both the G-7 as a whole (excluding Japan) and also between the countries that are now members of the Euro area. Reflecting this, we employ two VAR models, in order that the two types of cross-country changes can be clearly distinguished. One specification (which we term the 'International VAR') comprises the US, the Euro area, Canada and the UK, while the second (the 'Euro area VAR') examines the three Euro area countries that are members of the G-7, namely Germany, France and Italy (the so-called E-3). We consciously study the Euro area as an aggregate in the former, in order to recognise the international importance of this economic region, with aggregate output comparable to the US.

Our econometric methodology is based on the system multiple break tests of Qu and Perron (2007), but we develop this further by separating mean and covariance breaks. While the broad approach is similar to that employed by Doyle and Faust (2005), ours is more flexible in that we neither specify *a priori* the number of breaks nor are coefficient and covariance breaks required to be synchronous. Further, we adopt a new recursive method for decomposing covariance breaks into ones relating to volatility and correlation. Our results confirm that international business cycle affiliations remain largely unchanged during the three decades from 1970 to 2000, although European integration does play a role. Further, and perhaps more importantly, we are able to shed light on how these affiliations are affected by inclusion of the 2008/09 recession period.

The structure of this paper is as follows. Section 2 outlines our methodology, while Section 3 discusses our data and the results of a univariate analysis for each series. Our principal results on business cycle affiliations are then presented in Section 4, while Section 5 concludes.

2. Methodology

The framework for our analysis is a conventional VAR system for n countries

$$\mathrm{y}_t = \boldsymbol{\delta} + \sum_{i=1}^{p} \mathrm{A}_i \mathrm{y}_{t-i} + \mathrm{u}_t \qquad (1)$$

where $\mathbf{y}_t = [y_{1t},...,y_{nt}]'$ is a vector of quarterly output growth rates while the error term $\mathbf{u}_t$ has $E(\mathbf{u}_t) = \mathbf{0}$, covariance matrix $E(\mathbf{u}_t\mathbf{u}_t') = \boldsymbol{\Sigma}$ and is temporally uncorrelated. Defining **D** to be the diagonal matrix containing the standard deviations of $\mathbf{u}_t$ and **P** to be the corresponding correlation matrix, then (by definition) $\boldsymbol{\Sigma} = \mathbf{D}\ \mathbf{P}\ \mathbf{D}$. Our methodology seeks to date structural breaks in each of the three components of (1), namely dynamic output interactions as captured by the VAR coefficients $\mathbf{A}_i$ (i = 1, …, p), output volatility measured by **D**, and conditional contemporaneous output growth correlations in **P**.

Our structural break analysis builds upon the recent methodology of Qu and Perron (2007) to test for coefficient and covariance breaks in a VAR system. This provides us with tools to deal with three scenarios, namely breaks occurring simultaneously in both the VAR coefficients ($\mathbf{A}_i$, i = 1, …, p, and $\boldsymbol{\delta}$) and the covariance matrix $\boldsymbol{\Sigma}$, breaks occurring only in the VAR coefficients or breaks occurring only in the covariance matrix. Indeed, the previous literature concerning the univariate properties of output growth implies volatility declines might be anticipated in the early 1980s (see, e.g., Sensier and Dijk, 2004), whereas globalization may affect dynamic linkages from the latter part of the century (Kose, Otrok and Whiteman, 2008).

For those reasons, we test for (separate) breaks in the VAR coefficients and the covariance matrix, as outlined in subsection 2.1. Subsection 2.2 then describes the recursive procedure employed for separating volatility and correlation breaks, including Monte Carlo evidence on its performance.

2.1 Tests for dynamic and covariance breaks

For a given VAR order p in (1), the procedure of Qu and Perron (2007) is used to test the stability of the VAR coefficients (including the intercept) against the possibility of $m \leq M$ breaks, where m is unknown and the maximum number of breaks M is pre-specified. This is a test of the null hypothesis $H_0 : \mathbf{A}_{ij} = \mathbf{A}_i, \boldsymbol{\delta}_j = \boldsymbol{\delta}$ (j = 1, ..., m+1; i = 1,..., p) in

$$\mathrm{y}_t = \boldsymbol{\delta}_j + \sum_{i=1}^{p} \mathbf{A}_{ij} \mathbf{y}_{t-i} + \mathbf{u}_t , \tag{2}$$

for $t = T_{j-1} + 1, \ldots, T_j$, $j = 1, \ldots, m+1$, where T_j denote the break dates marking the m subsamples, with $T_0 = 0$ and $T_{m+1} = T$ (T being the total sample size), and where $\mathbf{u}_t$ satisfies $E(\mathbf{u}_t) = \mathbf{0}$ and $E(\mathbf{u}_t\mathbf{u}_s') = \mathbf{0}$, $t \neq s$. To allow the possibility of breaks in $\boldsymbol{\Sigma}$, $E(\mathbf{u}_t\mathbf{u}_t')$ is not allowed to be time-varying.

The overall null of no breaks is tested using the 'double maximum' statistic

$$WD \max F_T(M) = \max_{1 \leq m \leq M} a_m \left[\sup_{(\lambda_1, \ldots, \lambda_m \in \Lambda_\varepsilon)} F_T(m, q, \varepsilon) \right], \tag{3}$$

where λ_j (j = 1, …, m) indicate possible break dates as fractions of the sample size, with $0 < \lambda_1 < \ldots < \lambda_m < 1$ and $T_j = [T\lambda_j]$, and Λ_ε denotes all permissible sample partitions satisfying the requirement that a fraction of at least ε of the sample is contained in each segment, for some $0 < \varepsilon < 1$. The parameter $a_m = c(\alpha,1)/c(\alpha,m)$ with $c(\alpha,m)$ the asymptotic critical value (at a significance level of 100α percent) of the supremum statistic

$\sup_{(\lambda_1,\ldots,\lambda_m \in \Lambda_\ell)} F_T(m,q,\varepsilon)$ against a specific number of m breaks. For a total of q coefficients in (2), all of which are allowed to change,

$$F_T(m,q,\varepsilon) = \left(\frac{T-(m+1)q}{m} \right) \hat{\boldsymbol{\beta}}' \mathbf{R}' [\mathbf{R} \hat{\mathbf{V}}(\hat{\boldsymbol{\beta}}) \mathbf{R}']^{-1} \mathbf{R} \hat{\boldsymbol{\beta}}, \tag{4}$$

is a Wald-type test statistic for structural change at m known dates, $\hat{\boldsymbol{\beta}}$ is the stacked vector of estimated VAR coefficients given the m breaks with estimated robust covariance matrix $\hat{\mathbf{V}}(\hat{\boldsymbol{\beta}})$ [2], and $\mathbf{R}$ is the non-stochastic matrix such that $(\mathbf{R}\boldsymbol{\beta})' = (\boldsymbol{\beta}_1' - \boldsymbol{\beta}_2', \ldots, \boldsymbol{\beta}_m' - \boldsymbol{\beta}_{m+1}')$ where $\boldsymbol{\beta}_j$ is the vector of coefficients in the j-th segment.

