I Walked the Line: Identification of Fiscal Multipliers in SVARs

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Introduction

- Fiscal policy has been used to counter the impact of the financial crisis.
- Fiscal multipliers are crucial to understand the impact of fiscal interventions.
- Structural Vector Autoregressions (SVARs) based evidence:
  - Large qualitative and quantitative dispersion in results.

Motivation

- Blanchard & Perotti (2002): Contemporaneous restrictions.
- Mountford & Uhlig (2009): Sign restrictions (SR).

	1 qrt	4 qrts	8 qrts	12 qrts	20 qrts	Peak
BP identification scheme applied to MU (2009) VAR						
Def.-Fin. G↑	1.33*	1.40*	0.50	0.72*	-0.10	1.60* (3)
Def.-Fin. T↓	0.02	0.40	1.09*	1.58*	0.60	1.62* (13)
Bal.-Bud. G↑	1.31*	1.01*	-0.60	-0.87	-0.64	1.46 (3)
MU (2009)						
Def.-Fin. G↑	0.65*	0.27	-0.74*	-1.19*	-2.24*	0.65* (1)
Def.-Fin. T↓	0.28*	0.93	2.05*	3.41*	2.59*	3.57* (13)
Bal.-Bud. G↑	0.37*	-0.66*	-2.79*	-4.60*	-4.83*	0.37 (3)

Overview of the Paper

- Starting point: Restrictions on output elasticities of tax revenue and government spending.
- Describe analytical relation between elasticities and multipliers.
  - Small changes in elasticities induce large changes in multipliers.
- Recast identification schemes in terms of elasticities.
  - Explain differences in results.
- Compute elasticities from data.
  - Spending multipliers larger than tax multipliers.

Research Agenda

- Caldara & Kamps (2007): Explore implications of reduced-form specifications on multipliers.
- Caldara & Kamps (2010a): Analytics of structural VARs.
- Caldara & Kamps (2010b): Analytics of sign restrictions.

A Simple Bivariate Model

- Reduced-form VAR:

$$\begin{bmatrix} Y_t \\ T_t \end{bmatrix} = \mu + A(L) \begin{bmatrix} Y_{t-1} \\ T_{t-1} \end{bmatrix} + \begin{bmatrix} u_t^Y \\ u_t^T \end{bmatrix}, \text{ with } E\left(u_t u_t'\right) = \Sigma_u$$

.

- Structural model:

$$\begin{aligned} u_t^Y &= c_1 u_t^T + e_t^Y \\ u_t^T &= a_1 u_t^Y + e_t^T \end{aligned}$$

- B&P (2002) restrict the output elasticity of tax revenue (a_1)

- Find analytical solution:

$$c_1 = c_1\left(a_1; \Sigma_u\right)$$

VAR Model

- Country: U.S.
- Sample: 1947:Q1 - 2010:Q1
- Benchmark model: 3 equations (Y, T, G)
- Large model: 13 equations
  - Forward looking variables.
  - Romer and Romer (2010) and Ramey (2010) measures of fiscal news.

