I Walked the Line: Identification of Fiscal Multipliers in SVARs - Preliminary paper -

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What this paper is about (or my attempt to follow the walked line)

- To provide policy recommendations for fiscal interventions is today problematic since there is little agreement in the profession (both theoretically and empirically) about the qualitative effects of such interventions
- If we focus on the empirical SVAR literature, we have from the estimation of fiscal multipliers (i.e. dynamic impact of tax cut and increased government spending) that
  - Results are dispersed.
  - No stylized facts have emerged.

- Two contributions:
  1. Derive *analytical* mapping between output elasticities of fiscal variables and fiscal multipliers.
  2. Estimate robust fiscal multipliers.

  Two main results:
  1. Short run spending multipliers are larger than tax multipliers.
  2. In the short run, tax multipliers can be negative.

Contribution 1: Analytical mapping between output elasticities of fiscal variables and fiscal multipliers

$$A_0 X_t = \sum_{l=1}^{p} A_l X_{t-l} + \epsilon_t$$

$$A_0 u_t = \epsilon_t \quad (1)$$

Where $A_0 = \begin{pmatrix} 1 & -c_1 \\ -a_1 & 1 \end{pmatrix}$

$$\Sigma = E(u_t u_t') = A_0^{-1} E(\epsilon_t \epsilon_t') A_0^{-1'} = A_0^{-1} \Sigma_\epsilon A_0^{-1'} \quad (2)$$

Bivariate model

Equation (1) can for a simple bivariate model be written as

$$\begin{aligned} u_t^Y &= c_1 u_t^T + \epsilon_t^Y \\ u_t^T &= a_1 u_t^Y + \epsilon_t^T \end{aligned}$$

This implies that the impact response of GDP and tax revenue to the two structural shocks can be written as

$$\begin{bmatrix} u_t^Y \\ u_t^T \end{bmatrix} = \frac{1}{1 - a_1 c_1} \begin{bmatrix} 1 & c_1 \\ a_1 & 1 \end{bmatrix} \begin{bmatrix} \epsilon_t^Y \\ \epsilon_t^M \end{bmatrix}$$

Tax revenue elasticity of output:

$$\frac{\partial u_t^Y / \partial \epsilon_t^T}{\partial u_t^T / \partial \epsilon_t^T} = c_1 \text{ Prior:} c_1 \leq 0$$

Output elasticity of tax revenue:

$$\frac{\partial u_t^T / \partial \epsilon_t^Y}{\partial u_t^Y / \partial \epsilon_t^Y} = a_1 \text{ Prior:} a_1 \geq 1$$

Since

$$\Sigma = \begin{bmatrix} \sigma_{YY} & \sigma_{YT} \\ \sigma_{YT} & \sigma_{TT} \end{bmatrix}$$

The paper shows (?) that by using equation (2) that the analytical solution for c_1 can be written as

$$c_1(a_1; \Sigma) = \frac{\sigma_{YY} - a_1 \sigma_{YY}}{\sigma_{YY} - a_1 \sigma_{YY}}$$

The expression gives a non-linear mapping between output elasticity of tax revenue (a_1) and tax revenue elasticity of output (c_1).

If $c_1 = 0$, which is equivalent to a Cholesky factorization in which $[Y_t, T_t]$. Then we have that

$$\begin{bmatrix} u_t^Y \\ u_t^T \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \bar{a}_1 = \frac{\sigma_{YT}}{\sigma_{TT}} & 1 \end{bmatrix} \begin{bmatrix} \epsilon_t^Y \\ \epsilon_t^T \end{bmatrix}$$

If $a_1 = 0$, which is equivalent to a Cholesky factorization in which $[T_t, Y_t]$. Then we have that

$$\begin{bmatrix} u_t^Y \\ u_t^T \end{bmatrix} = \begin{bmatrix} 1 & \frac{\sigma_{YT}}{\sigma_{TT}} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \epsilon_t^Y \\ \epsilon_t^T \end{bmatrix}$$

Robust finding in the literature is that $\sigma_{YT} > 0$.

Henze, with this scheme a decline in taxes must be associated with a decline in GDP.

Tax Multipliers

Dollar response of output to a tax shock of size of one dollar

$$TM_0(a_1;\Sigma) = (-1)\frac{c_1(a_1;\Sigma)}{1 - a_1 c_1(a_1;\Sigma)}\frac{1}{\overline{T}/\overline{Y}}$$

$$TM_0(a_1;\Sigma) = \frac{a_1\sigma_{YT} - \sigma_{TT}}{a_1^2\sigma_{YY} - 2a_1\sigma_{YT} + \sigma_{TT}}$$

The impact tax multipliers is a bounded function of a_1 (reasonable range: -0,30 to 0,30).

Trivariate model

$$
\begin{aligned}
u_t^Y &= c_1 u_t^T + c_2 u_t^G + \epsilon_t^Y \\
u_t^T &= a_1 u_t^Y + a_2 u_t^G + \epsilon_t^T \\
u_t^G &= b_1 u_t^Y + b_2 u_t^T + \epsilon_t^G
\end{aligned}
$$

Where:

$$
A_0 = \begin{bmatrix} 1 & -c_1 & -c_2 \\ -a_1 & 1 & -a_2 \\ -b_1 & -b_2 & 1 \end{bmatrix}
$$

As before, it should be possible to use equation (2) to get analytical solution for the elasticity of the structural coefficients.

To simplify this expression, it is assumed in the paper that $b_2 = 0$ (tax revenue elasticity of government spending).

Government Multipliers

Dollar response of output to a government shock of size of one dollar

$$TM_0(b_1;\Sigma) = \frac{\sigma_{GY} - b_1\sigma_{YY}}{b_1^2\sigma_{YY} - 2b_1\sigma_{YG} + \sigma_{GG}} \frac{1}{\overline{G}/\overline{Y}}$$

Tax Multipliers

Dollar response of output to a tax shock of size of one dollar

$$TM_0(b_1;\Sigma) = \frac{a_1\sigma_{YY} - \sigma_{YT}}{a_1^2\sigma_{YY} - 2a_1\sigma_{YT} + \sigma_{TT}} \frac{1}{\overline{T}/\overline{Y}}$$

This results under some assumptions (?) also hold in the multivariate case.

Contribution 2: Estimate robust fiscal multipliers

In the paper, the output elasticity of tax revenue a_1 is treated as a random variables and estimated using the following tree methodologies (very preliminary):

- Estimating output elasticity of tax revenue:
  Median for $a_1 = 2, 10$ while credible set ranges between $2, 00$ and $2, 20$
- Back of the Envelope Calculation from DSGE models:
  Median for $a_1 = 1, 60$ while credible set ranges between $1, 20$ and $2, 20$
- Bayesian estimation:
  ?

I am at the moment unable to confirm the main results of the paper from these estimations.