I Walked the Line: Identification of Fiscal Multipliers in SVARs*

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Structural Vector Autoregressions (SVARs) have been used to estimate fiscal multipliers. Results in the literature are dispersed over a broad range and no stylized facts emerge. This paper makes two contributions. First, I derive an analytical mapping between output elasticities of fiscal variables and fiscal multipliers. I recast identification schemes into restrictions on the elasticities. I show that differences in elasticities account for differences in results. Second, I estimate robust fiscal multipliers. In contrast to previous findings two results emerge: First, in the short run spending multipliers are larger than tax multipliers; Second, in the short run tax multipliers can be negative.

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1 Introduction

The recent economic crisis and the policy measures adopted by most countries to counter-act its effects has brought new interest in the long lasting debate about the effectiveness of fiscal policy interventions in stabilizing the economy. In the profession there is little agreement even on the qualitative effects of fiscal interventions, both on theoretical and empirical grounds. Effectiveness of fiscal interventions is commonly measured in terms of fiscal multipliers, defined as the response of output to cuts in taxes (tax multiplier) and increases in public spending (spending multiplier) unrelated to the economic cycle. Structural Vector Autoregressions (SVARs) have been largely used to estimate fiscal multipliers. Table 1 reports fiscal multipliers estimated by two prominent studies in the literature: Blanchard and Perotti (2002), B&P henceforth, and Mountford and Uhlig (2009), M&U henceforth. B&P identify exogenous fiscal interventions using institutional information about the tax and transfer system. B&P find that both tax multipliers and spending multipliers are positive, similar in magnitude, and statistically significant. M&U identify exogenous fiscal interventions imposing sign restrictions on the responses of variables to such interventions[1]. M&U document negative spending multipliers. Consequently tax cuts are more effective than spending increases in stimulating output. Differences in results can be driven by two factors: the choice of reduced-form model and data series; and the choice of the scheme used to identify the SVARs. Table 2 reports estimates of fiscal multipliers obtained applying different identification schemes to the same reduced-form model and data. The use of identical statistical models and data *increases* differences in results.[2]

This paper investigates the relation between identification schemes and fiscal multipliers. The choice of the identification scheme can be thought as a model selection problem. Identification schemes select a structural model (or a sub-set of structural models) from the set of all admissible structural models[3]. In the literature model comparison is based on the analysis of impulse response functions, that is, on the analysis of the inference we are interested in. Instead, I propose a comparative framework based on the assumptions imposed to identify the structural models. The main difficulty is to construct a metric to compare SVARs identified imposing restrictions on structural coefficients, such as B&P, to SVARs identified imposing restrictions on impulse responses or variance decomposition, such sign restrictions, long-run restrictions, or simple Cholesky decompositions. My approach is based on two steps. First, I derive an analytical relation between output elasticities of fiscal variables, i.e. structural coefficients, and fiscal multipliers. Second, I recast different identification schemes used in the literature in terms of elasticities. This method is an ap-

[1] They also impose sign restrictions to identify movements in endogenous variables due to sources of business cycle fluctuations other than fiscal policy. The reader can refer to section 3 for a comprehensive discussion of sign restrictions.

[2] A detailed analysis on the effects of the choice of reduced-form models on the estimation of fiscal multipliers can be found in Caldara and Kamps (2008)

[3] In this paper we only focus on exactly-identified VARs

plication of the analytical analysis of SVARs developed in Caldara and Kamps (2010a,b).

The B&P identification scheme is already casted in terms of output elasticities of fiscal variables, and it constitutes a useful benchmark. B&P construct the elasticities using off-model information. Updating their calculations for the post-war period I compute an output elasticity of tax revenue of 2.10, while the output elasticity of government spending is fixed to zero. The "pure" sign restriction approach selects a set of models that are characterized by output elasticities of tax revenue that ranges between 1.2 and 21, and by output elasticities of spending between -2 and 2. The penalty function approach to sign restrictions[4], used by M&U, selects a structural model characterized by elasticities of 5.13 and 0.24 respectively. Different elasticities translate into different multipliers. The impact tax multiplier associated to an output elasticity of tax revenue of 2.10 is zero, while the tax multiplier associated to an elasticity of 4.03 is 0.5. Differences persist also at longer horizons. Similarly, the impact spending multiplier associated to an output elasticity of 0 is 0.57, while when the elasticity increases to 0.25 the multiplier drops to zero.

Next I construct a robust measure of fiscal multipliers. The output elasticities of tax revenue and government spending are random variables. I select the set of structural models of interest using information about the entire distribution of the elasticities rather than using only the point estimates as B&P.

To characterize the entire distribution of elasticities I follow the two-step methodology proposed by B&P. First, they estimate elasticities for different tax categories using the methodology developed by the OECD[5]. Second, they aggregate these elasticities constructing weights based on the relative contribution of each tax category to total tax revenue, *net of transfers*. I find that sampling uncertainty for the output elasticity of tax revenue is very small, with the 90% credible set ranging between 2.00 and 2.20. Then I propose two departures from the B&P construction of elasticities. First, I compute the elasticity of personal income tax revenue to earnings using TAXSIM, a micro-simulation model developed by the NBER and based on survey data, rather than using OECD data[6]. The distribution of the output elasticity of tax revenue shifts to values ranging between 1.67 to 1.78. Second, I construct weights to aggregate different elasticities based on data on tax revenue, without subtracting transfers. The distribution of the output elasticity of tax revenue shifts to values ranging between 1 and 1.1. In the current version of the paper I maintain the assumption that the output elasticity of spending is zero.

Two results emerge from the estimation of structural models selected based on the empirical distributions for the elasticities. First, spending multipliers are larger than tax multipliers up to two years after the policy interventions. Second, for values of the output elasticities below 2, tax multipliers are negative

[4]See section 3 for details

[5]As explained is section 4, elasticities of tax revenues to tax base are constructed from legislation.

[6]Section 4 explains why elasticities estimated using TAXSIM seem to be more reliable than estimates provided by the OECD.

up to one year after the policy intervention.