Tax Multiplier

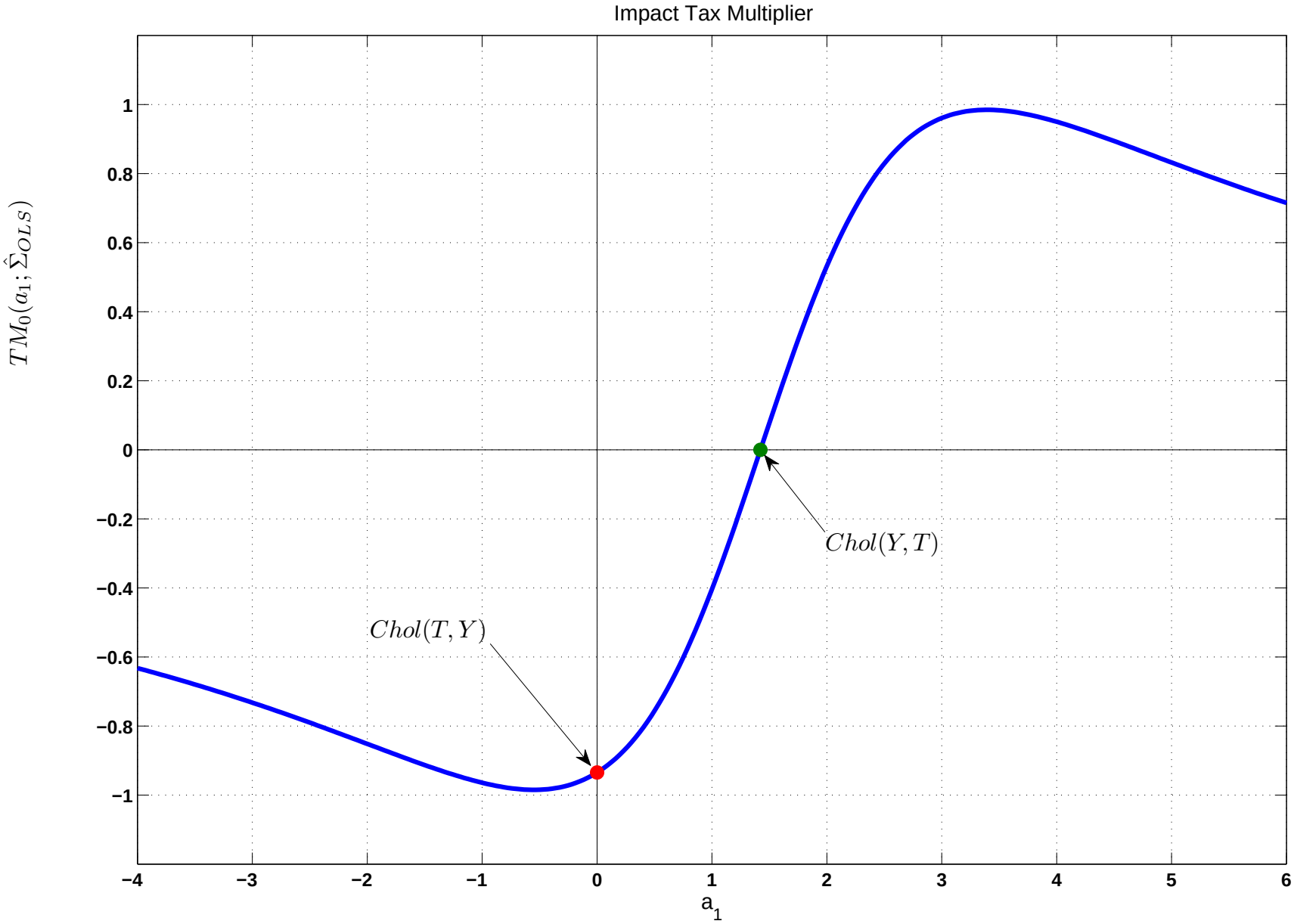
DEFINITION: The tax multiplier is the response in dollars of output to a negative tax shock of size one dollar.

- Impact tax multiplier:

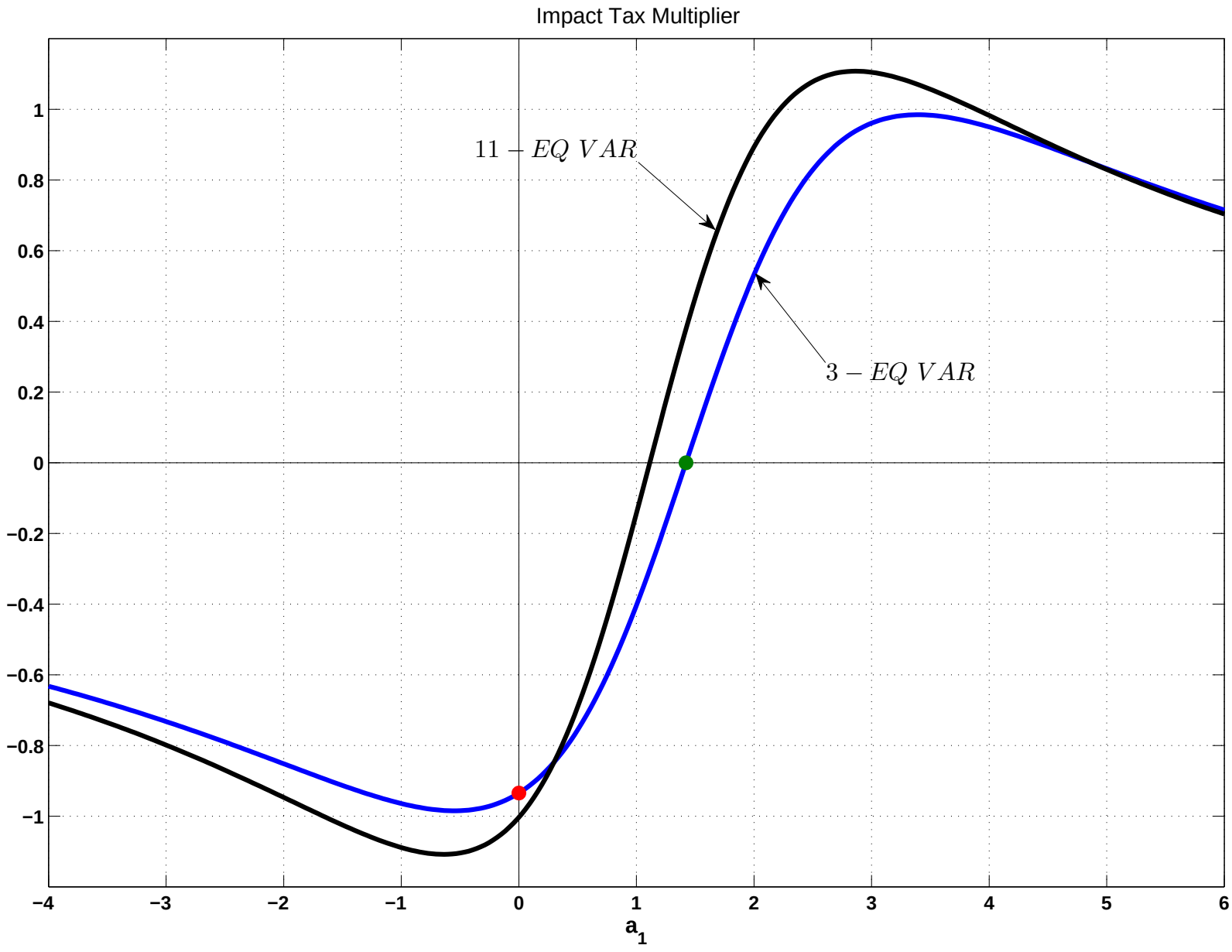
$$\begin{aligned} TM_0\left(a_1;\Sigma_u\right) &= (-1)\frac{c_1\left(a_1;\Sigma_u\right)}{1-a_1c_1\left(a_1;\Sigma_u\right)} \\ &= \frac{a_1\sigma_{YY}-\sigma_{TY}}{a_1^2\sigma_{YY}+\sigma_{TT}-2a_1\sigma_{TY}} \end{aligned}$$

- This relation generalizes to a $k-$equation VAR if we impose block recursion.

