

**The Seeds of the 2007-2009 Crisis -  
the Housing Market and the Business Cycle**

**On-going**

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## Motivation

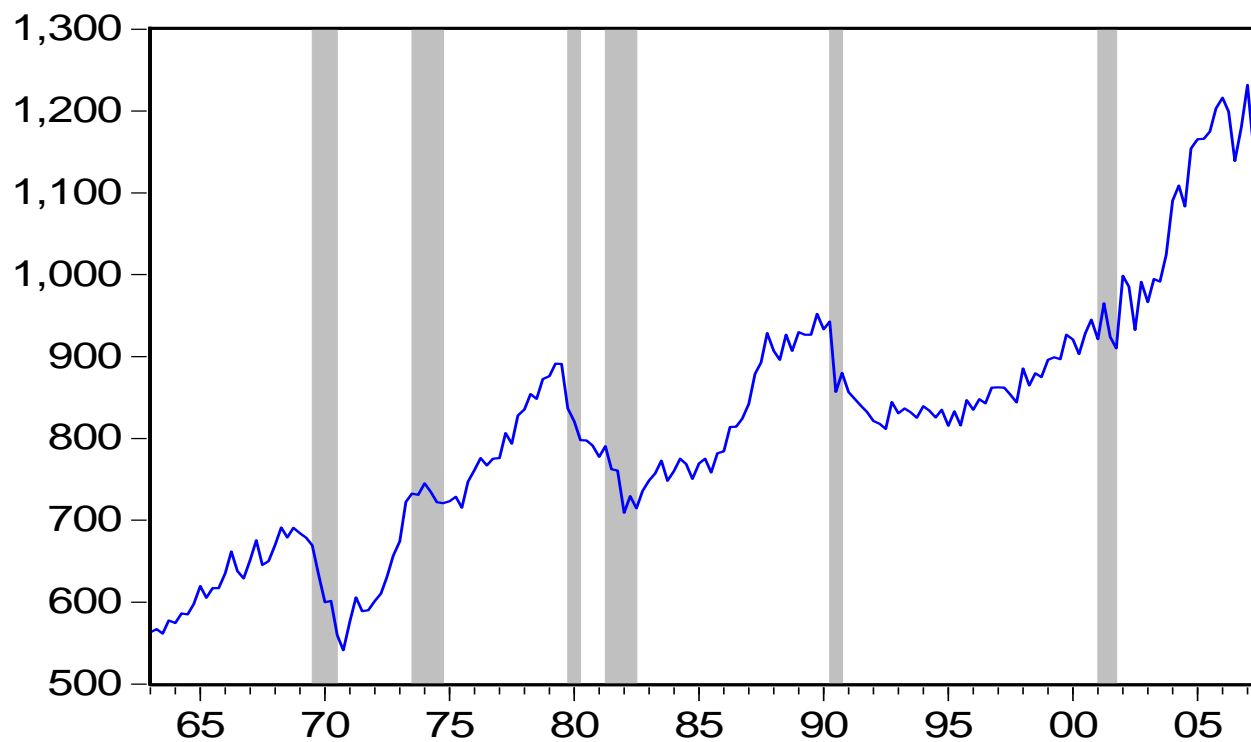
- Historical linkage between business cycle and the housing market cycle
- Understanding the relationship between the two cycles can shed light on the seeds of the housing crisis.
  - Crisis in housing market sector is associated with the particular features of the 2001 recession

### Goal

- This paper proposes a nonlinear two-factor model to represent the phases of the housing market cycle and the phases of the business cycle.
  - Model as a tool to investigate the historical relationship between the housing market and the business cycle, and the recent bubble-bust in the housing market.

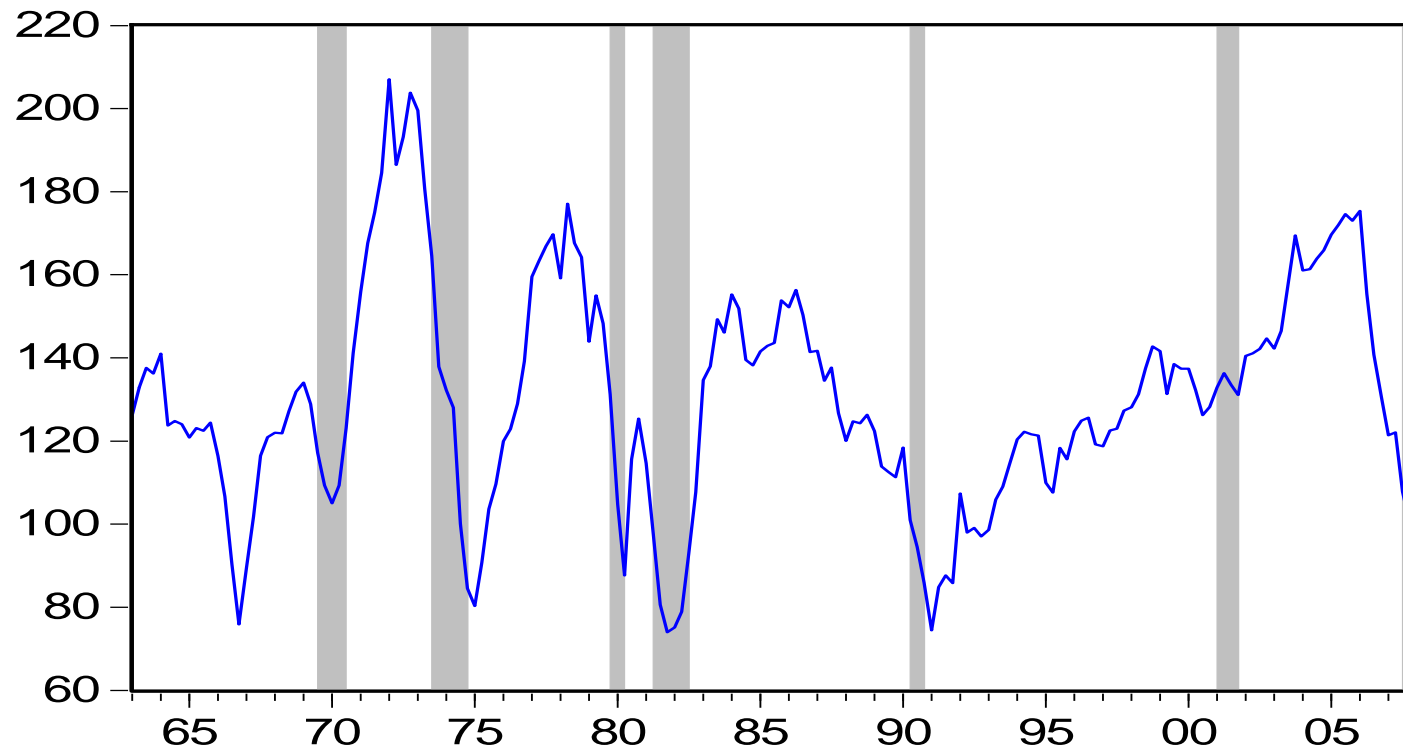
## Housing Market – Basic Facts

Median Real Price of Houses Sold - U.S.



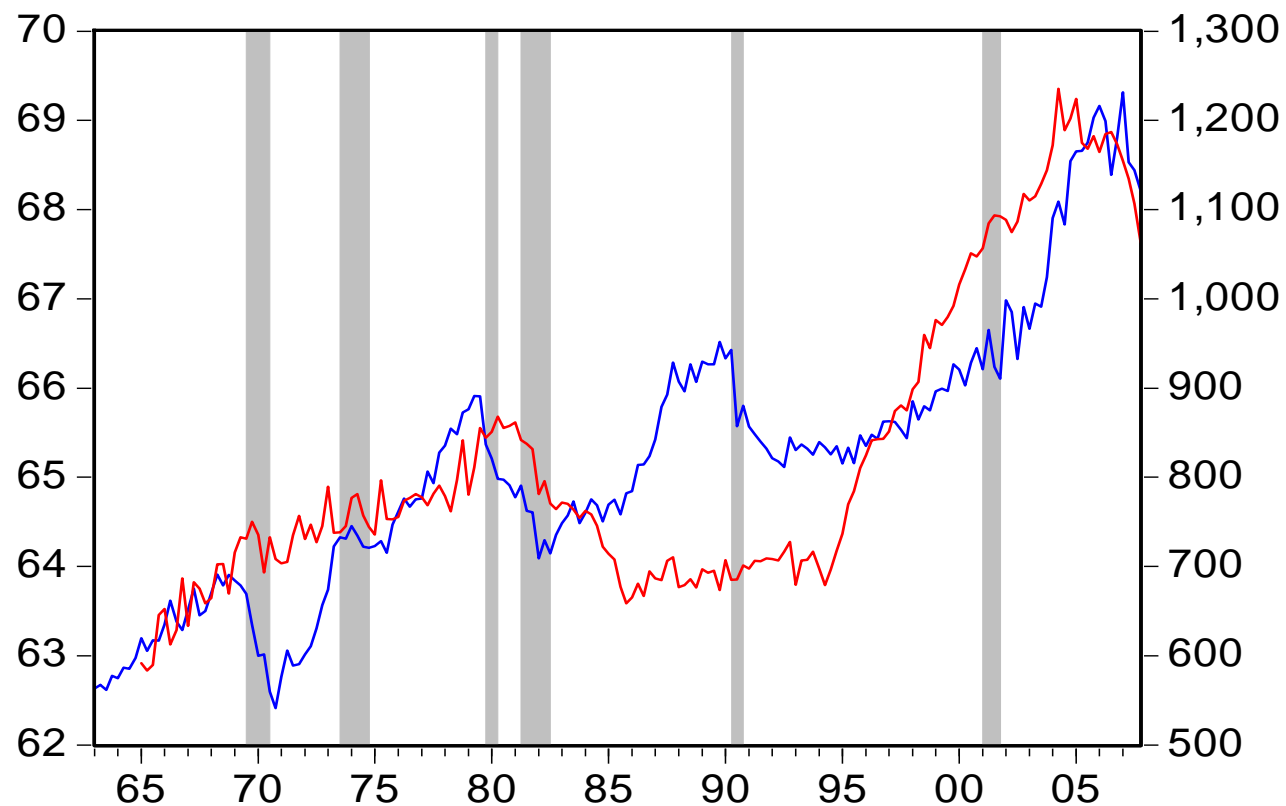
## Housing Market – Basic Facts

### U.S. Housing Starts



New Privately Owned Housing Units Started (Housing Starts). Source: U.S. Census Bureau