Results are based on the estimation of a large Bayesian VAR. Together with output and fiscal series, I include a set of forward-looking variables. Some authors, e.g. Leeper, Walker, and Yang (2008) have pointed out that fiscal foresight, the phenomenon that legislative and implementation lags ensure that private agents receive signals about future fiscal policies, produces equilibrium time series with a non-invertible moving average component, which misaligns the agents' and the the econometrician's information set in estimated VARs. If this is the case, the VAR does not span the set of structural fiscal shocks, and hence inference can be distorted[7]. Work by Giannone and Reichlin (2006) and Forni and Gambetti (2010) has shown that the inclusion of forward-looking variables should help to retrieve *unexpected* shocks, even in the presence of fiscal foresight. Together with forward-looking variables, the VAR includes the measure of news on discretionary tax changes constructed by Romer and Romer (2010), R&R henceforth, and the measure of news on changes in defense spending constructed by Ramey (forthcoming). The inclusion of the news series has modest quantitative effects on the estimate of fiscal multipliers. This result is in line with results by R&R.

The rest of the paper is organized as follows. In section 2 I derive the analytical framework. In section 3 I recast the sign restriction approach in terms of output elasticities. In section 4 I estimate probability distributions for the output elasticity of tax revenue. In section 5 I compute a probability distribution on elasticities based on back of the envelope calculations from a DSGE model. This framework is also useful to map deep parameters and shocks of a DSGE model into structural coefficients and shocks of the SVAR. Section 6 estimates robust multipliers. Section 7 concludes.

2 Analytics

In this section we derive the analytical relation between the output elasticities of tax revenue and government spending , and fiscal multipliers. We first focus on the tax multiplier using a simple bivariate model in output and tax revenue. We then look at the spending multiplier adding government spending to the VAR model. Finally, we study the response of private consumption to fiscal shocks.

Our basic structural VAR specification is

$$A_0 X_t = \sum_{l=1}^{p} A_l X_{t-l} + \epsilon_t \tag{1}$$

where X_t is a $n_x \times 1$ vector of endogenous variables and A_l is a $n_x \times n_x$ matrix of parameters for $0 \leq l \leq p$. ϵ_t is $n_x \times 1$ vector of exogenous structural shocks.

[7]For a detailed discussion of non-invertibility problems see Fernández-Villaverde, Rubio-Ramirez, Sargent, and Watson (2007)

The distribution of ϵ_t, conditional on the past information, is Gaussian with mean zero and diagonal covariance matrix Σ_ϵ. Deterministic terms are omitted for convenience. In appendix A we provide details of the reduced form VAR specification and data used for the empirical exercise.

Denote by u_t the $n_x \times 1$ vector of reduced-form residuals. The distribution of u_t is Gaussian with mean zero and diagonal covariance matrix Σ_u. The relation between reduced-form residuals and structural shocks is:

$$A_0 u_t = \epsilon_t \tag{2}$$

The matrix A_0 decomposes reduced-form residuals u_t into mutually independent structural shocks ϵ_t. This decomposition is needed because we are typically interested in impulse response functions to such structural shocks, given the estimated VAR. Notice that we can rewrite (2) as $u_t = A_0^{-1}\epsilon_t$, where the jth column of A_0^{-1} is an impulse vector, i.e. it represents the immediate impact on all variables of the jth structural shocks.

So far the only restriction on A_0 is given by the covariance structure of u_t and ϵ_t:

$$\begin{aligned} \mathbb{E}\left[u_t u_t'\right] &= \mathbb{E}\left[\epsilon_t \epsilon_t'\right] \\ \Sigma &= A_0^{-1}\Sigma_\epsilon A_0^{-1'} \end{aligned} \tag{3}$$

Simple accounting shows that there are $n_x\left(n_x - 1\right)/2$ degrees of freedom in specifying A_0, and hence further restrictions are needed to achieve identification. Restrictions can be imposed directly on the structural parameters of the model, i.e. on the elements of A_0, as in B&P. Alternatively, restrictions can be imposed on the impulse responses of the SVAR on impact (e.g. Cholesky decomposition), in the short-run (e.g. sign restrictions), or in the long-run. Restrictions on impulse responses are imposed directly on matrix A_0^{-1} or on selected impulse vectors.

In this section we characterize analytically SVARs when restrictions are imposed directly on A_0. In section 3 we will map restrictions imposed on A_0^{-1} into restrictions on A_0.

2.1 A Simple Bivariate Model

We start from a simple bivariate model in the logarithms of quarterly output, denoted by Y_t and tax revenue net of transfers (as in B&P), denoted by T_t, all in real, per capita terms.

Following the notation used by B&P, we write system (2) as:

$$u_t^Y = c_1 u_t^T + \epsilon_t^Y \tag{4}$$

$$u_t^T = a_1 u_t^Y + \epsilon_t^T \tag{5}$$

where

$$A_0 = \begin{bmatrix} 1 & -c_1 \\ -a_1 & 1 \end{bmatrix} \tag{6}$$

Unexpected movements in GDP $\left(u_t^Y\right)$ can be due to two factors: the response to unexpected movements in tax revenue, captured by $c_1 u_t^T$, and non-policy shocks ϵ_t^Y. Similarly, unexpected movements in tax revenue $\left(u_t^T\right)$ can be due to the response to unexpected movements in GDP, captured by $a_1 u_t^Y$, and structural tax shocks ϵ_t^T.

We can use a simple Real Business Cycle (RBC) model to interpret the structural shocks. ϵ_t^T represents a shock to a proportional tax rate levied on labor income and capital income. ϵ_t^Y represents a non-policy shock, for instance a shock to technology.

The impact response of GDP and tax revenue to structural shocks is:

$$\begin{aligned} u_t &= A_0^{-1}\epsilon_t \\ \begin{bmatrix} u_t^Y \\ u_t^T \end{bmatrix} &= \frac{1}{1-a_1c_1}\begin{bmatrix} 1 & c_1 \\ a_1 & 1 \end{bmatrix}\begin{bmatrix} \epsilon_t^Y \\ \epsilon_t^T \end{bmatrix} \end{aligned}$$

The first column of matrix A_0^{-1} is the impulse vector associated to a non-policy shock: the first row denotes the response of output and the second row the response of tax revenue. Similarly, the second column of A_0^{-1} denotes the impulse vector associated to a tax shock.