If the *WD*max test of (3) rejects the null of no breaks, a sequential *F*-type test is used to determine the number of breaks and their locations. In particular, this procedure makes use of the test statistic

$$SEQ_T(l+1|l) = \max_{1 \le j \le l+1} \left[\sup_{\tau \in \Lambda_{j,\varepsilon}} F_T(\hat{T}_1, \ldots, \hat{T}_{j-1}, \tau, \hat{T}_j, \ldots, \hat{T}_l) - F_T(\hat{T}_1, \ldots, \hat{T}_l) \right], \tag{5}$$

where $\Lambda_{j,\varepsilon} = \{\tau; \hat{T}_{j-1} + (\hat{T}_j - \hat{T}_{j-1})\varepsilon \le \tau \le \hat{T}_j + (\hat{T}_j - \hat{T}_{j-1})\varepsilon\}$, and F_T is defined as in (4). The statistic in (5) can be used to test the null of l breaks against the alternative of l+1 breaks, by testing for the presence of an additional break in each of the segments defined by the break dates $(\hat{T}_1, \hat{T}_2, \ldots, \hat{T}_l)$ obtained from estimating the model with l breaks. The test is applied sequentially for l = 1, 2, … until it fails to reject the null hypothesis of no additional break, or until the maximum number of permitted breaks, *M*, is reached.

Having estimated the number of structural breaks using (5), the break dates and VAR coefficients are estimated by maximizing a Gaussian quasi-likelihood function using the efficient dynamic programming algorithm outlined in Bai and Perron (2003) and Qu and Perron (2007). This also allows the construction of confidence intervals for the break dates.

[2] As there are potential breaks in the variance-covariance matrix in the residuals of (2), we use the Heteroskedasticity Consistent (HC) version when testing for the breaks in the conditional mean dynamics.

Testing for breaks in the conditional covariance matrix $\mathbf{\Sigma}$ proceeds along similar lines as the procedure for breaks in dynamics described above, and this is conducted conditional on the estimated coefficient breaks. The null hypothesis of no breaks, that is $H_0 : \mathbf{\Sigma}_j = \mathbf{\Sigma}_1$ (j = 2, ..., m+1) is tested for an unknown $m \leq M$ number of breaks, using a 'double maximum' likelihood ratio-type test statistic. In particular, the Sup$F$ statistic in (3) is replaced by the SupLR statistic defined as

$$\sup LR_T(m,q,\varepsilon) = \sup_{(\lambda_1,\ldots,\lambda_m \in \Lambda_\ell)} 2\ln\left(\frac{\hat{L}_T(T_1,\ldots,T_m)}{\tilde{L}_T}\right), \tag{6}$$

where $\ln \hat{L}_T(T_1,...,T_m) = -\frac{T}{2}(\ln 2\pi + 1) - \sum_{j=1}^{m+1} \frac{T_j - T_{j-1}}{2} \ln |\hat{\mathbf{\Sigma}}_j|$ and $\mathbf{\Sigma}_j = \frac{1}{T_j - T_{j-1}} \sum_{t=T_{j-1}+1}^{T_j} \hat{\mathbf{u}}_t \hat{\mathbf{u}}_t'$

with $\hat{\mathbf{u}}_t$ (t = 1, …., T) the residual vector from (2), while ~ represents the corresponding quantities computed under the null hypothesis of no covariance matrix breaks. Although m is used here to denote the number of covariance matrix breaks, as for the VAR coefficient break test in (3), it should be noted that the procedure we employ does not restrict either the number or dates of these two types of breaks to be the same.

If the null hypothesis of no covariance matrix breaks is rejected, the number of breaks is obtained using a similar procedure to that for the VAR coefficients, with the sequential test in (5) replaced by

$$SEQ_T(l+1|l) = \max_{1\leq j\leq l+1}\left[\sup_{\tau\in\Lambda_{j,\varepsilon}}\left(\ln\left(\frac{\hat{L}_T(T_1,\ldots,T_{j-1},\tau,T_j,\ldots,T_l)}{\hat{L}_T(T_1,\ldots,T_l)}\right)\right)\right] \tag{7}$$

Again the break dates are then estimated by maximizing a Gaussian quasi-likelihood function, which is also used for computing confidence intervals for these dates[3].

[3] In Bataa, Osborn, Sensier and van Dijk (2009), we develop a procedure for iterating between coefficient and covariance breaks, with finite sample inference adopted to confirm breaks. However, this procedure is not required in the current context since coefficient breaks are found to be almost entirely absent in this international business cycle analysis.

A practical problem in implementing this procedure is that the required asymptotic critical values have been tabulated for only very small systems. Therefore, the critical values employed here are obtained by simulation, using the method described in Bai and Perron (1998, p.57) and Perron and Qu (2006, p.389)[4].

2.2 Decomposing volatility and correlation breaks

As already discussed, covariance breaks can originate from changes in either volatility or correlations. For example, an increase in covariance could result from an increase in correlation or from a decline in volatility. Since these have quite different implications for the nature of international business cycle linkages, it is important to identify volatility or correlation as the source of a covariance break.

Using the identity $\mathbf{\Sigma} = \mathbf{D}\ \mathbf{P}\ \mathbf{D}$, we distinguish between volatility and correlation changes (that is, changes in $\mathbf{D}$ and $\mathbf{P}$ respectively) conditional on the breaks identified by the methodology of subsection 2.1, especially the covariance matrix break dates. This is achieved by firstly testing whether there is a significant volatility break at each covariance break date ($j = 1, \ldots, m$) and sequentially eliminating insignificant ones. Secondly, after standardizing for (significant) volatility changes, an analogous procedure is undertaken to determine whether correlation breaks occur at each of the covariance break dates.