## Housing Market – Basic Facts



U.S. Census Bureau Home Ownership Rate (---), U.S. Median House Prices (—)

## Housing Market Cycle and NBER-Dated Recessions

- Peaks towards the end of business cycle expansions
  - Beginning of low housing market phases slightly leads recessions
- Troughs in the end of business cycle recessions
  - Beginning of high housing market phases coincides or slightly lags the end of recessions

## Literature

### **State-level housing prices in U.S.**

- Vansteenkiste (2007), Dees, Dimauro Pesaran, and Smith (2005) and Pesaran, Schuermann, and Weiner (2004) – VAR; Negro and Otrok (2007) - Dynamic factor model: house price indexes (48 states in U.S)
- Kim and Bhattacharya (2007) Smooth Transition Autoregressive (STAR) model to examine nonlinear properties of housing prices (nominal median sales price of existing single-family homes) during 1969-2004 in the U.S. and its four regions.
- Fadiga and Wang (2009) multivariate state-space model

### **U.S. metropolitan-level housing market**

- Clayton, Miller, and Peng (2008) bivariate panel VAR model (114 metropolitan areas in the U.S.).
- Wheaton and Lee (2007) panel estimation (with 101 MSA market in U.S.)
- Wheaton and Nechayev (2007) housing price in 59 MSA during 1998 to 2005 in U.S.: demand fundamentals (population, income growth, and decrease in interest rates) fail to explain this period increase in housing prices.
- Goodman and Thibodeau (2008)

## Contributions of this Paper

- Proposes a new model of the joint dynamics of the housing market and the business cycle (extends Chauvet 1998/1999)
- Common variation and asymmetries in the phases of housing market and the business cycle are modeled in a flexible setting.
- Relationship between the two cycles is analyzed simultaneously in a unified framework. Combined information from the business cycle, interest rates, and the housing market to shed light on housing market bubble and bust
- Findings:
  - Strong correlation between business cycles and the housing market cycles
  - Relationship has changed over time

## Modeling Strategy

- We use a dynamic two-factor model with regime switching to characterize the housing market and the business cycle. Model distinguishes between low value and high value periods using Markov switching.
- The housing market factor follows a two-state Markov process representing low and high housing price phases
  - The factor is constructed from the median price of houses sold in the four regions of the U.S. (Northeast, South, Midwest, and West)
- The business market factor follows a two-state Markov process representing recessions and expansion phases
  - The factor is built from key coincident economic indicators
- The model is used to study the interrelationship between the two factors and the two cycles that they represent

## Baseline Model

$$Y_{it} = \lambda_i F_t^H + \gamma_i F_t^{BC} + Z_{it}$$

$$Z_{it} = \Psi_i Z_{it-1} + \varepsilon_{it} \quad \varepsilon_{it} \sim \text{i.i.d. } N(0, \sigma_i^2)$$

where:

- $Y_{it}$  is  $8 \times 1$  vector:
  - Growth rate of housing prices in the West, Northeast, Midwest, and South
  - Growth rate of industrial production, real personal income, payroll employment, and manufacturing and trade sales
- $F_t^H$  is a scalar latent dynamic factor representing housing market: common movements across regions
- $F_t^{BC}$  is a scalar latent dynamic factor representing common movements in the coincident economic indicators associated with the business cycle
- $Z_{it}$  is a vector of idiosyncratic terms

## The Baseline Model (cont.)

Latent factors follow autoregressive process

$$F_t^H = \mu_{St} + \phi^H F_{t-1}^H + \beta^{BC} F_{t-1}^{BC} + v_t^H$$

$$F_t^{BC} = \alpha_{St} + \phi^{BC} F_{t-1}^{BC} + \beta^H F_{t-1}^H + v_t^{BC}$$

- Linkages between the two sectors modeled through vector autoregressive process in the transition equations and the covariance structure of the factors.

- Each factor follows different Markov switching processes  $S_t^H$  and  $S_t^{BC}$  representing:

Low and high price phases in the housing market

$$\mu_{st} = \mu_1 S_t^H + \mu_0 (1 - S_t^H) \quad \mu_1 > 0, S_t^H = 0, 1$$

$\mu_{st}$  - mean growth rate of housing price in high ( $S_t^H=1$ ) and low phases ( $S_t^H=0$ )

Recessions and expansions phases

$$\alpha_{st} = \alpha_1 S_t^{BC} + \alpha_0 (1 - S_t^{BC}) \quad \alpha_1 > 0, S_t^{BC} = 0, 1$$

$\alpha_{st}$  - mean growth rate of business cycle in expansions ( $S_t^{BC}=1$ ) and recessions ( $S_t^{BC}=0$ )

with transition probabilities

$$P[S_t^H = 0 | S_{t-1}^H = 0] = q^H \quad P[S_t^H = 1 | S_{t-1}^H = 1] = p^H$$

$$P[S_t^{BC} = 0 | S_{t-1}^{BC} = 0] = q^{BC} \quad P[S_t^{BC} = 1 | S_{t-1}^{BC} = 1] = p^{BC}$$

- No a priori restriction is imposed on the relationship between  $S_t^H$  and  $S_t^{BC}$

## The Data

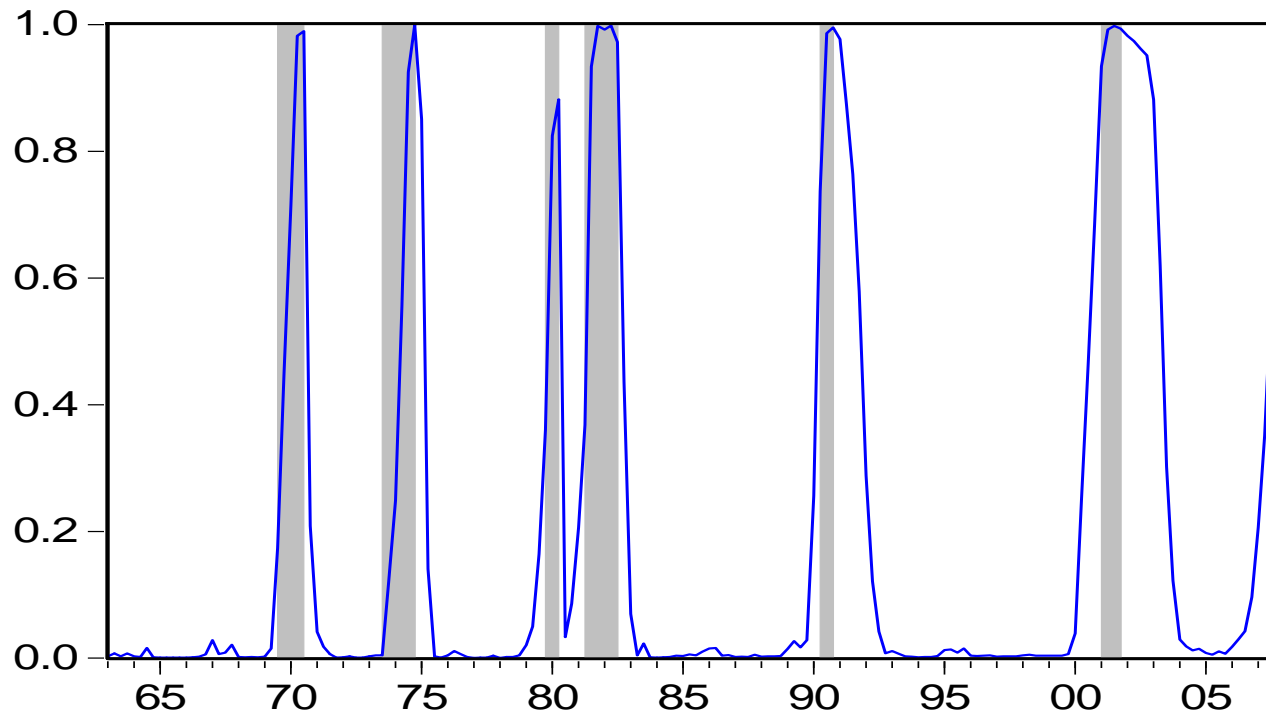
- Quarterly frequency: 1963q3 to 2007q4
- Economic indicators: Industrial Production (FRED®), manufacturing and trade sales (BEA), nonagricultural payroll (BLS), real personal income less transfer payments (BEA)
- Housing market: Median sales price of houses sold by region - West, Northeast, Midwest, and South. Source: Census Bureau
  - Deflated using CPI (Consumer Price Index for All Urban Consumers: All Items Less Food & Energy - BLS)
- Effective Federal Funds Rate (1963:1 on) and 30-Year Conventional Mortgage Rate (1971:2 on). Source: Board of Governors of the Federal Reserve System