To identify a bivariate SVAR we need to impose one restriction, either on c_1 or a_1. The coefficient c_1 captures the response of output to a 1% increase in tax revenue:

$$c_1 = \frac{\partial u_t^Y/\epsilon_t^T}{\partial u_t^T/\epsilon_t^T} \tag{7}$$

Theoretical models predict that $c_1 \leq 0$: an exogenous tax shock ϵ_t^T that increases tax revenue by 1% provokes a decline in output. [Add paragraph on Ricardian equivalence]. Restrictions on this coefficient are not imposed for two reasons. First, theoretical models do not have sharp predictions about the size of c_1. Second, estimates of c_1 can be used to validate and calibrate theoretical models.

The coefficient a_1 denotes the output elasticity of tax revenue:

$$a_1 = \frac{\partial u_t^T/\epsilon_t^Y}{\partial u_t^Y/\epsilon_t^Y} \tag{8}$$

In public finance literature this elasticity serves as an indicator of the overall progressivity of the tax system. A proportional income tax has an elasticity of 1.0, while progressive tax systems whose tax-income ratios increase with income have an elasticity greater than 1.0. Several international organizations and national governments provide figures for this elasticity estimating it using information from statutory tax rates. For instance, B&P estimate an output elasticity of tax revenue for the United States for the sample 1947 to 1997 of 2.08.[8] Hence SVAR models have been identified imposing restrictions on this elasticity. We would like to highlight that this elasticity is a random variable.

[8] In section 4 we show that using the Blanchard and Perotti (2002) methodology $a_1 = 2.1$ for the sample 1947 to 2010.

Equation (3) denotes a system of three non-linear equations (as many as the distinct elements of Σ_u) in four unknowns a_1, c_1, and the standard deviations of the structural shocks. As explained in the previous paragraph, we assume that a_1 is restricted, i.e. we have off-model information about this structural coefficient. The system becomes exactly identified and a unique solution exists. In the SVAR literature system (3) is solved numerically. Instead we solve system (3) analytically. The analytical derivation and the complete solution is reported in appendix C. Define the elements of the variance-covariance matrix of reduced-form residuals as:

$$\Sigma_u = \begin{bmatrix} \sigma_{YY} & \sigma_{YT} \\ \sigma_{YT} & \sigma_{TT} \end{bmatrix} \tag{9}$$

The solution for c_1 is:

$$c_1\left(a_1; \Sigma\right) = \frac{\sigma_{YT} - a_1\sigma_{YY}}{\sigma_{TT} - a_1\sigma_{YT}} \tag{10}$$

This analytical expression provides a non-linear mapping between the output elasticity of tax revenue a_1 and the tax revenue elasticity of output c_1. The mapping depends on the elements of the variance-covariance matrix Σ of reduced-form residuals. Before moving to results permitting inference, figure 1 plots $-c_1\left(a_1; \hat{\Sigma}_{OLS}\right)$, where the variance-covariance matrix has been fixed at the OLS point estimate. $-c_1$ indicates the response of output to a tax shock that decreases tax revenue by 1%. The function $c_1\left(a_1; \Sigma\right)$ has two interesting properties that hold for any Σ. First, c_1 is zero if and only if $a_1 \equiv \overline{a}_1 = \sigma_{YT}/\sigma_{YY}$. If $c_1 = 0$, the impact response of model variables to structural shocks become:

$$\begin{bmatrix} u_t^Y \\ u_t^T \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \sigma_{YT}/\sigma_{YY} & 1 \end{bmatrix} \begin{bmatrix} \epsilon_t^Y \\ \epsilon_t^T \end{bmatrix}$$

To impose $a_1 = \overline{a}_1$ is equivalent to identify the SVAR taking a Cholesky factorization of matrix Σ and assuming that GDP is ordered before tax revenue (Sims, 1980). In our sample $\overline{a}_1 = 2.06$, a value extremely close to 2.10, the point estimate of a_1 obtained using B&P methodology. If instead we impose $a_1 = 0$, we get $c_1 = \sigma_{YT}/\sigma_{TT}$ and

$$\begin{bmatrix} u_t^Y \\ u_t^T \end{bmatrix} = \begin{bmatrix} 1 & \sigma_{YT}/\sigma_{TT} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \epsilon_t^Y \\ \epsilon_t^T \end{bmatrix}$$

To impose $a_1 = 0$ is equivalent to identify the SVAR taking a Cholesky factorization of matrix Σ and assuming that tax revenue is ordered before GDP. Notice that if $a_1 < \overline{a}_1$ a 1% decline in tax revenue is associated to a *decline* in GDP. We can gain intuition examining the case $a_1 = 0$. A robust finding in the literature is that $\sigma_{YT} > 0$. When $a_1 = 0$ the tax shock need to explain the entire sample covariance σ_{YT}, and hence a decline taxes must be associated to a decline in GDP.

Second:

$$\begin{aligned}\lim_{a_1 \to a_{1,lim}^-} c_1(a_1;\Sigma) &= +\infty \\ \lim_{a_1 \to a_{1,lim}^+} c_1(a_1;\Sigma) &= -\infty\end{aligned}$$

where $a_{1,lim} = \sigma_{TT}/\sigma_{YT}$. Notice that $a_{1,lim} > \overline{a}_1$ for any Σ such that $|\rho_{YT}| < 1$, where ρ_{YT} is the correlation coefficient between u_t^Y and u_t^T. The restriction $a_{1,lim}$ does note define a SVAR model.

Some authors use $-c_1$ as measure of the impact tax multiplier (e.g. M&U). According to this definition, even if we exclude values of a_1 around the discontinuity and larger than $a_{1,lim}$, the tax multiplier can be potentially very large. This measure of the effectiveness of tax policy suffers from an important drawback: it does not consider the macroeconomic feedback in the economy, i.e. the presence of automatic stabilizers.