This recursive decomposition procedure employs the residuals from (2), which are computed recognising any coefficient breaks uncovered. Then set $\hat{T}_j^{(Cor)} = \hat{T}_j^{(Vol)} = \hat{T}_j^{(C)}$, $j = 1, \ldots, m = m^{(Vol)} = m^{(Cor)}$, where $\hat{T}_j^{(Cor)}$ and $\hat{T}_j^{(Vol)}$ are estimated correlation and volatility break dates respectively, with the number of corresponding breaks being $m^{(Vol)}$ and $m^{(Cor)}$. Using an obvious subscript notation to indicate the regime, a test of the null hypothesis $\mathbf{D}_j = \mathbf{D}_{j+1}$ is

[4] The Gauss program we employ for obtaining critical values has been modified from that used by Perron and Qu (2006) and available from Zhongjun Qu's website. The program of Qu and Perron (2007), also obtained from that website, is the basis of our program for testing and estimating coefficient and covariance breaks.

implemented through a test of equality[5] for the means of the vectors of squared residuals, $\hat{\mathbf{u}}_t^2 = (\hat{u}_1^2, ..., \hat{u}_p^2)'$, over each covariance regime j = 1, …, $m^{(Vol)}$. Employing finite sample bootstrap inference, each volatility break j = 1, …, $m^{(Vol)}$ is tested and, if one or more breaks is not significant, the least significant break is dropped and the remaining breaks are re-tested. This continues until all remaining volatility breaks are individually significant. Hence the algorithm delivers a set of $m^{(Vol)} \leq m$ estimated break volatility dates, which are a subset of the estimated covariance break dates $\hat{T}_1^{(C)}, ..., \hat{T}_m^{(C)}$.

Conditional on significant volatility breaks, the VAR residuals are standardized to unit variance and breaks in the correlation matrix $\mathbf{P}$ are examined by applying finite sample bootstrap inference to the statistic of Jennrich (1970), which is designed to test the equality of two correlation matrices. The procedure is analogous to that just outlined for volatility breaks, and hence begins with a (separate) test of $\mathbf{P}_j = \mathbf{P}_{j+1}$ for each j = 1, …, m. Again, by sequentially dropping the least significant break (according to the bootstrap p-value) until all remaining breaks are significant, this yields a set of $m^{(Cor)} \leq m$ correlation matrix breaks, which are also a subset of the estimated covariance break dates $\hat{T}_1^{(C)}, ..., \hat{T}_m^{(C)}$. The algorithms for separating volatility and correlation breaks are detailed in Appendix A.

Table 1 presents some Monte Carlo results on the performance of our procedure. This analysis employs a VAR(1) data generating process (DGP) for n = 3 variables, in which the mean is known to be zero (and hence an intercept is excluded) with constant VAR coefficient matrix

$$\mathbf{A} = \begin{bmatrix} 0.7 & 0.1 & 0.5 \\ 0.2 & 0.3 & 0.4 \\ 0 & 0 & 0.5 \end{bmatrix}$$

while the regime-specific correlation matrices and standard deviations are given by

[5] The test statistic computed is Hotelling's T^2 statistic for equality of the means of two samples. However, since this is simply a re-scaled F-statistic, identical finite sample results will be obtained using the latter.

$$\mathbf{P}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \mathbf{P}_2 = \begin{bmatrix} 1 & 0.5 & 0.2 \\ 0.5 & 1 & 0.2 \\ 0.2 & 0.2 & 1 \end{bmatrix},$$

$$\mathbf{D}_1^* = [0.02, 0.02, 0.035], \mathbf{D}_2^* = [0.015, 0.015, 0.015] \tag{8}$$

in which a vector **D*** presents the diagonal elements of the corresponding **D** matrix. The sample size in this experiment is set to T = 200, while the dates (but not the nature) of covariance breaks implied by (8) are assumed known at one or more of $0.25T$, $0.50T$ or $0.75T$. The nominal level of significance is 5 percent, with results based on 1,000 replications of the above DGP and 1,000 bootstrap replications used for testing.

- Table 1 about here -

Table 1 shows that both the volatility and correlation tests generally have good size when the break occurs in the other element of $\mathbf{\Sigma}$. This applies particularly to the volatility break test, while the correlation test is sometimes modestly over-sized in samples when the volatility and correlation breaks occur at different dates within the sample. The volatility test is also impressive in terms of the unit power it achieves in all cases. While power is lower for the correlation test, it is nevertheless always close to or exceeding 0.80, and appears to be unaffected by whether a volatility break occurs within the sample or not. Despite a suggestion in these results that it is (statistically) more difficult to establish the presence of correlation than volatility breaks, nevertheless these results, overall, are encouraging that a given covariance break can be reliably identified as being a volatility or correlation break, or both.

3. Data

Our analysis employs quarterly real GDP growth (measured as 100 times the difference of the log values) over the period 1970Q1 to 2010Q1[6] for seven economies (the Canada, Euro area, UK and US, together with the individual Euro area countries of France, Germany and Italy). All series for individual countries are obtained from the OECD database and are seasonally adjusted, with that for Germany taking account of the reunification in 1990.

Of course, the Euro area came into existence only in 1999 and its membership has expanded since that date. Although Greece was not formally a member until 2001, most Euro area analyses include Greece as its membership was anticipated. This is, however, far less evident for members who have joined since the beginning of 2007 (Cyprus, Malta, Slovenia and Slovakia). To avoid these problems, and other issues relating to the new member countries, our Euro area series is confined to the "Euro 12" as of January 2001 and is constructed as a weighted average of the GDP growth rates for these 12 countries, using the weights employed in the historical data of the Area Wide Model (AWM) of the European Central Bank (Fagan, Henry and Mestre, 2001).

Many business cycle analyses filter GDP growth rate data in order to remove very short run fluctuations and hence concentrate on the so-called business cycle frequencies. However, such filtering has substantial consequences for the dynamics of the process and hence we prefer to analyze unfiltered data.

Appendix B provides a univariate analysis of each of the series employed in the VAR models of the next section. This analysis decomposes the observed data into mean, dynamic and outlier components, and iteratively tests for breaks in these components and in the volatility of the series. Further, by identifying outliers within this iterative procedure, their detection takes account of structural breaks over the sample period.

[6] Although data are available from 1960 onwards, this earlier data is not reliable for some European countries and hence we our analysis employs data data from 1970.

It is notable from the results (which allow a maximum of five breaks in each mean, dynamic and volatility component) that breaks in the mean and dynamics of GDP growth are relatively rare: only France has a mean break and only the UK experiences a dynamic break. However, in line with previous analyses, volatility breaks are widespread, with all countries except France having at least one such break. Indeed, there is substantial communality in the dates of the identified volatility breaks, which typically occur in the early 1980s, then around 1990 and 2000. Outliers are identified for France, Italy and the UK (two). These outliers are removed (and replaced by the median of the six neighbouring non-outlier observations) prior to the substantive VAR analysis of the next section.

4. International Business Cycle Affiliations

We now turn to the principal interest of this paper, namely evidence for changes in international business cycle affiliations. All tests are performed at the 5 percent level of significance and a maximum of $M = 3$ breaks are permitted in both the VAR coefficients and covariance matrix, with a minimum of $\varepsilon = 20\%$ of sample observations required to be in each regime. These choices reflect the sample size available, with the aim of specifying ε to be large enough for the tests to have approximately correct size and small enough for them to have decent power. Moreover, this selection reflects the use of robust inference for the VAR coefficients in (2). See Bai and Perron 2003, pp.13-14) for a discussion of these issues.