## Results – Business Cycles

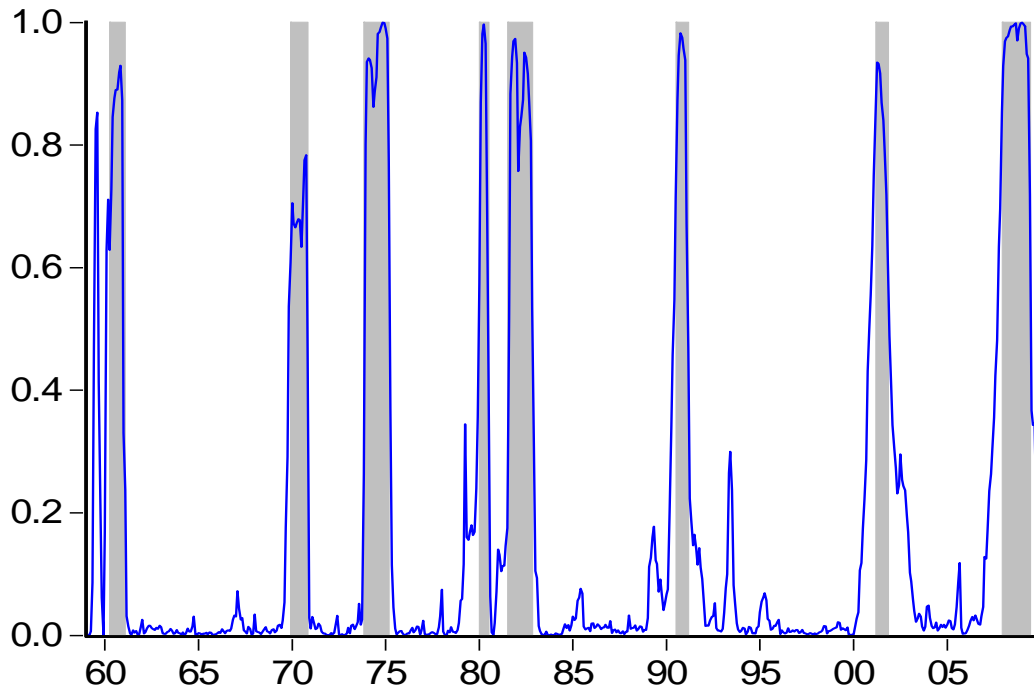
- Smoothed probabilities of recession closely match NBER-dated recessions.
- However, the 1990-1991 and especially the 2001 recessions were followed by slow recovery – probabilities of recession remained high
- Recovery after 2001 recession sluggish due to payroll employment: “jobless recovery”
- Probability of recession did not remain high during the recovery when other measures of employment are used. However, payroll is the series used by the NBER to date recessions.
  - Uncertainty in real time regarding the true state of the economy – NBER announced that the recession ended in November 2001 only in July 2003 (around 1 ½ years after the fact).

# U.S. Business Cycles

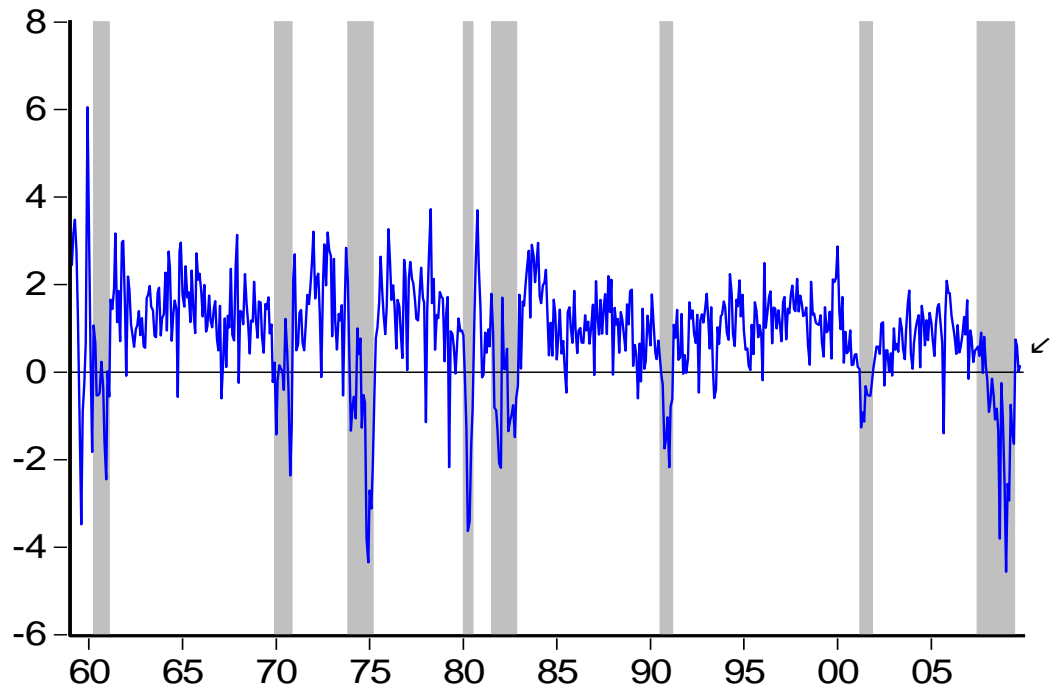
Probabilities of Recessions - U.S.