To better understand the role of automatic stabilization, define the response of tax revenue to a one dollar tax shock:

$$\begin{aligned}TR_0(a_1;\Sigma) &= (-1)\frac{1}{1-a_1c_1(a_1;\Sigma)} \\ TR_0(a_1;\Sigma) &= \frac{a_1\sigma_{YT}-\sigma_{TT}}{a_1^2\sigma_{YY}-2a_1\sigma_{YT}+\sigma_{TT}} \qquad (11)\end{aligned}$$

At $a_1 = 0$ the response of tax revenue $\left(u_t^T\right)$ to a tax shock $\left(\epsilon_t^T = -1\right)$ is -1. Inspecting equation (14) we see that tax revenue does not react to movements in GDP. When $a_1 = \overline{a}_1$ the response of tax revenue is also -1. The reason is that GDP does not react to movements in tax revenue, i.e. $c_1 = 0$. For $\overline{a}_1 < a_1 < a_{1,lim}$ the response of tax revenue increases in a_1. The larger a_1, the more GDP reacts to a given change in tax revenue (c_1), the smaller the decline in tax revenue. The intuition s that a tax cut increases the tax base. Hence the size of a tax shock ϵ_t^T necessary to generate a decline in tax revenue of size 1% is increasing in a_1, as can be seen in Figure 4. For $a_1 > a_{1,lim}$ a negative tax shock generates an increase in tax revenue. This is due to the fact that in this interval for a_1 GDP declines after a tax cut.

Tax Multiplier. Following B&P we define the tax multiplier as the dollar response of output to a tax shock of size one dollar:

$$\begin{aligned}TM_0(a_1;\Sigma) &= (-1)\frac{c_1(a_1;\Sigma)}{1-a_1c_1(a_1;\Sigma)}\frac{1}{\overline{T}/\overline{Y}} \\ TM_0(a_1;\Sigma) &= \frac{a_1\sigma_{YY}-\sigma_{TY}}{a_1^2\sigma_{YY}-2a_1\sigma_{YT}+\sigma_{TT}}\frac{1}{\overline{T}/\overline{Y}}\end{aligned}$$

where -1 denotes the size of the shock, and $\overline{T}/\overline{Y}$ denotes the mean tax-to-GDP ratio. This scaling factor transforms percentage changes into dollar changes. The following theorem states some key properties of the impact tax multiplier $TM_0(a_1;\Sigma)$.

Proposition 1 *For any positive-definite variance-covariance matrix Σ, the impact tax multiplier $TM_0(a_1;\Sigma)$ has the following properties:*

1. *It has a unique global minimum:*

$$TM_0\left(a_1^{min};\Sigma\right) = -\frac{\sigma_Y}{\sigma_T}\left(\frac{1}{2\left(1-\rho_{TY}^2\right)^{0.5}}\right)\frac{1}{\overline{T}/\overline{Y}} < 0$$

where $a_1^{min} = (\sigma_T/\sigma_Y)\left(\sqrt{1-\rho_{TY}^2}-\rho_{TY}\right)$.

2. *It has a unique global maximum:*

$$TM_0\left(a_1^{max};\Sigma\right) = \frac{\sigma_Y}{\sigma_T}\left(\frac{1}{2\left(1-\rho_{TY}^2\right)^{0.5}}\right)\frac{1}{\overline{T}/\overline{Y}} > 0$$

where $a_1^{max} = (\sigma_T/\sigma_Y)\left(\sqrt{1-\rho_{TY}^2}+\rho_{TY}\right)$.

3. *It has a unique zero:*

$$TM_0\left(\overline{a}_1;\Sigma\right) = 0 \Longleftrightarrow \overline{a}_1 = \frac{\sigma_{TY}}{\sigma_{YY}}$$

4. *It is such that:*

$$\lim_{a_1\to\pm\infty} TM_0\left(a_1;\Sigma\right) = 0 \tag{12}$$

Proof The proof is provided in Appendix C. ||

Figure 4 plots $TM_0\left(a_1;\hat{\Sigma}_{OLS}\right)$. The impact multiplier is a bounded function of a_1. The set of admissible values of the impact multiplier ranges between -0.68 and 0.68. The lower bound is associated to a value of the elasticity $a_1^{min} = -2.64$, while the upper bound is associated to a value of the elasticity $a_1^{max} = 6.75$. Since the bounds have opposite sign and $TM_0(a_1;\Sigma)$ is a continuous function in a_1, the zero multiplier is always included. Not surprisingly, the impact multiplier is zero at $a_1 = \overline{a}_1$, the elasticity that associated to $c_1 = 0$. In our numerical example $\overline{a}_1 = 2.06$. Values of $a_1 < \overline{a}_1$ are associated to negative impact multipliers, as it is the case for $-c_1(a_1;\Sigma)$. Values of $a_1 > \overline{a}_1$ are associated to positive impact multipliers. Notice that for plausible values of a_1, which for the moment we define between 1 and 3, the impact tax multiplier ranges between -0.3 and 0.3. A formal characterization of empirically relevant intervals for a_1 is given in section 4.

The upper panel of figure 4 plots the tax multiplier four quarters after the shock. The set of admissible multipliers is larger than for the impact multiplier: it ranges between -1.05 and 0.98. The multiplier is zero at $a_1 = 2.3$, a value close to $\overline{a}_1$ and to the elasticity estimated using the B&P approach. The multiplier eight quarters after the shock is zero at $a_1 = 2$, while the set admissible multipliers is smaller, ranging between -0.58 and 0.60. Finally, identification

problems seem to be less pervasive twelve quarters after the shock. The tax multiplier is zero at $a_1 = 0.4$, a value far from plausible estimates for the United States. The set of admissible multipliers ranges between -0.45 and 0.70.

In this section we have shown how to characterize analytically the identification problem in a bi-variate SVAR model. The main message is that sign and the size of the tax multiplier in the short-run depend on the choice of the output elasticity of tax revenue a_1. This elasticity is a random variable. In the next three sections we discuss how to compute distributions for $a_1$using three different approaches.

2.2 Government Spending

Much of the recent debate on the effects of fiscal policy focuses on the reaction of output and its components to exogenous increases in government spending. Consider three-equation structural model:

$$u_t^Y = c_1 u_t^T + c_2 u_t^G + \epsilon_t^Y \tag{13}$$
$$u_t^T = a_1 u_t^Y + a_2 u_t^G + \epsilon_t^T \tag{14}$$
$$u_t^G = b_1 u_t^Y + b_2 u_t^T + \epsilon_t^G \tag{15}$$

where:

$$A_0 = \begin{bmatrix} 1 & -c_1 & -c_2 \\ -a_1 & 1 & -a_2 \\ -b_1 & -b_2 & 1 \end{bmatrix} \tag{16}$$

Unexpected movements in government spending u_t^G can have contemporaneous effects on output (c_2) and tax revenue (a_2). Similarly, output and tax revenue can have contemporaneous effects on spending (b_1 and b_2 respectively). The structural coefficient b_1 represents the output elasticity of government spending. We restrict this coefficient and solve analytically for the spending multiplier as function of b_1. In order to exactly identify the SVAR we need to impose an additional restriction. For our analytical framework, restricting a_2 or b_2 is conceptually identical. It turns out that restricting b_2 allows to derive simpler analytical expressions.