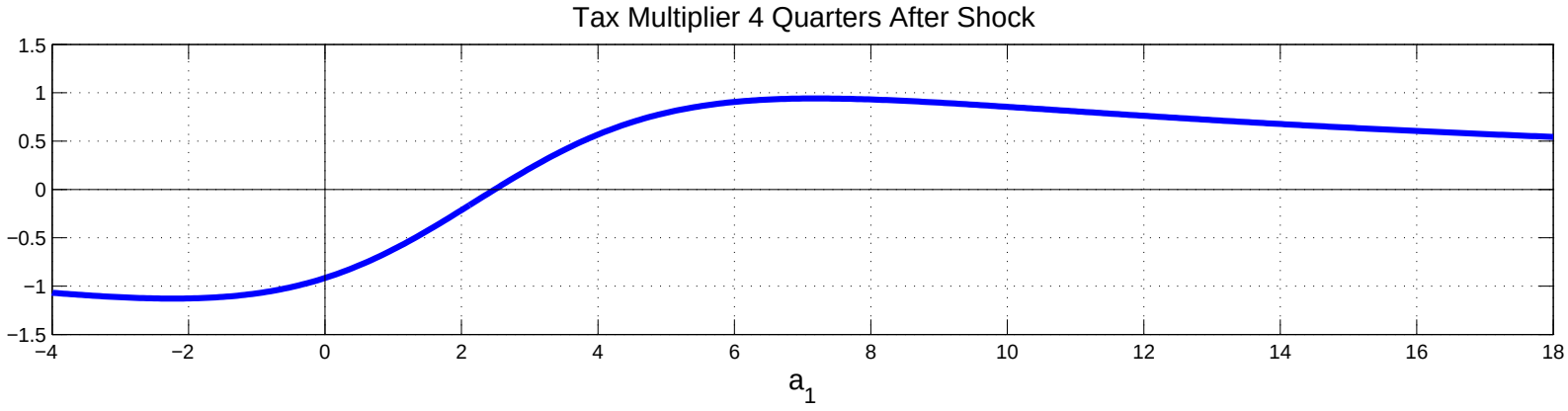
Impact Tax Multiplier

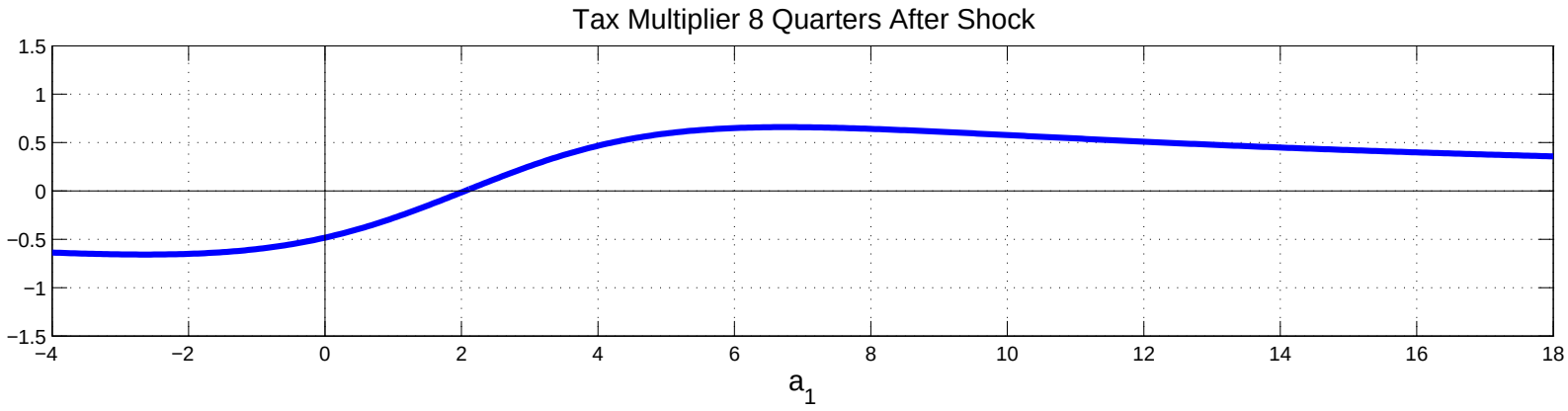


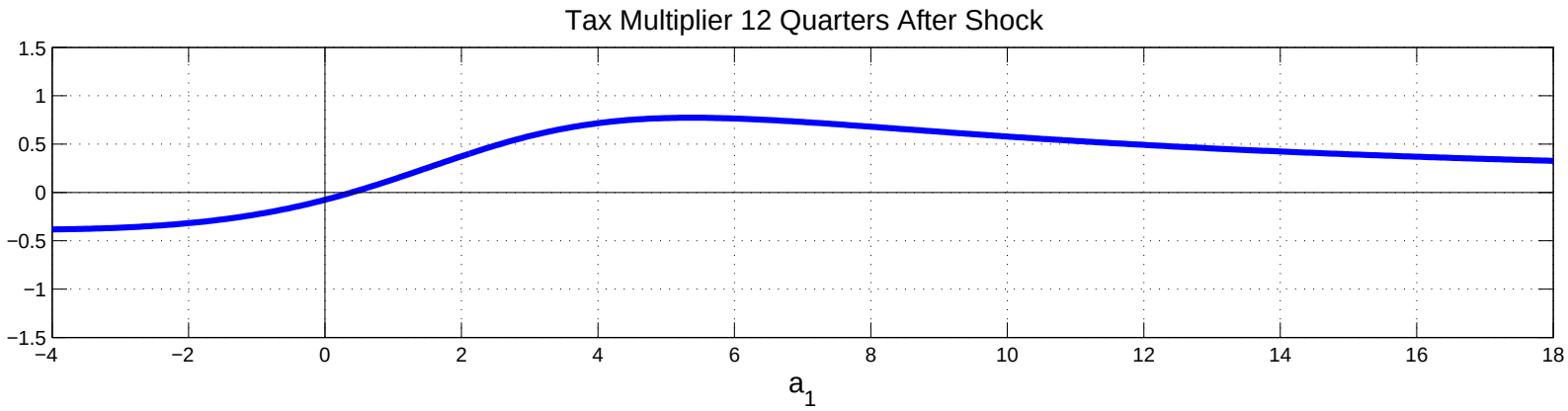
Impact Tax Multiplier



Tax Multipliers - Longer Horizon







Distribution of a_1 from SR

- Sign restrictions:

$$\begin{bmatrix} u_t^Y \\ u_t^T \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ + & ? \\ d_{21} & d_{22} \\ + & + \end{bmatrix} \begin{bmatrix} e_t^Y \\ e_t^T \end{bmatrix}$$

- We prove that sign restrictions are satisfied for

$$a_1 > 0$$

- What is the distribution of a_1?
  - Pure sign-restriction:

$$a_1 \in (0.84; 6.92)$$

  - Penaltu function:

$$a_1 = 2.14$$

Estimated Distribution for a_1

- The OECD estimates the output elasticity of tax revenue for four different tax categories i:

$$\eta_{T_i,y} = \eta_{T_i,TB_i}\eta_{TB_i,y}$$

where T_i is tax revenue, TB_i is the tax base , and y is the output gap

  - η_{T_i,TB_i} are constructed from legislation
  - $\eta_{TB_i,y}$ are estimated

- Aggregation:

$$a_1 = \sum_i w_i \eta_{T_i,y}$$

where w_i are weights.