4.1 Overall tests and break date estimation

Implementation of the procedure in subsection 2.1 requires prior specification of VAR lag order p. In practice, we employ the Hannan-Quinn criterion, which selects $p = 1$ for both the 'International VAR' (consisting of Canada, the Euro area, the UK and the US) and for the

'Euro Area VAR' (France, Germany and Italy). Further, the residuals in the subsamples indicated by coefficient breaks pass a system test for first-order serial correlation in both VARs[7].

The results of the system coefficient and covariance breaks tests are shown in Table 2. The WD*max* statistic for coefficient breaks is not significant in the International VAR, and hence no such breaks are considered in the subsequent analysis. However, this statistic finds evidence of change in the coefficients for the Euro area VAR, with the sequential test indicating the presence of only one such break, which is dated in 2002. On the other hand, both systems very strongly reject constancy for the covariance matrices, with the sequential tests pointing to three breaks in $\mathbf{\Sigma}$ for each system Indeed, it is remarkable that the dated breaks are synchronous across these systems. A further important point to note is that 2002Q1 is at the extreme end of the search interval for break dates, with 20% (eight years) of sample observations in the final covariance regime identified. Hence this last break, and the associated covariance regime, may be at least partly attributed to the communality of the 2008/9 recession[8].

– Table 2 about here –

Table 2 includes 90% confidence intervals for the break dates, computed as in Qu and Perron (2007). All intervals for the covariance breaks are relatively tight, covering between one and three years. However, the dating of the coefficient break for the Euro area appears to be less precise.

[7] The maximum lag considered in each case is eight. The serial correlation test applied is a multivariate extension of the Godfrey and Tremayne (2005) Lagrange Multiplier test robust to (conditional) heteroskedasticity.

[8] An analysis specifying a maximum of four breaks for the Euro Area VAR, with ε = 15% of the total observations, confirms the general findings reported in Table 2, and specifically the sequential test finds no evidence of a fourth covariance break. Perhaps not surprisingly, however, the (single) mean break and the third covariance break are dated later, namely at 2004Q1 and 2003Q4, respectively. Neverthtless, the substantive results reported in subsequent tables carry over to this case. With four equations in the International VAR, the number of parameters involved make it impractical to conduct a corresponding analysis for that specification.

It is notable that breaks in the international G-7 system are identified by Doyle and Faust (2005) in 1981Q1 and 1992Q2, which are closely aligned with the covariance break dates in Table 2. However, we find no evidence that changes in the intercepts or VAR coefficients apply around these dates.

4.2 VAR coefficients

Considering first the Euro area VAR, for which one coefficient break is dated in Table 2, a recursive "general to specific" procedure is adopted to investigate the nature of the break. Starting from the general model in which all coefficients (including the intercepts) are allowed to change at 2002Q1, and all covariances are allowed to change at the break dates identified for this component in Table 2, finite sample bootstrap inference is applied to test separately the null hypotheses $a_{h,k}^{(1)} = a_{h,k}^{(2)}$ for each element (h, k) of $\mathbf{A}_1$ and $\delta_h^{(1)} = \delta_h^{(2)}$ for each element h of $\boldsymbol{\delta}$ in (2). The bootstrap DGP uses the estimated VAR coefficients (obtained with the restriction under test imposed, but all other coefficients allowed to change at the break date) and with *i.i.d.* re-sampled residual vectors from within each covariance regime that applies for each t. The (intercept or VAR coefficient) restriction with the highest bootstrap p-value is sequentially imposed, one by one, and the test repeated, until all remaining restrictions are rejected at a 5% significance level.

This procedure leads to the conclusion that only a single coefficient of the Euro area VAR changes in 2002, with this being the intercept in the equation for France[9]. Indeed, no other coefficient has a bootstrap p-value less that 10% when it is dropped during this recursive procedure. Therefore, it appears that dynamic business cycle interactions are unchanged over the period, even for a sample including the recent recession.

[9] Although the date differs, the existence of such a break is consistent with the finding of a mean break for France in the univariate analysis of Appendix B.

Table 3 presents the estimated elements of $\mathbf{A}_1$ for both VAR systems, with that for the Euro area obtained allowing the France intercept (but no other coefficient) to change at 2002Q1. Bootstrap percentage *p*-values are also shown for the significance of each individual coefficient, with these computed conditional on the relevant covariance break dates in Table 2.

– Table 3 about here –

For the International VAR, these coefficients point to the US leading GDP growth in each of Canada, the UK and the Euro area. Otherwise, the only significant effect at 5% is the own lag for the Euro area, although lagged UK output is close to being significant for the US. Perhaps surprisingly, however, cross-country dynamic effects between the E-3 countries appear to be limited to France and Italy, with no evidence that Germany leads the business cycle in these countries. On the other hand, the magnitude of coefficients indicates possible effects from these other countries to Germany, although these fail to be significant. Further, (significant) own dynamic effects apply in France and Italy, but not Germany, with these compatible with the own lag applying for the Euro area as a whole in the International VAR.

4.3 Volatility and correlations

Tables 4 and 5 present results relating to volatility and conditional correlations, respectively, for both VARs. These results are based on VARs whose coefficient estimates are shown in Table 3. The break dates considered are those detected for $\mathbf{\Sigma}$ in the relevant system of Table 2.

Once again, there is remarkable agreement across the two VAR specifications. For volatility, Table 4 points to significant breaks in both 1984 and 1993, but not 2002. Thus, the well-established break in US output growth volatility around 1984 (see, for example, McConnell and Perez-Quiros, 2000) is not confined to that country. However, the nature of this break varies, with the UK also showing a substantial decline in volatility, even in the VAR context of Table 4, whereas volatility for both Canada and the Euro area aggregate are largely unchanged at this date. Nevertheless, for the individual E-3 countries, Italy also shows a substantial decline. In line with the volatility breaks found in the univariate analysis of Table A.1 for Canada, the UK, the Euro area and Germany around 1993, these countries continue to exhibit such declines within the relevant VARs of Table 4, so that the international linkages captured here are not sufficient to explain this feature.

– Table 4 about here –

Turning to the correlation results in Table 5, there is (not surprisingly) agreement that the covariance break of 2002 represents a correlation break. However, within the International VAR, there is no compelling statistical evidence that correlation breaks occur in either 1984 or 1993, thus indicating that any globalization effects that have applied are not sufficiently strong to render a significant change in the disturbance correlation matrix for the system[10]. However, the effect of economic integration within Europe is evidenced by the correlation break for the Euro area system in 1984, but no further break is found.