As in the bivariate case, we solve analytically system (3). For ease of exposition we only focus on the analytical expression for the spending multiplier. To further simplify the analysis we assume $b_2 = 0$. This coefficient represents the tax revenue elasticity of government spending. In the United States tax revenue and spending display a low correlation (0.04 in our sample). Hence setting $b_2 = 0$ has a minimal impact on the size of the spending multiplier. Of course this assumption might not be innocuous for countries other than the United States, and its impact on the results should be checked carefully.

Spending Multiplier. We define the spending multiplier as the dollar response of output to a spending shock of size one dollar:

$$GM_0\left(b_1;\Sigma\right) \quad = \quad \frac{\sigma_{GY}-b_1\sigma_{YY}}{b_1^2\sigma_{YY}-2b_1\sigma_{YG}+\sigma_{GG}}\frac{1}{\overline{G}/\overline{Y}}$$

where $G/\overline{Y}$ denotes the mean spending-to-GDP ratio. This scaling factor transforms percentage changes into dollar changes. First, notice that the spending multiplier does not depend on a_1, the output elasticity of tax revenue, nor on any term associated to taxation. Second, the spending multiplier function has identical theoretical properties to the tax multiplier function, which in the interest of space we will not re-examine further[9].

Figure 6 plots $GM_0\left(b_1;\hat{\Sigma}_{OLS}\right)$. The impact multiplier is a bounded function of b_1. The set of admissible values of the impact multiplier ranges between -1.6 and 1.6. The lower bound is associated to a value of the elasticity $b_1^{min}=1.82$, while the upper bound is associated to a value of the elasticity $b_1^{max}=-2.05$. Since the bounds have opposite sign and $GM_0\left(b_1;\Sigma\right)$ is a continuous function in b_1 In our numerical example the impact spending multiplier is zero at $\bar{b}_1=0.28$. Values of $b_1>\bar{b}_1$ are associated to negative impact multipliers, while values of $b_1<\bar{b}_1$ are associated to positive impact multipliers. B&P calibrate $b_1=0$, as they argue that there are not components of government consumption and investment that react automatically to the business cycle. The OECD adopts the same assumption. Notice however that the function GM_0 is very steep around $b_1=0$. Hence small changes in b_1 have large implications for the spending multiplier.

2.3 Consumption Response

1. Analytical results can be extended to larger VARs. Our key assumption is that additional variables do not affect output, tax revenue, and government spending. Then the impact response of variable i to shock a tax shock is:

$$TMi_0\left(a_1;\Sigma\right)=\frac{a_1\sigma_{Yi}-\sigma_{Ti}}{a_1^2\sigma_{YY}-2a_1\sigma_{YT}+\sigma_{TT}}\frac{1}{\overline{T}/\overline{i}}=$$

   while the response to spending shock is:

$$GMi_0\left(b_1;\Sigma\right) \quad = \quad \frac{\sigma_{Gi}-b_1\sigma_{Yi}}{b_1^2\sigma_{YY}-2b_1\sigma_{YG}+\sigma_{GG}}\frac{1}{\overline{G}/\overline{i}}$$

2. Figure 7 plots the consumption response to a negative tax shock of size of size one dollar as a function of the output elasticity of tax revenue a_1. The slope of the function is small, and also the maximum and minimum responses are close to zero (± 0.25). Notice that the response is zero at $a_1=2$, which again is very close to the point estimate obtained using the B&P approach.

[9] Notice that the size of the spending shock is 1, while the size of the tax multiplier is -1. This change of sign simply flips the function over the $x-$axes.

3. Figure 8 plots the consumption response to a spending shock of size one dollar as a function of the output elasticity of spending b_1. The response is zero at $b_1 = 0$. For elasticities larger than zero the response of consumption turns negative, although it might not be statistically significant when sampling uncertainty is introduced.

[TO BE COMPLETED]

3 Sign Restrictions

The sign restriction approach has been used by M&U to identify fiscal policy shocks. These authors identify two shocks. First, a business cycle shock is identified to capture movements in model variables not attributable to fiscal policy. In our small model this shock is ϵ_t^Y, the GDP shock. We assume that GDP and tax revenue increase on impact following a business cycle shock[10]. Second, they identify a tax shock. We assume that tax revenue increases on impact following a tax shock. The response of output to a tax shock is left unrestricted. The impact response of model variables to structural shocks:

$$\begin{aligned} A_0^{-1} &= \frac{1}{1-a_1c_1}\begin{bmatrix} 1 & c_1 \\ a_1 & 1 \end{bmatrix} \\ A_0^{-1}(a_1;\Sigma) &= \frac{1}{\det(A_0)}\begin{bmatrix} \sigma_{TT}-a_1\sigma_{TY}, & \sigma_{TY}-a_1\sigma_{YY} \\ a_1(\sigma_{TT}-a_1\sigma_{TY}), & \sigma_{TT}-a_1\sigma_{TY} \end{bmatrix} \\ &= \begin{bmatrix} + & ? \\ + & + \end{bmatrix} \end{aligned}$$

where $\det(A_0) = a_1^2\sigma_{YY} + \sigma_{TT} - 2a_1\sigma_{TY}$. The first column of matrix A_0^{-1} is the impulse vector associated to a tax shock: the first row denotes the response of tax revenue to a tax shock and the second row the response of GDP. Similarly, the second column of denotes the impulse vector associated to a GDP shock. In our setting the restriction imposed to identify the tax shock is simply a normalization. Assume that we draw an impulse vector where the second row is negative (and it is orthogonal to the business cycle shock). This is classified as a negative tax shock. We simply flip the sign of the vector and we obtain a positive tax shock satisfying sign restrictions. Hence, the only binding constraints we impose are on the business cycle shock.