Estimated Distribution of a_1

B&P method of constructing a_1:

$$a_1 \in (2.00; 2.20)$$

Four methodological issues:

1. Aggregation of sub-elasticities.
2. Elasticity of personal income tax revenue to earnings.
3. Elasticity of wage bill to output.

$$a_1 \in (1.08; 1.12)$$

4. Lack of data for period 1947-1960 $\rightarrow$ work in progress ($a_1 \uparrow$)

Distributions of a_1

Method	Distribution of a_1		$p\left(TM_0 \mid \hat{\Sigma}_{u,OLS}\right)$	
	Median	90% C.S.	Median	68% C.S.
Estimation	1.10	(1.08; 1.12)	-0.31	(-0.32; -0.29)
Sign Restrictions	2.31	(0.84; 6.92)	0.41	(-0.90; 0.98)
Sign Restriction + P.F.	2.14	-	0.63	-

Spending Multiplier

DEFINITION: The spending multiplier is the response in dollars of output to a positive tax shock of size one dollar.

- Impact spending multiplier:

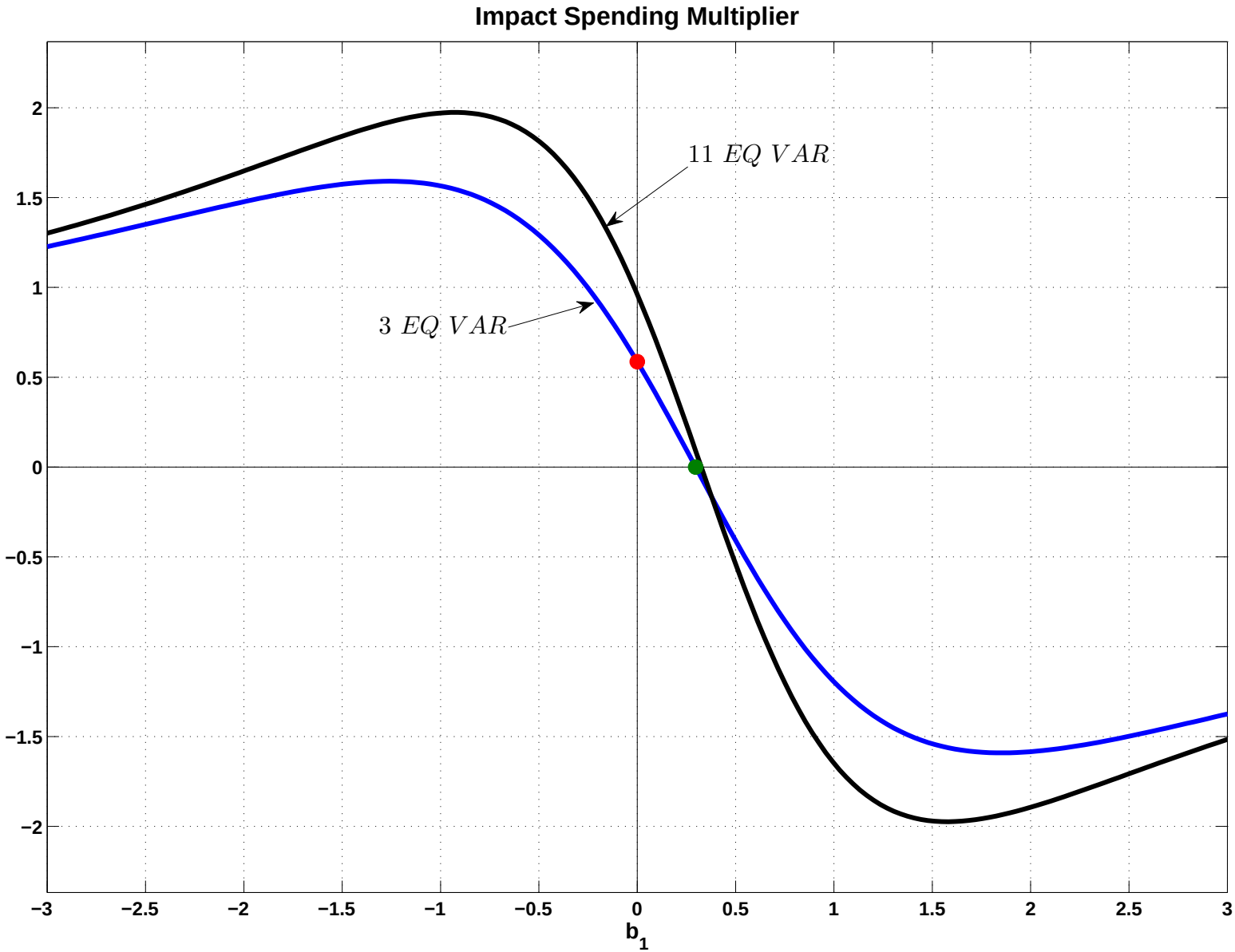
$$\begin{aligned} GM_0(b_1;\Sigma_u) &= \frac{c_2(b_1;\Sigma_u)}{1-b_1 c_2(b_1;\Sigma_u)} \\ &= \frac{\sigma_{GY}-b_1\sigma_{YY}}{b_1^2\sigma_{YY}+\sigma_{GG}-2b_1\sigma_{GY}} \end{aligned}$$