**Smoothed Probabilities of Recession**



**U.S. Business Cycle Indicator**

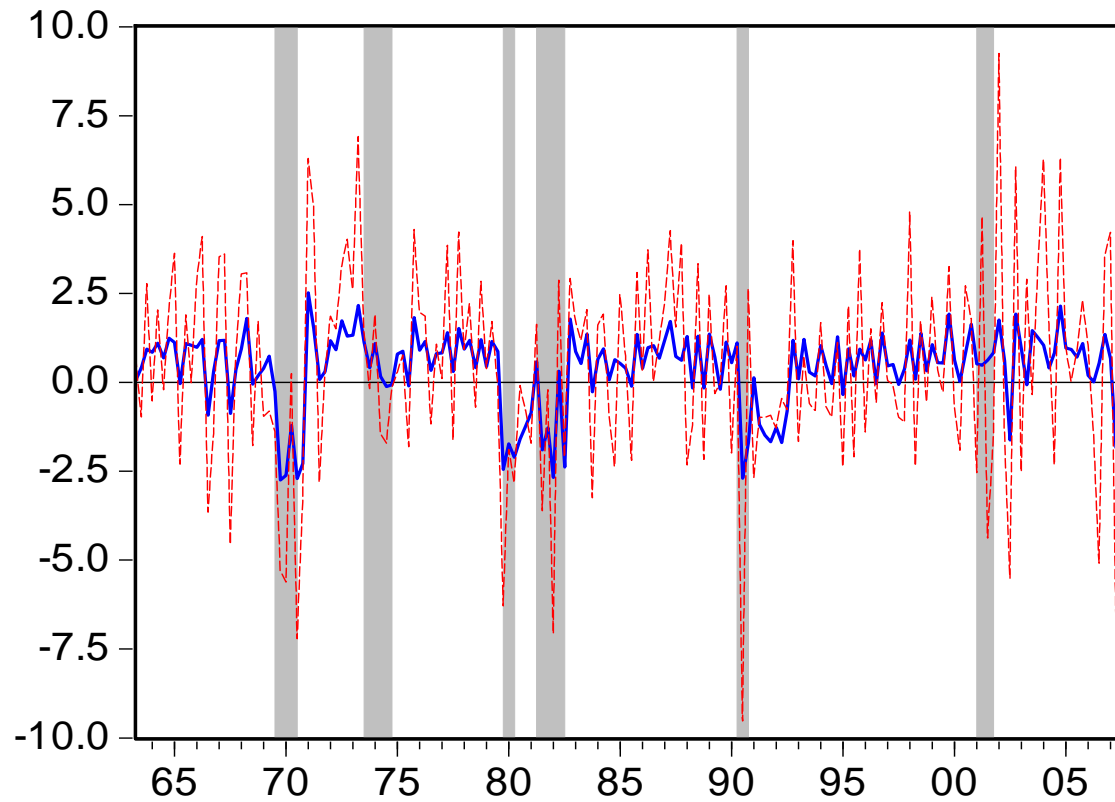


**Table 1: Maximum Likelihood Estimates – Basic Model**

<b>Parameters</b>	<b>Business Cycle</b>	<b>Parameters</b>	<b>Housing Cycle</b>
$\mu_1^{BC}$	1.50 (0.20)	$\mu_1^H$	3.52 (1.06)
$\mu_0^{BC}$	-0.54 (0.16)	$\mu_0^H$	-2.47 (0.79)
$p_{11}^{BC}$	0.93 (0.01)	$p_{11}^H$	0.97 (0.02)
$p_{00}^{BC}$	0.78 (0.03)	$p_{00}^H$	0.88 (0.06)
$\sigma_{\eta_{BC}}^2$	1.43 (0.21)	$\sigma_{\eta_H}^2$	1.19 (0.18)
$\phi^{BC}$	0.46 (0.05)	$\phi^H$	-0.28 (0.16)
$\lambda_{Production}$	1	$\lambda_{Northeast}$	1
$\lambda_{Income}$	0.56 (0.02)	$\lambda_{West}$	1.60 (0.50)
$\lambda_{Sales}$	0.78 (0.03)	$\lambda_{Midwest}$	1.41 (0.43)
$\lambda_{Employment}$	0.41 (0.02)	$\lambda_{South}$	1.26 (0.36)
$\sigma_{v,Production}^2$	0.32 (0.02)	$\sigma_{v,Northeast}^2$	36.92 (3.93)
$\sigma_{v,Income}^2$	0.11 (0.01)	$\sigma_{v,West}^2$	8.85 (1.30)
$\sigma_{v,Sales}^2$	0.78 (0.05)	$\sigma_{v,Midwest}^2$	23.09 (2.60)
$\sigma_{v,Employment}^2$	0.13 (0.01)	$\sigma_{v,South}^2$	8.21 (1.05)
$\beta^H$	0.16 (0.07)	$\beta^{BC}$	0.17 (0.05)
$\Psi_{Production}$	0.17 (0.05)	$\Psi_{Northeast}$	-0.37 (0.07)
$\Psi_{Income}$	0.16 (0.04)	$\Psi_{West}$	-0.40 (0.08)
$\Psi_{Sales}$	-0.24 (0.04)	$\Psi_{Midwest}$	0.43 (0.07)
$\Psi_{Employment}$	-0.49 (0.08)	$\Psi_{South}$	-0.32 (0.08)
$\sigma_{BC,SM}$	0.40 (0.16)		
<b>Log L</b>	-3470.22		

Asymptotic standard errors in parentheses

## Results - Housing Market Cycle



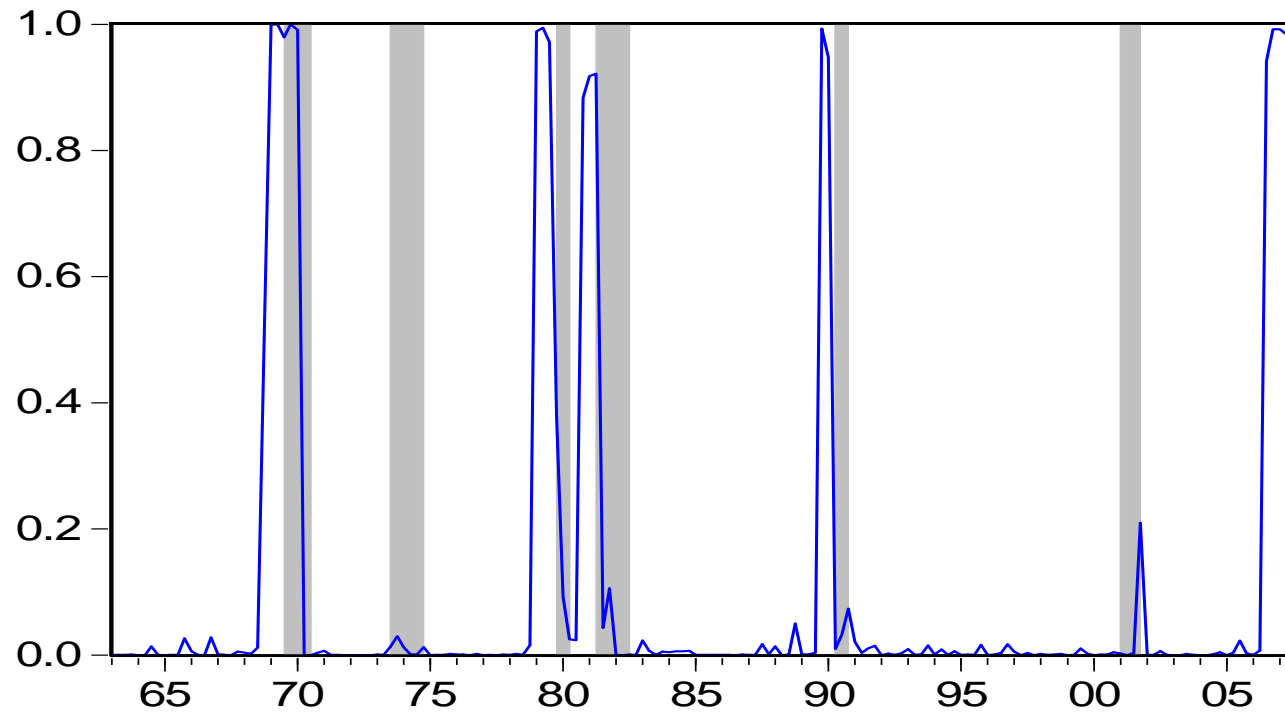
Housing Factor (—), U.S. Median Price of Houses Sold (---), and NBER Recessions (Shaded Area)

## Housing Market Cycles

- Probabilities of low housing market phase did not increase during the 1974-1975 and 2001 recessions
- Housing market did not exhibit regular cycle around these recessions
  - Variables measuring housing supply, housing demand, and prices did not fall very much during the 2001 recession (prices, housing starts, house completion, ownership rate, etc.)

# U.S. Housing Market Cycles

Probabilities of Low Housing Price - U.S.

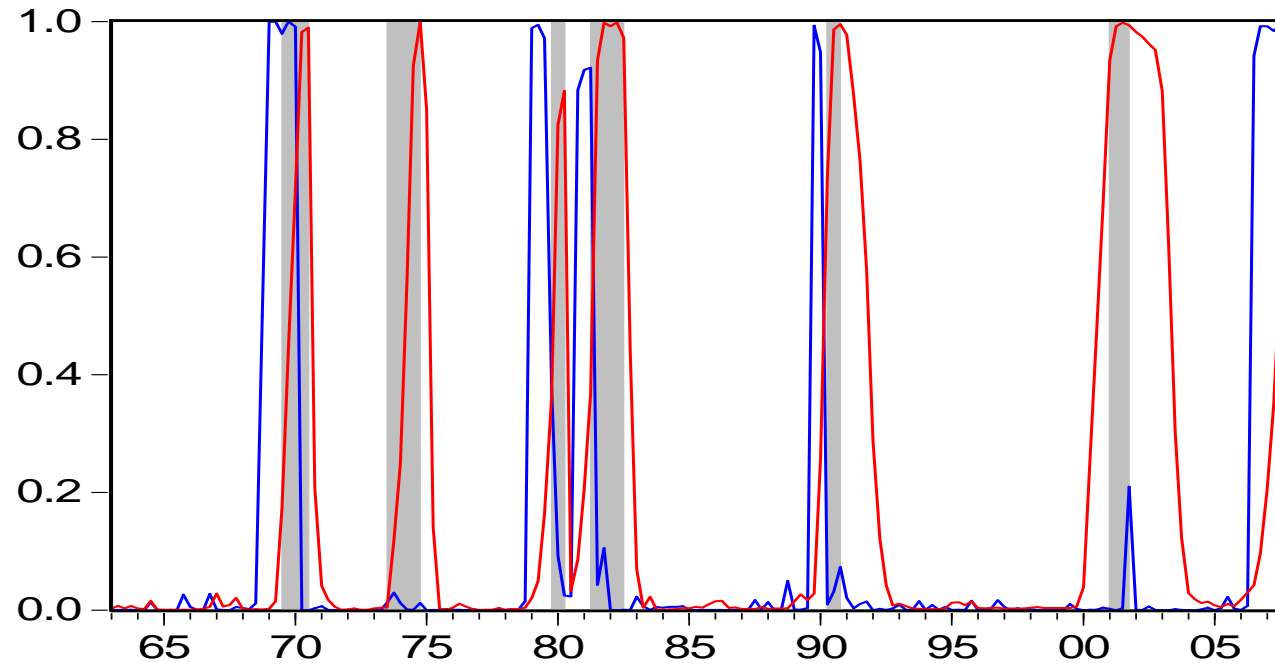


## Results – Housing Market Cycles

- Smoothed probabilities of low housing price closely associated with NBER-dated recessions with a lead
  - average of 2 quarters for peaks
- Estimated correlation parameter between house market factor and business cycle factor around 40% for full sample
- $\beta^{BC} > 0$      $F_{t-1}^{BC} \rightarrow F_t^H$
- $\beta^H > 0$      $F_{t-1}^H \rightarrow F_t^{BC}$
- Coefficients only capture linear average behavior. Around beginning and end of recessions  $\beta^{BC} < 0$