Sign restrictions are satisfied if and only if

$$a_1 > 0 \tag{17}$$

Recall from section 2 that the tax multiplier is zero when $a_1 = \overline{a}_1$, where σ_{TY}/σ_{YY}. For any variance covariance matrix Σ such that $\sigma_{YT} > 0$ we know

[10] Mountford and Uhlig (2009) identify the business cycle shock imposing additional restrictions on consumption and investment. Furthermore restrictions are imposed up to four quarters after the shocks.

that:

$$\overline{a}_1 > 0$$

Since $\overline{a}_1$ is strictly included in the set of sign restriction solutions, the sign of the multiplier cannot be determined. That is, there always exists an arbitrarily small δ such that

$$\begin{aligned} TM_0\left(a_1 = \overline{a}_1 - \delta; \Sigma\right) &< 0 \\ TM_0\left(a_1 = \overline{a}_1 + \delta; \Sigma\right) &> 0 \end{aligned}$$

and sign restrictions are satisfied.

We have shown that sign restrictions can be mapped into an interval for the output elasticity of tax revenue a_1. The following step is to characterize the distribution of a_1 over such interval. All algorithms implementing sign restrictions are based on orthonormal matrices, which are assumed to be uniformly distributed. An uniform distribution over orthonormal matrices does not imply an uniform distribution of the elasticity. Table 1 reports statistics that characterize the distribution of a_1 implied by sign restrictions imposed at different horizons. The VAR coefficients and the variance-covariance matrix Σ have been fixed at the OLS estimates. These statistics are based on 45000 accepted draws for the orthonormal matrix. When restrictions are imposed only on impact, the median elasticity is 5.11. When restrictions are imposed for more than one quarter after the shocks, analytical results are of little help, since autoregressive coefficients enter the solution. For this case, we provide a numerical evaluation of the distribution of a_1. When restrictions are imposed up to 4 or 8 quarters after the shock, the elasticity goes down to values around 3.6. These are very large values compared to standard estimates provided in public finance literature. Distribution are very wide. The 68% credible set includes values between 1.61 and 16.4 when restrictions are imposed on impact, and between 1.2 and 21.5 when restrictions are imposed for 4 and 8 quarters. Table 4 reports the distribution of the impact spending multiplier implied by the distribution on a_1 from sign restrictions. The median multiplier is 0.41, and the 68% credible set ranges between -0.46 and 0.63. The impact multiplier associated to an elasticity of 2.16, as in B&P, is zero.

Table 5 reports estimates of the output elasticity of government spending b_1 implied by sign restrictions imposed at different horizons. Results are obtained estimating an SVAR in output, tax revenue, and spending. Following M&U, to identify a business cycle shock no restrictions are imposed on the response of government spending. The spending shock is identified restricting the response of government spending. When restrictions are imposed only on impact, the median elasticity is 0.25. When restrictions are imposed up to 4 or 8 quarters after the shock, the elasticity goes down to 0.16 and -0.07 respectively . Distributions are very wide. The 68% credible sets include values between -2.3 and 2.2 when restrictions are imposed on impact and for 4 quarters, and between -2.3 and 2.46 when restrictions are imposed for 8 quarters.

Table 6 reports the distribution of the impact spending multiplier implied by the distribution on b_1 from sign restrictions. The median multiplier is 0.15,

and the 68% credible set ranges between -1.44 and 1.45. The impact multiplier associated to an elasticity of 0, as in B&P, is 0.57.

Penalty Function Approach

M&U do not report results for the pure sign restriction approach. Instead they select an element form the set of sign restriction solutions using a criterion function. This approach is known as the "penalty function" approach. This penalty function selects the solution that maximizes the forecast error variance of output and tax revenue up to last restricted quarter explained by ϵ_t^Y. The analytical solution to this problem is characterized in Caldara and Kamps (2010b). Table 3 reports the output elasticity of tax revenue implied by the penalty function approach. When restrictions are imposed only on impact, the elasticity is 5.13, a value almost identical to the median elasticity. When restrictions are imposed for more quarters, the elasticity drops to values around 4. Table 4 shows that the impact tax multiplier associated to an elasticity of 4 is 0.48, while the multiplier associated to an elasticity of 2.16, the value used by B&P is zero. Differences in elasticities explain why tax multipliers estimated by M&U are larger than multipliers estimated by B&P.

Table 5 reports the output elasticity of government spending implied by the penalty function approach. When restrictions are imposed only on impact, the elasticity is 0.25, a value almost identical to the median elasticity. When restrictions are imposed for four and eight quarters, the elasticity drops to 0.07 and 0.03 respectively. Table 6 shows that the impact tax multiplier associated to an elasticity of 0.07 is 0.43, while the multiplier associated to an elasticity of 0, the value used by B&P is 0.57. Small differences in elasticities explain why spending multipliers estimated by M&U are smaller than multipliers estimated by B&P.

4 Estimating Elasticities

This section builds on the seminal work of B&P. The OECD[11] estimates the output elasticity of tax revenue for four different tax categories: personal income tax; social security contributions corporate income tax and indirect taxes. In addition they also estimate the output elasticity of transfers. B&P aggregate these elasticities to obtain a point estimate for a_1 according to the following aggregator:

$$a_1 = \sum_i \eta_{T_i,x} \frac{T_i}{\tilde{T}}$$

where $\eta_{T_i,x}$ denotes the elasticity of tax category i to potential output x, and $\tilde{T}$ denotes tax revenue net of government transfers.

The elasticity $\eta_{T_i,x}$ can be separated in two components, an elasticity of tax revenue with respect to the relevant tax base, η_{T_i,TB_i}, and an elasticity of the

[11] Giorno, Richardson, Roseveare, and Van den Noord (1995); Girouard and André (2005); Van den Noord (2000)

tax base relative to the cyclically adjusted indicator, $\eta_{TB_i,x}$:

$$\eta_{T_i,x} = \eta_{T_i,TB_i}\eta_{TB_i,x} \tag{18}$$

The elasticities of taxes with respect to their base are extracted from tax legislation and related fiscal data, while the sensitivity of the different tax bases with respect to the output gap is estimated using time-series data.

We first obtain a measure of uncertainty around a_1 following the B&P methodology. In particular, we estimate the output elasticities of tax bases using Bayesian linear regressions. The first row of Table 7 reports the median and the 90% credible set for a_1. Sampling uncertainty is small. The median value for a_1 is 2.10, while the 90% credible set ranges between 2.00 and 2.20. In what follows we argue that even if the distribution of $a_1$is narrow, it can shift substantially depending on two factors. First, how the elasticity of personal income tax to output is estimated. Second, how sub-elasticities are aggregated.