– Table 5 about here –

Within the first correlation regime for both VARs in Table 5, whether this extends to 1984 or 1993, the individual cross-country correlations are typically relatively small, with the notable exceptions of those between the US and Canada and also between France and Germany. Indeed, the main effect of the 1984 Euro area break is to increase the correlation between France and Italy, which then exceeds that between France and Germany. Although all correlations increase in the final correlation regime from 2002, those between the E-3 countries are particularly high, at 0.74 or above. While the contemporaneous correlations within the International system are generally a little lower over this period, the high correlation of the UK with the Euro area is comparable to those between the E-3 countries.

[10] Interestingly, a VAR system for the US with France, Germany and Italy finds two covariance breaks, in 1984 and 1999, with only the former being a significant volatility break but both significant as correlation breaks. This system finds the US to be effectively contemporaneously uncorrelated with the individual E-3 countries between 1984 and 1999, with France having a correlation with the US of 0.11 and those for Germany and France being negative and less than 0.10 in absolute value.

5. Conclusions

This study provides formal evidence about changes in international business cycle affiliations since 1970. In doing so, the results largely agree with the findings of Heathcote and Perri (2004), Kose, Otrok and Whiteman (2008), Del Negro and Otrok (2008), Stock and Watson (2005) and Doyle and Faust (2005) that there is little evidence of increased international synchronization of the business cycle over this period, whether as measured by increased dynamic coefficients for cross-country effects or by the contemporaneous correlations of the disturbances in the VAR system. While the most recent period is an exception in that there is strong evidence of increased correlations due at least partly to the 2008/09 recession, dynamic effects (as captured by the VAR coefficients) appear to have remained stable over this extraordinary time. Economic integration within the Euro area also appears to have had its major effect through contemporaneous correlations rather than dynamics and, in this context, we find evidence of increased synchronization from 1984 with GDP growth in Italy becoming much more closely correlated with that in France. However, in common with Canova *et al.* (2007), we find little evidence that the 1990s saw increased co-movement among the three major Euro area countries.

Methodologically, the principal novelty of the paper is the procedure employed for identifying whether system covariance breaks can be attributed to variance or correlation breaks, or both. While the simulation evidence we provide indicates that volatility breaks of realistic magnitudes can be reliably identified with quarterly data for around 50 years, breaks may be more difficult to identify for the correlation matrix as a whole, perhaps particularly when not all correlations change. Therefore, an issue for further investigation is whether a procedure that focuses on individual elements of the correlation matrix may provide evidence of additional breaks in the international business cycle.

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Appendix A:
Algorithms for Volatility and Correlation Breaks

The procedure works from the identified break dates in the covariance matrix $\mathbf{\Sigma}$, and is designed to distinguish these breaks as volatility and/or correlation breaks. If any coefficient breaks are uncovered, the bootstrap inference employed here uses these $i = 1, \ldots, \ell$ identified VAR coefficient breaks and the associated coefficient break dates $\hat{T}_1^{(B)}, ..., \hat{T}_\ell^{(B)}$.

A.1 Identifying Variance Breaks

Having estimated coefficient breaks in the VAR of (2), with any required restrictions imposed on the coefficients, obtain the residuals for t = 1, …, T and allocate these to the covariance regimes j = 1, …, m. To start the recursive decomposition procedure for $\mathbf{\Sigma}_j$, set $\hat{T}_j^{(Cor)} = \hat{T}_j^{(Vol)} = \hat{T}_j^{(C)}$, j = 1, …, $m = m^{(Vol)} = m^{(Cor)}$, where $\hat{T}_j^{(Cor)}$ and $\hat{T}_j^{(Vol)}$ are estimated correlation and volatility break dates respectively, with the number of corresponding breaks being $m^{(Vol)}$ and $m^{(Cor)}$.

Now consider the identity $\mathbf{\Sigma}_j = \mathbf{D}_j\mathbf{P}_j\mathbf{D}_j$, where $\mathbf{D}_j$ is the diagonal matrix of standard deviations of the elements of $\mathbf{u}_t$ for $t = \hat{T}_{j-1}^{(Vol)} + 1, ..., \hat{T}_j^{(Vol)}$ and $\mathbf{P}_j$ is the corresponding correlation matrix. As indicated in Section 2.2, structural change in volatility is tested through $H_0 : \boldsymbol{\mu}_j = \boldsymbol{\mu}_{j+1}$ where $\boldsymbol{\mu}_j$ and $\boldsymbol{\mu}_{j+1}$ are vectors of the means of squared residuals before and after break j, respectively. With $\hat{T}_j^{(Vol)} - \hat{T}_{j-1}^{(Vol)}$ and $\hat{T}_{j+1}^{(Vol)} - \hat{T}_j^{(Vol)}$ observations in the respective subsamples, the test statistic used is

$$L_j = \frac{\left(\hat{T}_j^{(Vol)} - \hat{T}_{j-1}^{(Vol)}\right)\left(\hat{T}_{j+1}^{(Vol)} - \hat{T}_j^{(Vol)}\right)}{\hat{T}_{j+1}^{(Vol)} - \hat{T}_{j-1}^{(Vol)}} (\overline{\boldsymbol{\mu}}_j - \overline{\boldsymbol{\mu}}_{j+1})' \mathbf{S}_j^{-1} (\overline{\boldsymbol{\mu}}_j - \overline{\boldsymbol{\mu}}_{j+1}) \qquad \text{(A.1)}$$

where $\overline{\boldsymbol{\mu}}_j = \sum_{t=\hat{T}_{j-1}^{(Vol)}+1}^{\hat{T}_j^{(Vol)}} \hat{\mathbf{u}}_t^2 \Big/ \left(\hat{T}_j^{(Vol)} - \hat{T}_{j-1}^{(Vol)}\right)$, $\mathbf{S}_j = \dfrac{1}{\hat{T}_{j+1}^{(Vol)} - \hat{T}_{j-1}^{(Vol)} - 2} (\mathbf{W}_j + \mathbf{W}_{j+1})$,and

$$\mathbf{W}_j = \sum_{t=\hat{T}_{j-1}^{(Vol)}+1}^{\hat{T}_j^{(Vol)}} (\hat{\mathbf{u}}_t^2 - \overline{\boldsymbol{\mu}}_j)(\hat{\mathbf{u}}_t^2 - \overline{\boldsymbol{\mu}}_j)', \ \hat{\mathbf{u}}_t^2 = (\hat{u}_{1,t}^2, ..., \hat{u}_{n,t}^2)'.$$

Under normality, L_j is asymptotically distributed as Hotelling's T^2 statistic with n and $\hat{T}_{j+1} - \hat{T}_{j-1} - 2$ degrees of freedom, but our procedure employs finite sample inference using a bootstrap (see Rencher, 2002, p. 122).