## Housing Market and the Business Cycle

Housing Cycle and the Business Cycle

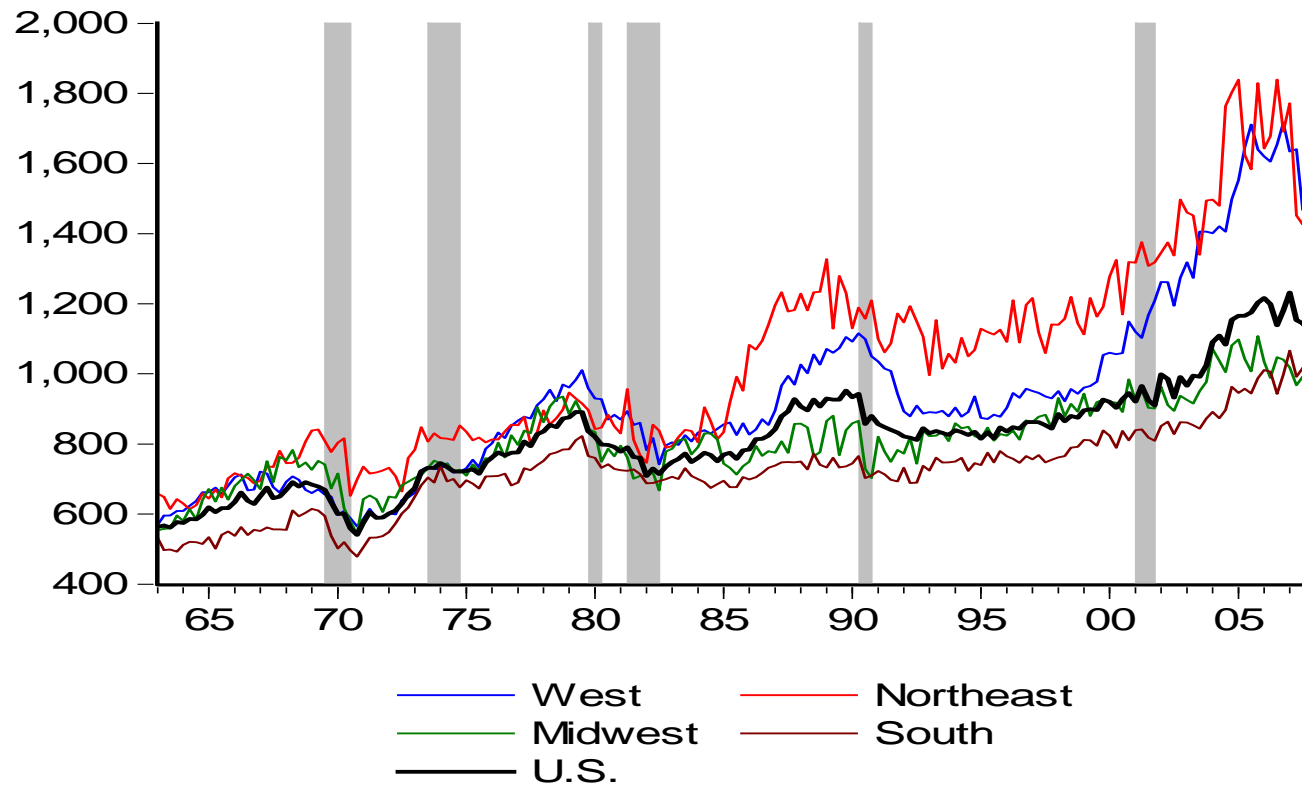


## Housing Market During the 2001 Recession

- Relationship between housing market and business cycle:
  - Missed link between housing cycle and the business cycle in the early 2000s
  - Continuous increase from mid-1990 to 2006, especially between 2002-2004.
  - In the West and Northeast there was a steeper increase in housing prices even during the 2001 recession. U.S. prices, ownership, house completion, and housing starts had steep increase from 2002 on.

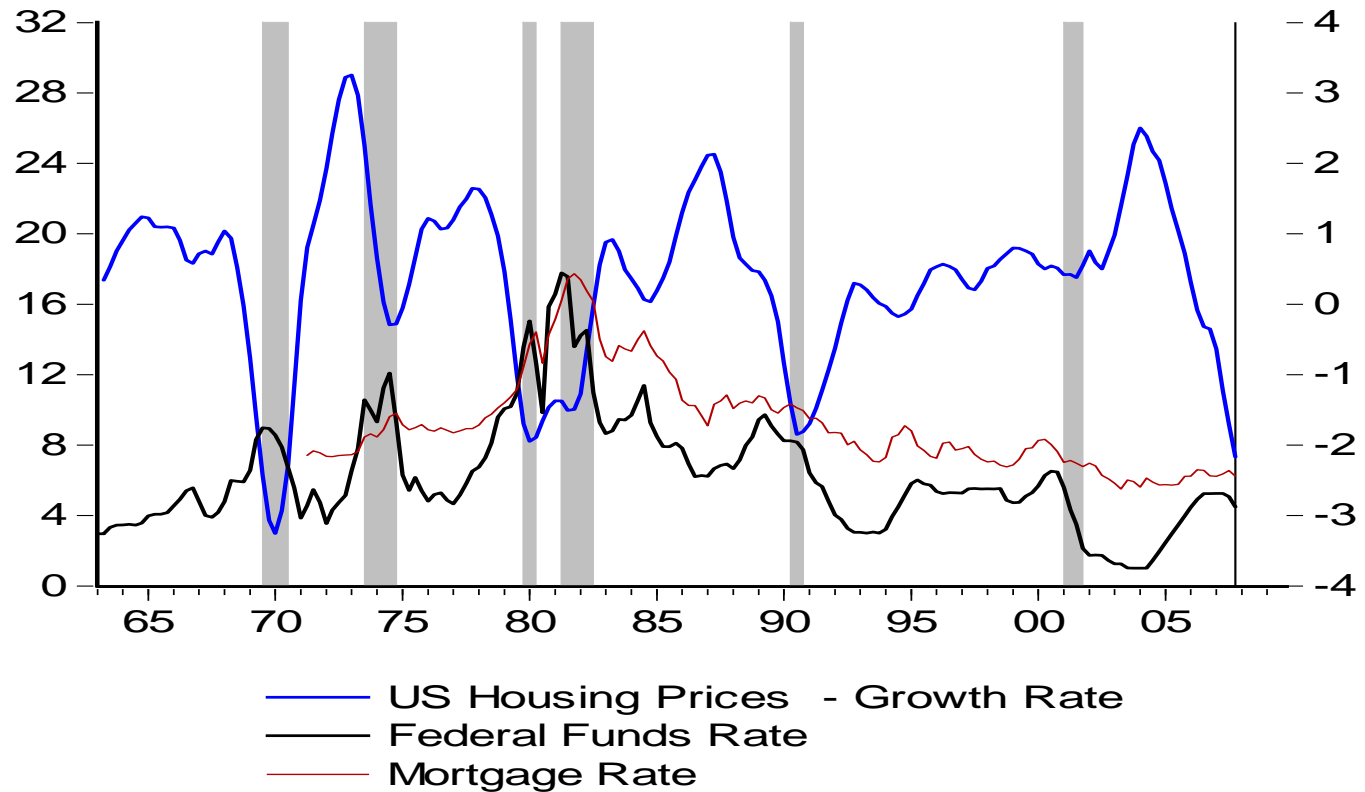
## Housing Market During the 2001 Recession

Median Prices by Region



# Link between Housing Market and the Business Cycle

Housing Market, Interest Rates, and Business Cycles



## Housing Market and Interest Rates

- Inverse relation with the level and changes in interest rates
- Interest rates movements lead housing market cycle
  - Increases in interest rates are related to subsequent decreases in house prices and housing starts
  - Increases in interest rates are related to subsequent decreases in economic activity
- Common element in 1974-75 and 2001 recessions: relatively low level of mortgage rates compared to historical record.

## Augmented Model

$$Y_{it} = \lambda_i F_t^H + \gamma_i F_t^{BC} + z_{it}$$

$$Z_{it} = \Psi_i z_{it-1} + \varepsilon_{it} \quad \varepsilon_{it} \sim \text{i.i.d. } N(0, \sigma_i^2)$$

$$F_t^H = \mu_{St} + \phi^H F_{t-1}^H + \beta^{BC} F_{t-1}^{BC} + \varphi_{St}^{H*} r_{t-1} + v_t^H$$

$$F_t^{BC} = \alpha_{St} + \phi^{BC} F_{t-1}^{BC} + \beta^H F_{t-1}^H + \varphi_{St}^{BC*} r_{t-1} + v_t^{BC}$$

- where  $r_t$  is the Federal Funds Rate. **Now, linkages also through their relationship with interest rates.**

- Interest rates coefficients follow independent Markov switching processes:

$$\begin{aligned} \varphi_{St}^{H*} &= \varphi_1^{H*} S_t^{H*} + \varphi_0^{H*} (1 - S_t^{H*}) & \varphi_1^{H*} > 0, S_t^{H*} = 0, 1 \\ \varphi_{St}^{BC*} &= \varphi_1^{BC*} S_t^{BC*} + \varphi_0^{BC*} (1 - S_t^{BC*}) & \varphi_1^{BC*} > 0, S_t^{BC*} = 0, 1 \end{aligned}$$

with transition probabilities:

$$P[S_t^{H*} = 0 | S_{t-1}^{H*} = 0] = q^{H*} \quad P[S_t^{H*} = 1 | S_{t-1}^{H*} = 1] = p^{H*}$$

$$P[S_t^{BC*} = 0 | S_{t-1}^{BC*} = 0] = q^{BC*} \quad P[S_t^{BC*} = 1 | S_{t-1}^{BC*} = 1] = p^{BC*}$$