4.1 Output Elasticity of Tax Revenue

4.1.1 Elasticity of Personal Income Tax to Output

1. The elasticity of personal income tax with respect to the tax base is the product of two elasticities, the elasticity of tax revenue with respect to per capita earnings, and the elasticity of the wage bill with respect to (potential output).

2. Figure 9 shows the elasticity of tax revenue with respect to per-capita earnings constructed by the OECD. The elasticity changes dramatically over time. From 1992 to 1996 it decreases three-fold (from 3.9 to 1.3). The largest fiscal bill passed during these years is OBRA 1993, signed by President Clinton, which has increases progressivity of the tax system (CBO papers 1993). This large change can be due to measurement error or to change in methodology (not reported by the OECD).

3. Alternatively values for the elasticity of tax revenue with respect to personal income tax can be estimated using TAXSIM, a micro simulation model estimated by the NBER and based on survey data. Figure 9 shows the elasticity estimated using TAXSIM. The sample coverage is larger, and values are less variable over time compared to the OECD estimates. Importantly, the mean elasticity is 2.58 according to OECD figures, while 1.68 according to NBER figures. Row 2 of Table 7 reports estimates of $a_1$obtained using an average elasticity of tax revenue with respect to personal income tax of 1.68. The distribution of a_1 shifts to the left, with the median declining to 1.73 from 2.10.

4. The elasticity of the wage bill with respect to output is estimated by the OECD using the following regression:

$$\triangle \log (W_t L_t / X_t^*) = \alpha_0 + \alpha_1 \triangle \log (X_t / X_t^*) \tag{19}$$

where X_t^* is potential output, W is the wage rate and L is employment. The median elasticity is 0.7, and the 90% credible set is $(0.65, 0.75)$. In DSGE models with a Cobb Douglas production function this elasticity is fixed to 1. Rows 3 and 4 of Table 7 show the distribution of a_1 when the elasticity is fixed to 1 instead of being estimated. The distributions shift upward, with a median $a_1 = 2.51$ when OECD data are used to compute the elasticity of tax revenue with respect to personal income tax, and with a median $a_1 = 1.92$ when TAXSIM data are used.

4.1.2 Aggregation

1. B&P aggregate sub-elasticities constructing weights based on a measure of tax revenue net of transfers.

2. An alternative is to construct weights for the sub-elasticities based on tax revenue, without subtracting transfers. Rows 6 to 9 in Table 7 report estimates of a_1 based on this weighting scheme. The elasticity becomes smaller. Using OECD data, the median a_1 is 1.21 or 1.43, while using TAXSIM data the elasticity is 1 or 1.09

3. Bottom line. It seems that sampling uncertainty is small, while how sub-elasticities are constructed and aggregate plays a substantial role. An elasticity of 1 instead of 2 implies a negative tax multiplier in the short-run.

ADD MATERIAL

4.2 Output Elasticity of Government Spending

1. B&P assume that the output elasticity of government consumption and investment is zero. This assumption seems to be appropriate for the United States, where defense spending are a large fraction of overall federal spending. At the same time, the importance of state and local spending in total spending has increased over time. Further research should estimate the output elasticity of state spending, which might be different from zero, since states need to run a balanced-budget, and hence the current conjuncture might affect the spending capacity.

5 Back of the Envelope Calculation from DSGE models.

What is the output elasticity of tax revenue in a DSGE model? How large is it?

Assume that the production sector consists of a continuum of firms operating in a competitive market. Further assume that firms produce output X_t using a Cobb-Douglas production function:

$$X_t = a_t K_t^{\alpha} L_t^{1-\alpha}$$

where a_t denotes an exogenous technology process. For simplicity assume that prices and wages are fully flexible. Cost minimization implies that capital and labor are paid a constant share of output:

$$R_t K_t = \alpha X_t \tag{20}$$
$$W_t L_t = (1-\alpha) X_t \tag{21}$$

Assume that the government taxes labor income $W_t L_t$ at a rate $\tau_{L,t}$ and capital income $R_t K_t$ at a rate $\tau_{L,t}$. Capital income is taxed after an allowance for depreciation. The tax revenue Tax_t is equal to:

$$Tax_t = +\tau_{K,t} R_t K_t + \tau_{L,t} W_t L_t \tag{22}$$

Taxes are used to finance public consumption G_t:

$$Tax_t = G_t$$

Plugging in (20) and (21) into (22) and log-linearizing we get:

$$\hat{Tax}_t = \frac{\tau_K \alpha}{\tau_K \alpha + \tau_L (1-\alpha)} \left[\hat{X}_t + \hat{\tau}_{K,t}\right] + \frac{\tau_L (1-\alpha)}{\tau_K \alpha + \tau_L (1-\alpha)} \left[\hat{X}_t + \hat{\tau}_{L,t}\right] \tag{23}$$

where $\hat{var}_t$ denote percentage deviations from steady state and τ_K and τ_L denote steady state tax rates.

Assume that tax rates follow an exogenous i.i.d. process. To compute the output elasticity of tax revenue assume that the economy is hit by a technology shock that increases output by 1% of his steady state value. Then tax revenue increases by

$$\begin{aligned} \hat{Tax}_t &= \frac{\tau_K \alpha}{\tau_K \alpha + \tau_L (1-\alpha)} + \frac{\tau_L (1-\alpha)}{\tau_K \alpha + \tau_L (1-\alpha)} \\ &= 1 \end{aligned}$$

Hence the output elasticity of tax revenue is one.