The algorithm to identify the volatility breaks proceeds as follows:

1. For each volatility break j = 1, …, $m^{(Vol)}$ calculate the statistic L_j (A.1) for the null hypothesis $\boldsymbol{\mu}_j = \boldsymbol{\mu}_{j+1}$ in adjacent regimes j and j+1.
2. To obtain the finite sample distribution of L_j under the null hypothesis, the residual vectors $\hat{\mathbf{u}}_t$ for $t = \hat{T}_{j-1}^{(Vol)} + 1, ..., \hat{T}_j^{(Vol)}, ..., \hat{T}_{j+1}^{(Vol)}$ are randomly *i.i.d.* re-sampled, with a wild

bootstrap[11] employed in other volatility regimes, to create the bootstrap residuals $\hat{\mathbf{u}}_t^*$. By resampling the vector of residuals, the contemporaneous correlation structure is kept intact. Using these bootstrap residuals, together with the associated VAR coefficient estimates, a pseudo dataset $\mathbf{y}_t^*$ is generated recursively from randomly chosen starting values. The VAR coefficients are re-estimated and associated individual coefficient tests applied using $\mathbf{y}_t^*$, but for computational feasibility the coefficient break dates are assumed known at $\hat{T}_1^{(B)}, ..., \hat{T}_\ell^{(B)}$. The pseudo residuals and the L_j statistic for $\boldsymbol{\mu}_j = \boldsymbol{\mu}_{j+1}$ in (A.1) are calculated, assuming volatility break dates are known, yielding the value L^*. This procedure is replicated N times and the resulting distribution used to obtain the empirical p-value for L_j.

3. If one or more volatility breaks $j = 1, \ldots, m^{(Vol)}$ examined in step 2 is not significant, the number of breaks is reduced to $m^{(Vol)}$ - 1 by removing the least significant break and the analysis in step 1 is repeated. This is iterated until all volatility breaks are individually significant.

The algorithm delivers a set of $m^{(Vol)} \leq m$ estimated break volatility dates, which are a subset of the estimated covariance break dates $\hat{T}_1^{(C)}, ..., \hat{T}_m^{(C)}$.

A.2 Identifying Correlation Breaks

An analogous recursive procedure to that just described is applied in order to identify which covariance matrix breaks represent correlation breaks. Here we test H_0: $\mathbf{P}_j = \mathbf{P}_{j+1}$ versus H_A: $\mathbf{P}_j \neq \mathbf{P}_{j+1}$ where $\mathbf{P}_j$ is the correlation matrix that applies for $t = \hat{T}_{j-1}^{(Cor)} + 1, ..., \hat{T}_j^{(Cor)}$. The test applied is that of Jennrich (1970). First let

$$\overline{\mathbf{P}}_j = \frac{\left(\hat{T}_j^{(Cor)} - \hat{T}_{j-1}^{(Cor)}\right)\hat{\mathbf{P}}_j + \left(\hat{T}_{j+1}^{(Cor)} - \hat{T}_j^{(Cor)}\right)\hat{\mathbf{P}}_{j+1}}{\hat{T}_{j+1}^{(Cor)} - \hat{T}_{j-1}^{(Cor)}},$$

and define

$$\mathbf{Z}_j = \sqrt{\frac{\left(\hat{T}_j^{(Cor)} - \hat{T}_{j-1}^{(Cor)}\right)\left(\hat{T}_{j+1}^{(Cor)} - \hat{T}_j^{(Cor)}\right)}{\hat{T}_{j+1}^{(Cor)} - \hat{T}_{j-1}^{(Cor)}}}\, \overline{\mathbf{P}}_j^{-1}(\hat{\mathbf{P}}_j - \hat{\mathbf{P}}_{j+1}).$$

Then the test statistic is

[11] The wild bootstrap sets $\hat{\mathrm{u}}_t^* = \eta\, \hat{\mathrm{u}}_t$ where η is randomly chosen as +1 or -1 with equal probabilities. The use of the wild bootstrap here allows the covariance matrix to differ across regimes, with constant variance imposed by the *i.i.d.* bootstrap only for regimes j and j+1. Gonçalves and Kilian (2004) show that the wild bootstrap has good performance for testing dynamic coefficients in the presence of heteroskedasticity.

$$J_j = \frac{1}{2} tr(\mathbf{Z}_j{}'\mathbf{Z}_j) - dg(\mathbf{Z}_j)\boldsymbol{\Delta}_j^{-1}\, dg(\mathbf{Z}_j), \qquad \text{(A.2)}$$

where the matrix Δ_j has typical $(x, y)^{\text{th}}$ element $\{\delta_{x,y} + r_{xy}r^{xy}\}$, in which $\delta_{x,y}$ is the Kronecker delta, $\{r_{xy}\}$ and $\{r^{xy}\}$ are indicated elements of $\overline{\mathbf{P}}_j$ and $\overline{\mathbf{P}}_j^{-1}$, respectively, while *tr* and *dg* denoting the trace and the diagonal, respectively. Although (A.2) is asymptotically distributed as $\chi^2(n(n-1)/2)$ under the null hypothesis of constant correlation, our inference again relies on a bootstrap procedure, which proceeds as follows.

First, employing the same VAR residuals as for the volatility test, standardize them to account for volatility breaks using $\hat{\mathbf{u}}_t'^{(Vol)} = \hat{\mathbf{u}}_t'\hat{\mathbf{D}}_j^{-1}$, $t = \hat{T}_{j-1}^{(Vol)}+1, ..., \hat{T}_j^{(Vol)}$ and $j = 1, \ldots, m^{(Vol)}+1$. Then:

1. For each potential correlation break $j = 1, \ldots, m^{(Cor)}$, calculate the Jennrich (1970) statistic J_j as in (A.2).
2. Using an *i.i.d.* bootstrap for the residual vector at each time period in the adjacent correlation regimes j and j+1, hence $t = \hat{T}_{j-1}^{(Cor)}+1, ..., \hat{T}_{j+1}^{(Cor)}$, and a vector wild bootstrap for all other t, obtain standardized pseudo residuals, $\hat{\mathbf{u}}_t'^{*}$, $t = 1, \ldots, T$. Then re-apply the volatility effects to form $\hat{\mathrm{u}}_t'^{**} = \hat{\mathrm{u}}_t'^{*}\hat{\mathrm{D}}_h$, $t = 1, \ldots, T$ and $h = 1, \ldots, m^{(Vol)} + 1$. Generate recursively a pseudo dataset $\mathbf{y}_t^*$ using randomly chosen initial values, together with the coefficients $\hat{\boldsymbol{\beta}}_i$, $i = 1, \ldots, \ell$ and residuals $\hat{\mathbf{u}}_t^{**}$. Allowing for ℓ coefficient breaks at dates $\hat{T}_1^{(B)}, ..., \hat{T}_\ell^{(B)}$, estimate the VAR coefficients and apply the associated individual coefficient tests using $\mathbf{y}_t^*$, to obtain the bootstrap residuals $\hat{\mathbf{u}}_t^{***} = \mathbf{y}_t^* - (\mathbf{I} \otimes \mathbf{z}_t'^{*})\mathbf{S}\hat{\boldsymbol{\beta}}_i^*$, $i = 1, \ldots, \ell$. Clear the residuals of volatility changes using the given volatility break dates, then obtain an empirical correlation break statistic using (A.2); denote the value obtained as J_j^*. Repeat this process N times and use the resulting distribution to obtain the empirical p-value for J_j.
3. If one or more of the volatility breaks for $j = 1, \ldots, m^{(Cor)}$ examined using the procedure of the previous step is not significant, the number of breaks is reduced to $m^{(Cor)}$-1 by removing the least significant break and the analysis in step 1 is repeated. This is iterated until all $m^{(Cor)}$ remaining correlation breaks are individually significant.