**Table 2: Maximum Likelihood Estimates – Augmented Model with Interest Rates**

Parameters	Business Cycle	Parameters	Housing Cycle
$\mu_1^{BC}$	1.48 (0.20)	$\mu_1^H$	3.64 (1.04)
$\mu_0^{BC}$	-0.53 (0.16)	$\mu_0^H$	-2.45 (0.76)
$p_{11}^{BC}$	0.92 (0.01)	$p_{11}^H$	0.96 (0.02)
$p_{00}^{BC}$	0.78 (0.03)	$p_{00}^H$	0.89 (0.05)
$\sigma_{\eta_{BC}}^2$	1.41 (0.21)	$\sigma_{\eta_H}^2$	1.16 (0.16)
$\phi^{BC}$	0.45 (0.05)	$\phi^H$	-0.27 (0.15)
$\lambda_{Production}$	1	$\lambda_{Northeast}$	1
$\lambda_{Income}$	0.54 (0.02)	$\lambda_{West}$	1.61 (0.51)
$\lambda_{Sales}$	0.79 (0.03)	$\lambda_{Midwest}$	1.39 (0.41)
$\lambda_{Employment}$	0.42 (0.02)	$\lambda_{South}$	1.31 (0.34)
$\sigma_{v,Production}^2$	0.35 (0.02)	$\sigma_{v,Northeast}^2$	37.26 (3.91)
$\sigma_{v,Income}^2$	0.11 (0.01)	$\sigma_{v,West}^2$	8.17 (1.31)
$\sigma_{v,Sales}^2$	0.77 (0.05)	$\sigma_{v,Midwest}^2$	23.42 (2.58)
$\sigma_{v,Employment}^2$	0.12 (0.01)	$\sigma_{v,South}^2$	8.23 (1.10)
$\beta^H$	0.15 (0.06)	$\beta^{BC}$	0.14 (0.02)
$\Psi_{Production}$	0.16 (0.05)	$\Psi_{Northeast}$	-0.38 (0.06)
$\phi^{BC}$	-0.08 (0.02)	$\phi^H$	-0.09 (0.03)
$\Psi_{Income}$	0.16 (0.04)	$\Psi_{West}$	-0.41 (0.08)
$\Psi_{Sales}$	-0.21 (0.04)	$\Psi_{Midwest}$	0.43 (0.06)
$\Psi_{Employment}$	-0.53 (0.08)	$\Psi_{South}$	-0.31 (0.08)
$\sigma_{BC,SM}$	0.41 (0.16)		
<b>Log L</b>	-3412.98		

**Table 3: Maximum Likelihood Estimates – Augmented Model with Markov Switching in the Interest Rates Coefficients**

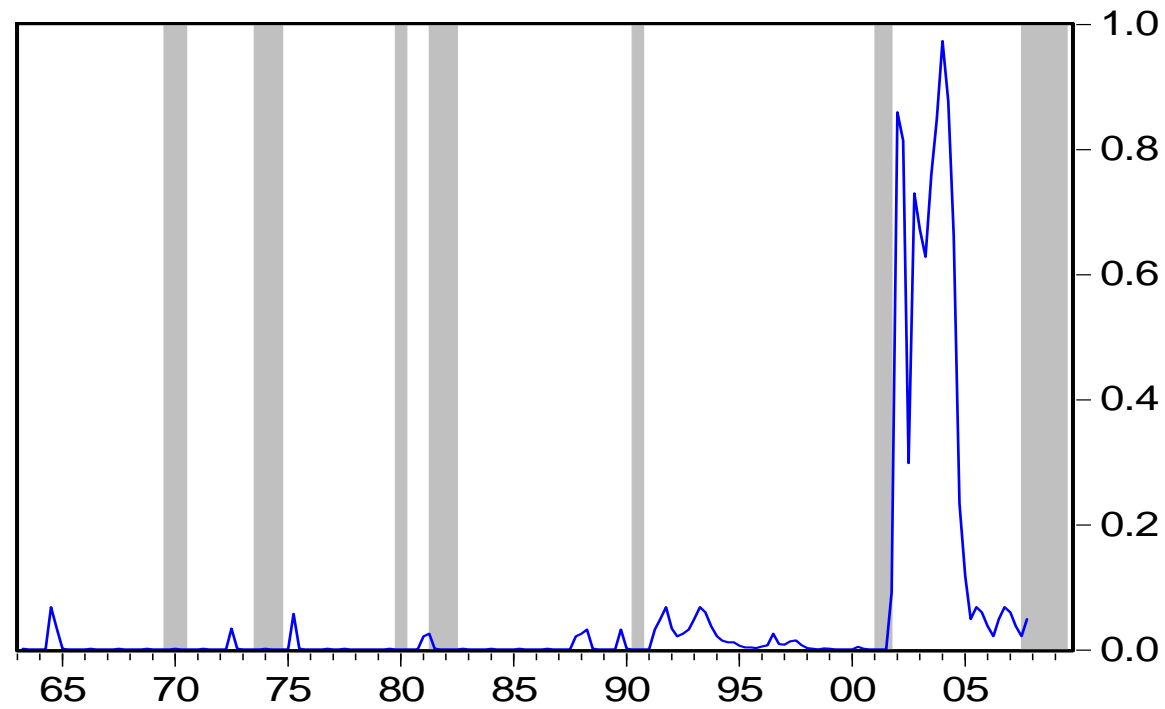
Parameters	Business Cycle	Parameters	Housing Cycle
$\mu_1^{BC}$	1.46 (0.19)	$\mu_1^H$	3.56 (1.01)
$\mu_0^{BC}$	-0.52 (0.15)	$\mu_0^H$	-2.42 (0.72)
$p_{11}^{BC}$	0.92 (0.01)	$p_{11}^H$	0.95 (0.02)
$p_{00}^{BC}$	0.79 (0.03)	$p_{00}^H$	0.88 (0.05)
$\sigma_{\eta_{BC}}^2$	1.42 (0.20)	$\sigma_{\eta_H}^2$	1.16 (0.14)
$\phi^{BC}$	0.43 (0.04)	$\phi^H$	-0.28 (0.14)
$\lambda_{Production}$	1	$\lambda_{Northeast}$	1
$\lambda_{Income}$	0.53 (0.02)	$\lambda_{West}$	1.63 (0.49)
$\lambda_{Sales}$	0.81 (0.03)	$\lambda_{Midwest}$	1.36 (0.41)
$\lambda_{Employment}$	0.45 (0.02)	$\lambda_{South}$	1.33 (0.33)
$\sigma_{v,Production}^2$	0.32 (0.02)	$\sigma_{v,Northeast}^2$	38.14 (3.92)
$\sigma_{v,Income}^2$	0.13 (0.01)	$\sigma_{v,West}^2$	8.12 (1.31)
$\sigma_{v,Sales}^2$	0.78 (0.04)	$\sigma_{v,Midwest}^2$	22.37 (2.56)
$\sigma_{v,Employment}^2$	0.11 (0.01)	$\sigma_{v,South}^2$	8.01 (1.12)
$\beta^H$	0.14 (0.05)	$\beta^{BC}$	0.12 (0.02)
$\Psi_{Production}$	0.18 (0.05)	$\Psi_{Northeast}$	-0.36 (0.05)
$\phi_1^{BC}$ (pre-2002)	-0.08 (0.02)	$\phi_1^H$ (2001 on)	-0.18 (0.08)
$\phi_0^{BC}$ (2002 on)	-0.02 (0.01)	$\phi_0^H$ (pre-2001)	-0.09 (0.03)
$\Psi_{Income}$	0.15 (0.03)	$\Psi_{West}$	-0.42 (0.08)
$\Psi_{Sales}$	-0.24 (0.04)	$\Psi_{Midwest}$	0.41 (0.05)
$\Psi_{Employment}$	-0.51 (0.08)	$\Psi_{South}$	-0.35 (0.08)
$\sigma_{BC,SM}$	0.43 (0.15)		
<b>Log L</b>	-3368.62		

Asymptotic standard errors in parentheses

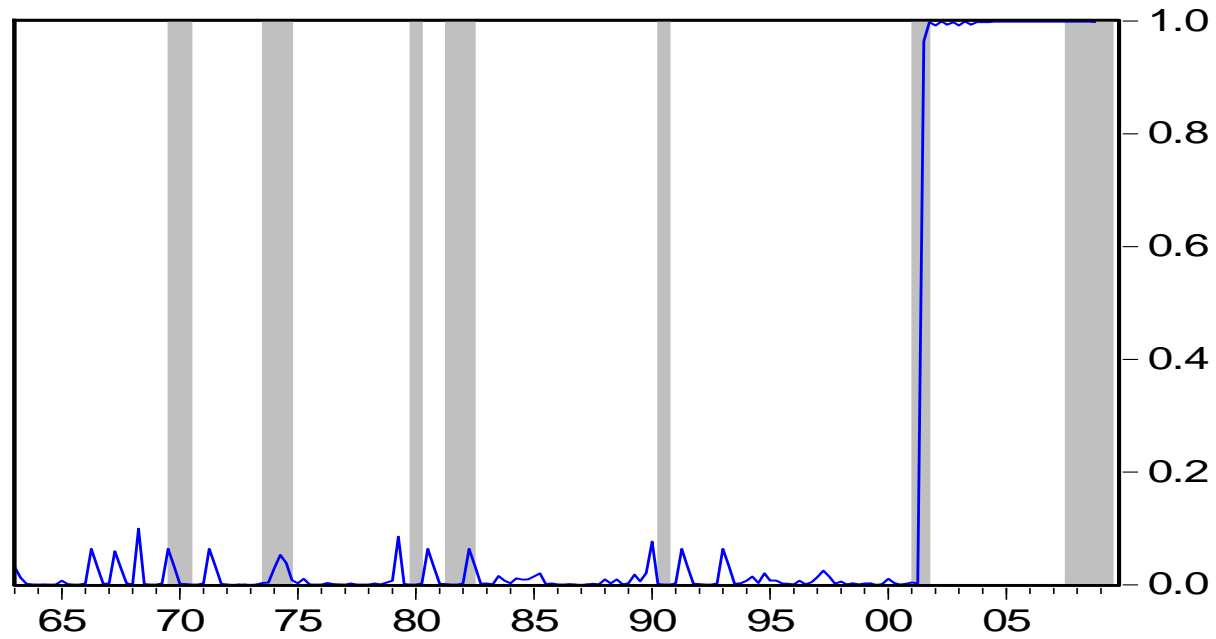
## Link between Housing Cycle Factor and Business Cycle Factor