  b_1: output elasticity of spending.

- $b_1 = 0$:

$$GM_0(b_1=0;\Sigma_u) = \frac{\sigma_{YG}}{\sigma_{GG}}$$

Impact Spending Multiplier



Distributions of b_1

Method	Distribution of b_1		$p\left(GM_0 \mid \hat{\Sigma}_{u,OLS}\right)$	
	Median	68% C.S.	Median	68% C.S.
Blanchard&Perotti	0.00	-	0.59	-
Sign Restrictions	0.16	(-2.26,2.23)	0.15	(-1.41; 1.45)
Sign Restriction + P.F.	0.08	-	0.44	-

- Small changes in elasticities account for large changes in multipliers.

Policy Experiments

- Mountford and Uhlig (2009): linear combination of spending and tax shocks.
- Elasticities: $a_1 = 1.1$; $b_1 = 0$.
- Three experiments (1 dollar in size):
  - Deficit financed spending increase.
  - Deficit financed tax cut.
  - Balanced-budget spending increase.

Results from 3 Eq VAR

	1 qrt	4 qrts	8 qrts	12 qrts	20 qrts	Maximum
Def.-Fin. G↑	0.55*	0.24	0.49	0.69*	0.50	0.73* (10)
Def.-Fin. T↓	-0.30*	-0.57*	-0.38	-0.14	0.11	0.11 (20)
Bal.-Bud. G↑	0.85*	0.81*	0.87*	0.83*	0.41	1.33* (3)

- Negative tax multipliers in the short-run.
- Two explanations:
  - Omitted variables.
  - Downward bias in a_1.

Results from 11 Eq VAR

	1 qrt	4 qrts	8 qrts	12 qrts	20 qrts	Maximum
Def.-Fin. G↑	0.84*	1.05*	1.59*	1.60*	0.85*	1.94* (10)
Def.-Fin. T↓	-0.20*	-0.03	0.59	1.17*	0.55	1.17* (20)
Bal.-Bud. G↑	1.04*	1.08*	1.00*	0.45	0.33	1.45* (3)

- Multipliers are larger than in 3-eq model.
- Explanations:
  - σ_{GG} & $\sigma_{TT} \downarrow \rightarrow$ Multipliers $\uparrow$
  - Change in covariance structure

Results from 11 Eq VAR

- Effects of a deficit-financed spending increase on output components:

	1 qrt	4 qrts	8 qrts	12 qrts	20 qrts	Maximum
Consumption	0.15*	0.39*	0.84*	0.79*	0.45*	0.94*(10)
Res. Invest.	-0.03	0.10	0.26*	0.10	-0.11	0.27*(9)
Non Res. Invest.	-0.01	0.05	0.13	0.31*	0.11	0.34*(11)

Robustness: Distribution for b_1

- State and local spending: 52% of G.
- Rodden and Wibbels (2010): data on S&L spending for 49 US states.
  - OLS regression. Elasticity is 0.17^{***}
- Assume elasticities of other spending categories are zero.
  - Overall outpute elasticity of spending: $b_1 = 0.1$.
- Work in progress: estimate elasticity of public sector wage bill.
  - Gomes (2009): Procyclicality of public sector wages.