This procedure yields $m^{(Cor)} \leq m$ estimated correlation break dates, which are a subset of the estimated covariance break dates $\hat{T}_1^{(C)}, ..., \hat{T}_m^{(C)}$.

Appendix B:

Univariate Analysis of Breaks

A preliminary univariate analysis is employed to correct the data for outliers and to investigate breaks in the individual series. The univariate procedure is again based on testing for breaks using the methodology of Qu and Perron, but applied here to distinguish between shifts in the level (mean), persistence and volatility. Further, outlier detection is also undertaken in this iterative framework. The methodology is identical to that employed to analyze inflation in Bataa *et al.* (2008), except that seasonality is not relevant in the present case as seasonally adjusted data are employed for our business cycle analysis.

The univariate decomposition for an observed series x_t can be written as:

$$x_t = L_t + O_t + y_t \qquad \text{(A.3)}$$

$$L_t = \mu_j \qquad t = T_{k_1 - 1} + 1, \ldots, T_{k_1}; \quad k_1 = 1, \ldots, m_1 + 1 \qquad \text{(A.4)}$$

$$y_t = \sum_{i=1}^{p} \phi_{k_3, i} y_{t-i} + u_t \qquad t = T_{k_2 - 1} + 1, \ldots, T_{k_2}; \quad k_2 = 1, \ldots, m_2 + 1 \qquad \text{(A.5)}$$

$$\operatorname{var}(u_t) = \sigma_{k_3}^2 \qquad t = T_{k_3 - 1} + 1, \ldots, T_{k_3}; \quad k_3 = 1, \ldots, m_3 + 1 \qquad \text{(A.6)}$$

where $T_0 = 0$; $T_{m_i} = T$ $(i = 1, 2, 3)$ and T denotes the total sample size. Note that the number and timing of structural breaks in the level (L_t) and dynamic (y_t) components in (A.4) and (A.5), and also the volatility in (A.6), are not constrained to be equal.

The univariate procedure iterates between testing for structural breaks in the level, dynamics and volatility components as well as testing for the presence of outliers. In these iterations, all components except the one under study are removed using the latest estimates. Thus, for example, level and dynamic components are removed when outliers are examined. To account for possible interaction between dynamic and volatility breaks, an additional 'inner loop' iterates between testing for breaks in the autoregressive coefficients of the dynamic component y_t and its conditional volatility. To be precise, after removing outliers and mean components, the sub-loop tests for breaks in dynamics; in the first iteration this employs heteroskedasticity robust inference, but subsequently volatility-regime estimates are used[12]. If any break is detected, the AR model is estimated allowing for these breaks, with variance breaks then investigated using the resulting residuals. If volatility breaks are detected, the variances are estimated over the implied segments. Once this 'inner loop' has converged, we return to the main loop. Further details of this procedure can be found in Bataa *et al*. (2008).

Break detection for each of L_t, y_t and σ^2 uses the Qu and Perron (2007) methodology, while outliers are defined as observations more than a given distance (measured in terms of the interquartile range) from the median, using the procedure described in Stock and Watson (2003). By embedding the outlier analysis within the iterative procedure, changes in other characteristics are taken into account when testing and correcting for outliers.

Since fewer parameters are estimated in the univariate analysis compared with the multivariate models, the maximum number of breaks allowed in each of (A.4) to (A.6) is $M = 5$, while the minimum sample proportion in a regime set to $\varepsilon = 0.15$. Outliers are detected as observations lying beyond four times the interquartile range from the "local" (regime-dependent) median, while the order p in (A.5) is specified using the Hannan-Quinn criterion for the whole sample and checking the adequacy via subsample heteroskedasticity robust tests for serial correlation.

[12] Pitarakis (2004) uses Monte Carlo simulations to assess the properties of mean breaks in the presence of volatility break, uncovering an extreme size distortion that actually increases with the sample size. He then provides evidence on improvements offered by a GLS transformation in that context.

Table A.1. Univariate GDP Growth Decomposition

Canada	UK	Euro Area	US	France	Germany	Italy
A. Mean Breaks						
Break Dates						
				1978Q4		
Regime Means						
0.72	0.49	0.57	0.71	1.02	0.50	0.49
				0.47		
B. Outliers Detected						
	1973Q1			1974Q4		1970Q1
	1979Q2					
C. Dynamics						
AR Order						
1	3	1	1	2	0	1
Dynamic Break Dates						
	1989Q4					
Persistence in Each Dynamic Regime						
0.42	0.31	0.52	0.31	0.56	0.06	0.53
	0.71					
D. Volatility						
Volatility Break Dates						
1977Q4	1980Q1	1980Q1	1983Q4		1983Q4	1983Q4
1990Q4	1989Q4	1992Q4			1992Q4	
2000Q2	2000Q1	2001Q4			2001Q3	
Standard Deviation in Each Volatility Regime						
1.10	1.23	0.64	1.17	0.47	1.05	0.96
0.83	0.67	0.58	0.55		1.26	0.56
0.40	0.31	0.29			0.59	
0.52	0.52	0.57			0.94	
E. Number of Iterations (Inner Loop)						
2 (2)	19* (2)	3 (2)	2 (2)	3 (2)	2 (3)	4 (2)

Notes: Decomposition into mean, outlier and dynamic components uses the iterative method outlined in the text. Breaks are detected using the Qu and Perron (2007) structural break test (Mean: trimming 15%, max breaks 5). Panel A shows the estimated break dates for the level component and the mean quarterly growth rates in the various subsamples determined by the level breaks. The dates of detected outliers are given in Panel B, where an outlier is defined as being four times the inter-quartile range from the median. Panel C indicates the autoregressive order of the dynamic component, selected according to the HQ information criterion, and used at entry to the dynamic/volatility sub-loop. If the selected order is zero, this is replaced by an order of one. The estimated break dates in the AR coefficients and estimated persistence, defined as the sum of autoregressive coefficients, based on sub-samples defined by the break dates, are also reported. Panel D shows the estimated volatility breaks and the corresponding estimated standard deviations of the errors. Finally, Panel E shows the number of iterations required for convergence of the main loop, with * indicating that the iteration converged to a two cycle oscillation and choice between these is made based on HQ criterion; the number of iterations required for convergence of the volatility/persistence loop is shown in parentheses.