- Estimated interest rate coefficient in the transition equations for the housing factor and business cycle factor highly significant
- Subsample analysis for 1995-2007 shows substantially higher interest rate parameter in the housing factor transition equation and substantially lower interest rate parameter in the business cycle factor transition equation
- Smoothed probabilities for the coefficients show:
  - Weaken link between interest rates and the business cycle during this period
  - Stronger link between interest rate and housing cycle during this period

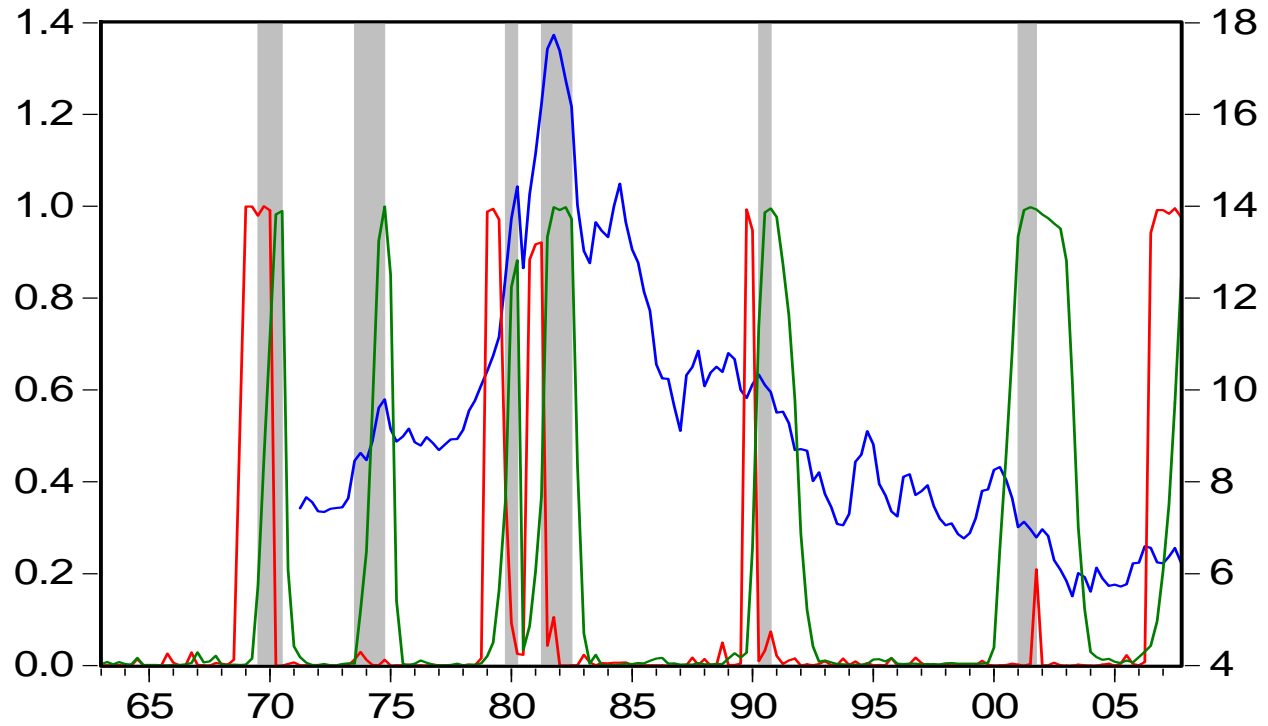
## Probabilities of Low Interest Rate Coefficient on Business Cycle Factor Equation



## Probabilities of High Interest Rate Coefficient on Housing Market Factor Equation



## Housing Cycle, Business Cycle, and Mortgage Rates



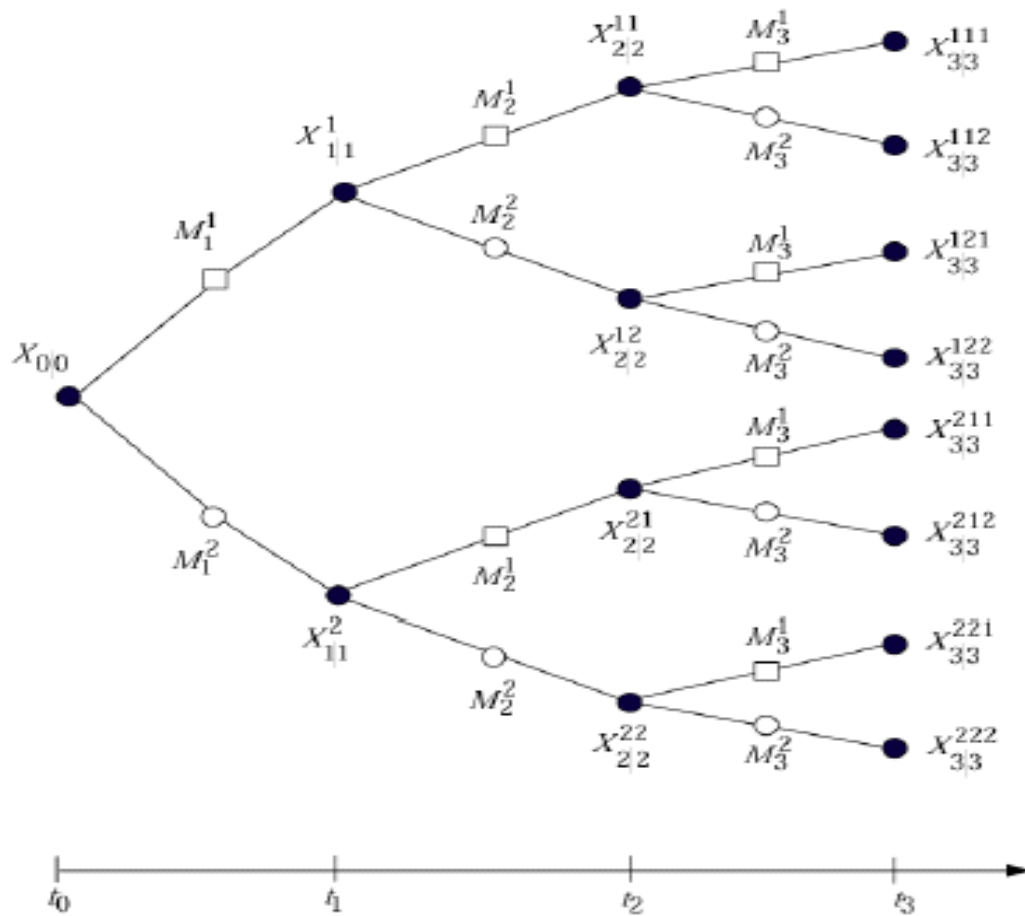
## Housing Market, Interest Rates, and the 2001 Recession

- Linkage between business cycle factor and interest rate weakened during the 2001-2004 recovery.
- Low level of interest rates during this expansion period due to uncertainty regarding the end of 2001 recession
  - Slow recovery in 2002-2003 led the Fed to keep interest rates at lowest level
  - Mortgage rates at lowest levels
- Bubble: full recovery in the economy starting in 2003 with still low level of interest rates catalyzed abrupt increase in housing prices, housing starts, and ownership
- Bust: higher interest rates from mid 2004 on associated with subsequent drastic decreases in housing market in 2006-2007

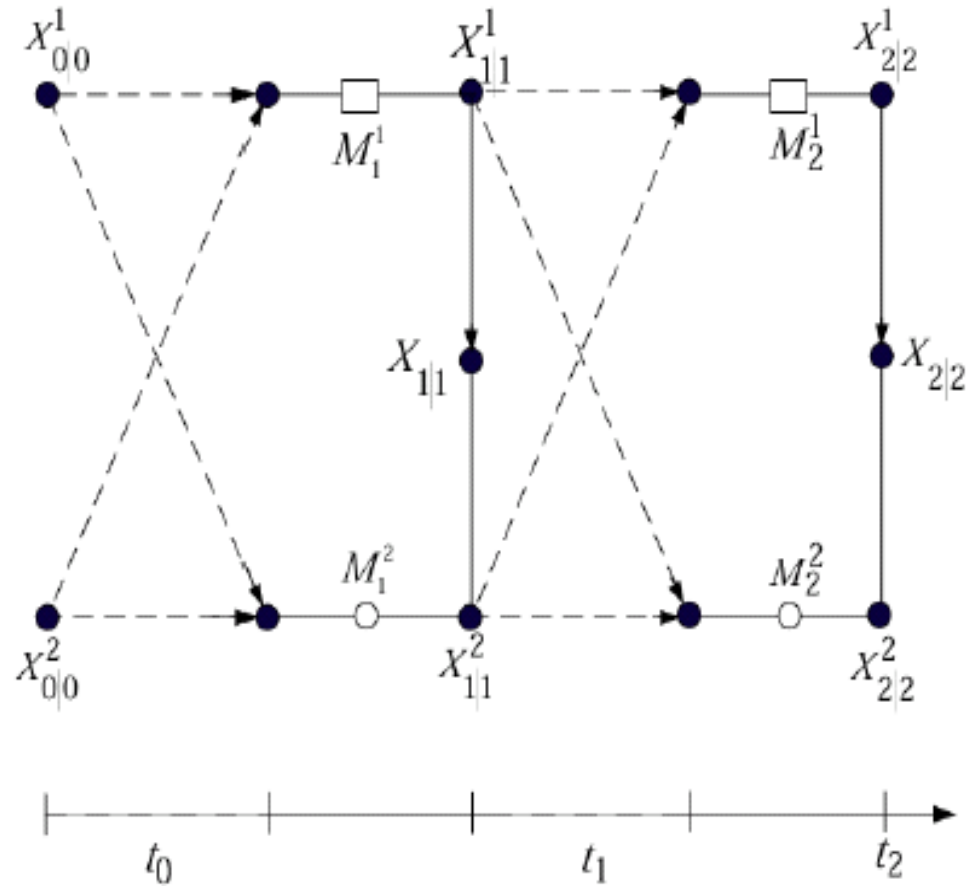
## Conclusions

- Strong historical linkage between business cycles and housing market cycles
- Strong linkage between business cycle and interest rates, and housing market cycle and interest rates
- Relationship changed during the 2001 recession and subsequent recovery
- Missed link between 2001 recession and housing market cycle a seed to the subsequent housing market bubble and bust