Results from 11 Eq VAR

- Robustness: $b_1 = 0.1$

	1 qrt	4 qrts	8 qrts	12 qrts	20 qrts	Maximum
Def.-Fin. G↑	0.64*	0.88*	1.49*	1.45*	0.67	1.82*(10)
Def.-Fin. T↓	-0.20*	-0.10	0.55*	1.14*	0.48	1.14*(12)
Bal.-Bud. G↑	0.84*	0.98*	0.95*	0.34	0.21	1.28*(3)

- No qualitative change in results.

Conclusions

- Estimates of fiscal multipliers are sensitive to restrictions on output elasticities of fiscal variables.
- Disagreement in the literature can be explained by different restrictions on elasticities.
- Provide inference over fiscal multipliers.

Analytical Solution

- System of equations:

$$\Sigma_u = A_0^{-1} \Sigma_e \left(A_0^{-1}\right)')$$

where

$$A_0 = \begin{bmatrix} 1 & -a_1 \\ -c_1 & 1 \end{bmatrix}$$

- Solution:

$$\begin{aligned} c_1 &= \frac{a_1 \sigma_{YY} - \sigma_{YT}}{a_1 \sigma_{YT} - \sigma_{TT}} \\ \varepsilon_{TT} &= \sigma_{TT} + a_1^2 \sigma_{YY} - 2 a_1 \sigma_{YT} \\ \varepsilon_{YY} &= \frac{\left(\sigma_{TT} + a_1^2 \sigma_{YY} - 2 a_1 \sigma_{YT}\right)\left(\sigma_{TT}\sigma_{YY} - \sigma_{YT}^2\right)}{\left(\sigma_{TT} - a_1 \sigma_{YT}\right)^2} \end{aligned}$$

Analytical IRFs

- A^{-1} gives the impact response of model variables to structural shock:

$$A_0^{-1}(a_1;\Sigma_u) = \frac{1}{1-a_1 c_1(a_1;\Sigma_u)} \begin{bmatrix} 1 & c_1(a_1;\Sigma_u) \\ a_1 & 1 \end{bmatrix}$$

- Substituting $c_1(a_1;\Sigma_u)$ we obtain:

$$A_0^{-1}(a_1;\Sigma_u) = \frac{1}{\det(A_0)} \begin{bmatrix} \sigma_{TT} - a_1\sigma_{TY}, & \sigma_{TY} - a_1\sigma_{YY} \\ a_1(\sigma_{TT} - a_1\sigma_{TY}), & \sigma_{TT} - a_1\sigma_{TY} \end{bmatrix}$$

where $\det(A_0) = a_1^2\sigma_{YY} + \sigma_{TT} - 2a_1\sigma_{TY}$.

Results from Original Papers

- Blanchard & Perotti (2002): Contemporaneous restrictions.
- Mountford & Uhlig (2009): Sign Restrictions.

	1 qrt	4 qrts	8 qrts	12 qrts	20 qrts	Peak
BP (2002)						
Def.-Fin. G↑	0.96*	0.57	0.79	1.17*	0.85*	1.21* (14)
Def.-Fin. T↓	0.87*	1.79*	1.92*	1.11*	0.33	1.97* (7)
Bal.-Bud. G↑	0.09	-1.22*	-1.13	0.06	0.52	0.71 (17)
MU (2009)						
Def.-Fin. G↑	0.65*	0.27	-0.74*	-1.19*	-2.24*	0.65* (1)
Def.-Fin. T↓	0.28*	0.93	2.05*	3.41*	2.59*	3.57* (13)
Bal.-Bud. G↑	0.37*	-0.66*	-2.79*	-4.60*	-4.83*	0.37 (3)