Table 1. Monte Carlo Results for Separation of Volatility and Correlation Breaks

	$T^{(Vol)} = 0.00T$		$T^{(Vol)} = 0.25T$		$T^{(Vol)} = 0.50T$		$T^{(Vol)} = 0.75T$	
	Cor.	Vol.	Cor.	Vol.	Cor.	Vol.	Cor.	Vol.
$T^{(Cor)} = 0.00T$	N.A.	N.A.	0.049	**1.000**	0.051	**1.000**	0.049	**1.000**
$T^{(Cor)} = 0.25T$	**0.868**	0.065	**0.867**	**1.000**	**0.807** 0.128	0.050 **1.000**	**0.871** 0.084	0.048 **1.000**
$T^{(Cor)} = 0.50T$	**0.926**	0.043	0.071 **0.919**	**1.000** 0.050	**0.925**	1.000	**0.909** 0.075	0.056 **1.000**
$T^{(Cor)} = 0.75T$	**0.820**	0.046	0.064 **0.833**	**1.000** 0.042	0.096 **0.786**	**1.000** 0.044	**0.817**	**1.000**

Notes: $T^{(Vol)}$ and $T^{(Cor)}$ indicate volatility and correlation breaks, respectively, at the indicated dates. Each cell has one or two rows corresponding to the number of distinct break dates. The reported values are the fraction of times a specific volatility or correlation break is significant (at 5%) according to the correlation or volatility test, as appropriate employing a sample of T = 200 and the data generating process provided in the text (subsection 2.2). Numbers in bold represent power, whereas others represent size.

Table 2. System Structural Break Test Results

A. International VAR		B. Euro Area VAR	
Coefficients	**Covariance Matrix**	**Coefficients**	**Covariance Matrix**
Asymptotic WD*max* Test Statistics [and Critical Values]			
32.32 [45.42]	**149.82** [27.91]	**44.35** [32.78]	**75.42** [21.09]
Asymptotic Sequential Test Statistics [and Critical Values]			
N.A.	**58.96** [29.55]	**10.38** [32.31]	**50.61** [22.06]
	60.94 [30.71]		**24.30** [22.98]
Break Dates and Confidence Intervals			
	1984Q2 1983Q2 1984Q3		**1984Q2** 1983Q3 1984Q3
	1993Q1 1991Q3 1993Q2		**1993Q1** 1990Q2 1993Q2
	2002Q1 2001Q2 2004Q3	**2002Q1** 1996Q3 2003Q2	**2002Q1** 2001Q2 2002Q2

Notes: Panel A relates to the International VAR (Canada, UK, Euro area, and US), while Panel B relates to the Euro Area VAR (France, Germany, Italy). A VAR order of $p = 1$ is used in both cases. The sequential test statistics compare $\ell+1$ versus ℓ breaks, beginning with $\ell = 1$. N.A. indicates not appropriate, since the overall test statistic is not significant. Asymptotic critical values for all tests are given in brackets, using a 5% significance level. Estimated break dates (in bold) are followed by the 90 percent confidence interval for this date.

Table 3. Individual VAR Coefficients

A. International VAR					B. Euro Area VAR			
	Canada	**UK**	**Euro Area**	**US**		**France**	**Germany**	**Italy**
Canada	**0.15** (22.56)	**-0.02** (86.99)	**0.07** (37.96)	**0.10** (35.93)	**France**	**0.35** (0.22)	**0.37** (25.66)	**0.32** (7.59)
UK	**0.02** (83.46)	**0.07** (56.36)	**-0.06** (35.54)	**0.18** (5.87)	**Germany**	**-0.04** (30.24)	**-0.15** (48.98)	**0.02** (68.54)
Euro Area	**0.19** (12.51)	**0.20** (22.00)	**0.45** (0.00)	**0.07** (58.01)	**Italy**	**0.27** (0.01)	**0.31** (9.74)	**0.40** (0.73)
US	**0.39** (0.02)	**0.25** (3.59)	**0.18** (1.82)	**0.18** (8.41)				

Notes: Columns represent equations. The first value (in bold) of each cell is the estimated coefficient, while the second value, in parentheses, is the bootstrap *p*-value (expressed as percentage) for the null hypothesis that the corresponding true value is zero. The Euro area VAR is estimated allowing an intercept break for France at 2002Q2.

Table 4. Volatility Test Results

	A. International VAR					B. Euro Area VAR			
Subsample	**Break *p*-Value**	**Canada**	**UK**	**Euro Area**	**US**	**Break *p*-Value**	**France**	**Germany**	**Italy**
1970Q1-1984Q2		0.88	1.14	0.55	1.15		0.48	1.05	0.95
1984Q3 - 1993Q1	0.00	0.72	0.64	0.62	0.41	0.73	0.46	1.24	0.53
1993Q2 - 2002Q1	0.00	0.41	0.47	0.43	0.54	0.32	0.43	0.81	0.57
2002Q2 - 2010Q1	43.99					55.82			

Notes: For each VAR, the break test *p*-value reports the empirical significance (as a percentage) of the volatility structural break as measured by the test of no change in the diagonal elements of $\mathbf{\Sigma}$ over adjacent Covariance Matrix subsamples identified in Table 2, with the result placed against the dates of the later subsample. The values reported are the final ones computed in the respective general to specific procedures (see subsection 2.3). The corresponding sub-sample residual standard deviations are also reported, where these are computed after merging subsamples based on the break test results (using 5% significance).

Table 5. Correlation Results

	A. International VAR					B. Euro Area VAR			
Subsample	**Break *p*-Value**	**Economy**	**Canada**	**UK**	**Euro Area**	**Break *p*-Value**	**Economy**	**France**	**Germany**
1970Q1-1984Q2		**UK**	0.20				**Germany**	0.42	
		Euro	0.13	0.29			**Italy**	0.00	0.18
		US	0.40	0.18	0.13				
1984Q3 - 1993Q1	24.92					0.06	**Germany**	0.30	
							Italy	0.58	0.18
1993Q2 - 2002Q1	83.25					22.69			
2002Q2 - 2010Q1	2.66	**UK**	0.45			0.05	**Germany**	0.74	
		Euro	0.41	0.75			**Italy**	0.79	0.83
		US	0.59	0.65	0.60				

Notes: As for Table 4, except that the results relate to the correlation matrix rather than volatility.