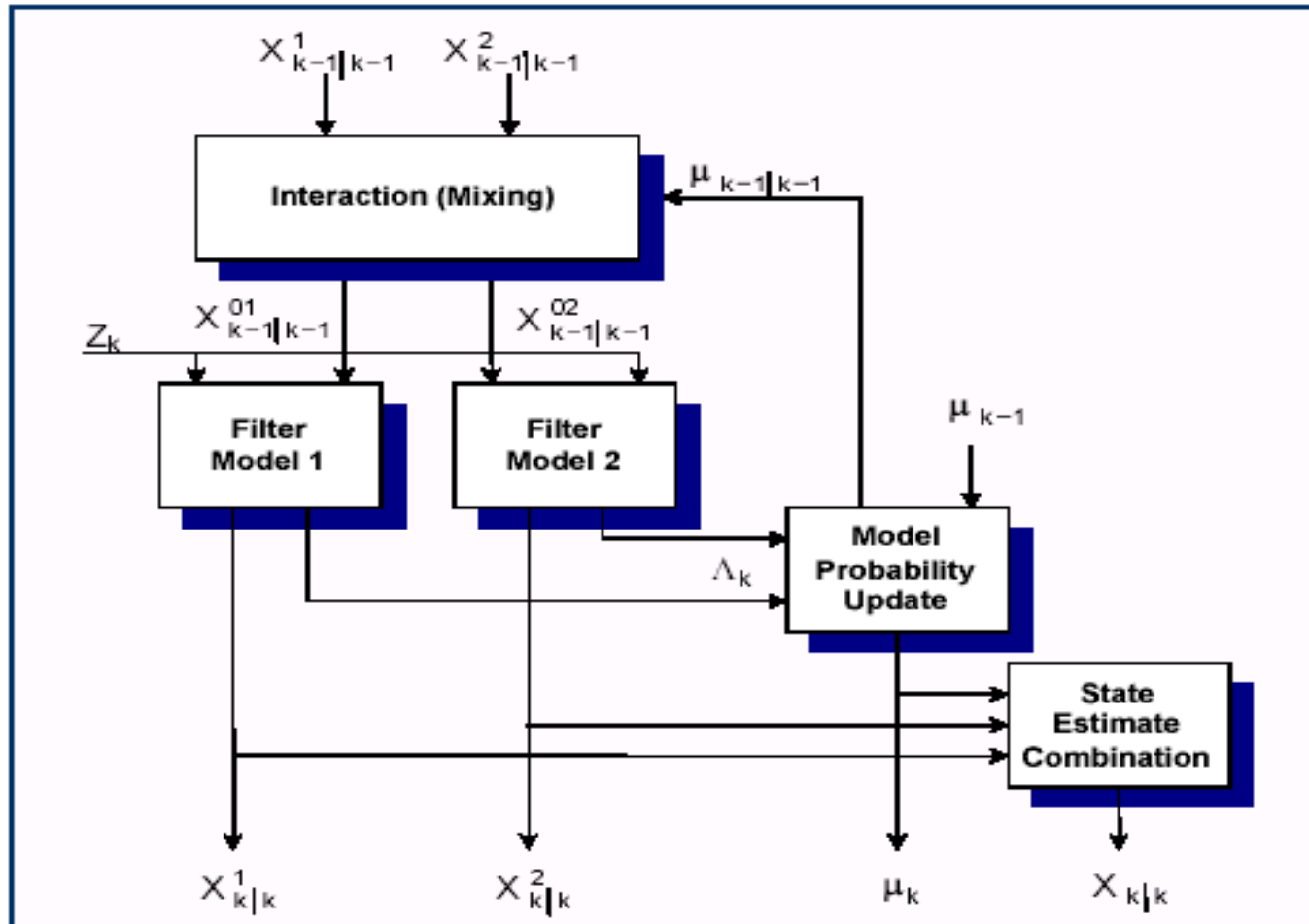
## Hypothesis Management



## Hypothesis Management of the Algorithm



## Algorithm with Two Models



## FILTER – STEP 1

- Step 1. Mixing of State Estimates
- Starting with  $X_{k-1|k-1}^j$  Mixed estimate for the filter matched to  $M^i$

$$X_{k-1|k-1}^{0i} = \sum_{j=1}^r X_{k-1|k-1}^j \mu_{k-1|k-1}^{j|i}$$

where  $\mu_{k-1|k-1}^{j|i}$  is computed as

$$\mu_{k-1|k-1}^{j|i} = \frac{\mu_{k-1}^j}{\mu_{k-1|k-1}^i} p_{ji} \quad \mu_{k-1|k-1}^i = \sum_{j=1}^r \mu_{k-1}^j p_{ji}$$

- The covariance is given by

$$P_{k-1|k-1}^{0i} = \sum_{j=1}^r \mu_{k-1|k-1}^{j|i} [P_{k-1|k-1}^j + (X_{k-1|k-1}^j - X_{k-1|k-1}^{0i})(X_{k-1|k-1}^j - X_{k-1|k-1}^{0i})^T]$$

## FILTER – STEP 2

### Step 2: Model Conditioned Updates

The  $r$  filters implementing the Kalman filtering equations are used to perform the model updates associated with the  $r$  models. Let

$$F_k^j = F_k(\theta_{k+1} = j), G_k^j = G_k(\theta_{k+1} = j) \quad \text{and} \quad H_k^j = H_k(\theta_k = j)$$

Then, for  $M_k^i$  with input  $X_{k-1|k-1}^{0i}$  and  $P_{k-1|k-1}^{0i}$ ,

$$X_{k|k-1}^i = F_{k-1}^i X_{k-1|k-1}^{0i}$$

$$P_{k|k-1}^i = F_{k-1}^i P_{k-1|k-1}^{0i} (F_{k-1}^i)^T + G_{k-1}^i Q_{k-1}^i (G_{k-1}^i)^T$$

$$S_k^i = H_k^i P_{k|k-1}^i (H_k^i)^T + R_k$$

$$K_k^i = P_{k|k-1}^i (H_k^i)^T (S_k^i)^{-1}$$

$$\tilde{Z}_k^i = Z_k - H_k^i X_{k|k-1}^i$$

$$X_{k|k}^i = X_{k|k-1}^i + K_k^i [\tilde{Z}_k^i]$$

$$P_{k|k}^i = [I - K_k^i H_k^i] P_{k|k-1}^i$$

## FILTER – STEP 3

### Step 3. Model Likelihood Computations

Likelihood corresponding to the  $r$  filters are computed as

$$\begin{aligned}\Lambda_t^i &= f[Z_t | M_t^i, Z^{t-1}] = f[Z_t | M_t^i, X_{t-1|t-1}^{0i}, P_{t-1|t-1}^{0i}] \\ &= f[Z_t | X_{t|t-1}^i, P_{t|t-1}^i]\end{aligned}$$

The likelihood of  $M_t^i$  is computed with filters residuals  $\tilde{Z}_t^i$ , the covariance of the residuals  $S_t^i$ , and the assumption of Gaussian statistics. The likelihood of  $M_t^i$  is given by

$$\Lambda_t^i = \frac{1}{\sqrt{|2\pi S_t^i|}} \exp[-0.5(\tilde{Z}_t^i)^T (S_t^i)^{-1} \tilde{Z}_t^i]$$

## FILTER – STEP 4

### Step 4: Model Probabilities Update

The Model Probabilities for output are computed as follows

$$\begin{aligned}\mu_t^i &= P[M_t^i | Z_t, Z^{t-1}] = \frac{P\{M_t^i | Z_t, Z^{t-1}\}}{f[Z_t | Z^{t-1}]} = \frac{f[Z_t | M_t^i, Z^{t-1}]}{f[Z_t | Z^{t-1}]} P\{M_t^i | Z^{t-1}\} \\ &= \frac{\Lambda_t^i}{c} \mu_{t-1|t-1}^i\end{aligned}$$

where

$$c = \sum_{j=1}^r \Lambda_t^j \mu_{t-1|t-1}^j$$

## FILTER – STEP 5

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Step 5: Combination of State Estimates

For output only, the state estimate is computed according to

$$X_{t|t} = \sum_{i=1}^r X_{t|t}^i \mu_t^i$$
$$\sum_{j=1}^r \Lambda_t^j \mu_{t-1|t-1}^j$$

The output covariance is given by

$$P_{t|t} = \sum_{i=1}^r \mu_t^i [P_{t|t}^i + (X_{t|t}^i - X_{t|t})(X_{t|t}^i - X_{t|t})^T]$$