Let us assume that tax policy follows a simple rule, as in Leeper, Plante, and Traum (2009):

$$\hat{\tau}_{K,t} = \varphi_K \hat{X}_t + \epsilon_t^{\tau_K}$$
$$\hat{\tau}_{L,t} = \varphi_L \hat{X}_t + \epsilon_t^{\tau_L}$$

Tax rates respond to the cyclical position of the economy. This fiscal rule captures the automatic stabilization role of the tax system. What is the output elasticity of tax revenue? Plugging the fiscal rules in the expression for the tax revenue and simplifying we get:

$$\hat{Tax}_t = \frac{\tau_K \alpha}{\tau_K \alpha + \tau_L (1-\alpha)} (1+\varphi_K) + \frac{\tau_L (1-\alpha)}{\tau_K \alpha + \tau_L (1-\alpha)} (1+\varphi_L)$$

Leeper, Plante, and Traum (2009) estimates a a DSGE model of fiscal policy using Bayesian techniques. To get a rough idea of the distribution of the output elasticity of tax revenue implied by a theoretical model we follow their specification of prior distributions over structural parameters. The capital share of production α is calibrated to 0.3. The steady state capital income tax rate and labor income tax rate are set to 0.184 and 0.223 respectively. The calibration of these parameters is standard and it does not depend on the exact specification of the model. The parameter φ_K is assumed to have a gamma distribution with mean 1 and standard deviation 0.3. The parameter φ_L is assumed to have a gamma distribution with mean 0.5 and standard deviation 0.25. The median is 1.60, and the 90% credible set ranges between 1.20 and 2.20. Notice that both the value of $a_1$used by B&P and the value of a_1 consistent with the penalty function approach lie outside this credible set.

6 Bayesian Estimation

In this section we provide results of a Bayesian estimation of two different VAR models. We first estimate a three-equation model in output, taxes, and spending. We then move to a large scale VAR model. In the current version of the paper we estimate a 7 equation model, but we plan to estimate an 18-equation VAR. The goal is to mitigate non-invertibility problems pointed out in Leeper, Walker, and Yang (2008) and Leeper, Walker, and Yang (2009), and recently studied, among the others, by Forni and Gambetti (2010) Mertens and Ravn (2010b),Mertens and Ravn (2010a), Yang (2007).

Following M&U we report results for three different fiscal policies, constructed as linear combinations of the basic tax and spending shocks. We consider three policy experiments; A deficit-financed spending increase, which is designed as a sequence of fiscal shocks such that government spending rises by 1 dollar and tax revenues remain unchanged for four quarters following the initial shock; A deficit-financed tax-cut, which is designed as a sequence of fiscal shocks such that tax revenues decline by 1 dollar and government spending remain unchanged for four quarters following the initial shock; and A balanced-budget spending increase, which is designed as a sequence of fiscal shocks such that government spending and tax revenues rise by 1 dollar for four quarters following the initial shock. We report results for policy scenarios in order to be able to compare our results with the evidence reported in the existing literature and summarized in tables 1 and 2.

6.1 Benchmark Estimation

Table 8 reports results for the three policy experiments described above using a 3-equation VAR model and two different distributions for the output elasticity of tax revenue a_1. The output elasticity of spending b_1 is assumed to be zero. There are two important results. First, a spending increase is more effective in stimulating the economy than a tax cut up to 12 quarters after the policy

intervention. These result holds irrespectively of the distribution of the output elasticity of tax revenue considered. Second, for values of a_1 estimated using the B&P methodology and OECD data, tax cuts are ineffective up to 8 quarters. For elasticities estimated using TAXSIM data and the aggregator proposed in section 4.1.2, tax cuts depress output up to four quarters after the shock. The effects of tax shocks are delayed. Output is stimulated only 12 quarters after the shock, and the peak multiplier is obtained only after four years. Third, the effects of spending increases are persistent and hump-shaped. The peak multiplier is obtained 10 quarters after the shock.

6.2 Large(r) Scale Model

Table 9 reports results for policy experiments for a 7-equation VAR model. The four variables that we added to the system are consumption, the Standard & Poor's 500 Index, the R&R measure of anticipated changes in tax revenue, and the Ramey (forthcoming) measure of anticipated changes in defense spending. Consumption and the stock market index are forward-looking variables, while R&R and Ramey's series are a good forecast of future fiscal policy. The inclusion of these variables should help detect/mitigate problems of fiscal foresight. [EXPLAIN]. Our qualitative results remain unchanged. Additional variables seem to have only quantitative effects on the estimates of multipliers. Interestingly, multipliers in this large system tend to be larger than multipliers estimated using the 3-equation VAR.

In the next revision of the paper we plan to estimate the multipliers associated to the fiscal episodes identified by R&R and Ramey. We want to test whether in our VAR model we confirm the results in Favero and Giavazzi (2010). That is, SVAR and narrative-approach propose two alternative methods to obtain instruments to estimate fiscal multipliers. Different instruments might well imply similar multipliers when shocks are propagated in the same reduced-form model, as they find for taxation.

7 Conclusions

[TO BE ADDED]

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Appendix

A The Ruduced-Form VAR

The structural VAR specification is

$$A_0 X_t = \sum_{l=1}^{p} A_l X_{t-l} + Cz + \epsilon_t, \text{ for } 1 \leq t \leq T, \tag{A.1}$$

where:

- p is the lag length,
- T is the sample size,
- X_t is an $n_x \times 1$ vector of endogenous variables,
- z is an $n_z \times 1$ vector of deterministic terms,
- ϵ_t is a $n_x \times 1$ vector of exogenous structural shocks,
- A_l is an $n_x \times n_x$ matrix of parameters for $0 \leq l \leq p$, and
- C is an $n_z \times n_x$ matrix of parameters.

The initial conditions $X_0, \ldots X_{1-p}$ are taken as given. The distribution of ϵ_t, conditional on the past information, is Gaussian with mean zero and diagonal covariance matrix Σ_ϵ.

The reduced-form representation implied by the structural model (A.1) is

$$X_t = \sum_{l=1}^{p} B_l X_{t-l} + Dz + u_t$$

where $B_l = A_0^{-1} A_l$ for $1 \leq l \leq p$, $D = A_0^{-1} C$, and u_t is a $n_x \times 1$ vector of reduced-form residuals with mean zero and symmetric covariance matrix Σ_u.

We follow Mountford and Uhlig (2009) and choose a lag length p of six quarters. We also include a constant.

We estimate the model using a Bayesian approach. We impose the Minnesota Prior proposed by Doan, Litterman, and Sims (1984).

B The Data

The data are collected from multiple sources.

B.1 Data for Estimation of the VAR Models.

[TO BE ADDED]

B.2 Data for Estimation of Elasticities.

[TO BE ADDED]

C Analytical Derivation and Proofs

[TO BE ADDED]

	1 qrt	4 qrts	8 qrts	12 qrts	20 qrts	Maximum
MU (2009)						
Def.-Fin. Spending Increase	0.65*	0.27	-0.74*	-1.19*	-2.24*	0.65* (1)
Def.-Fin. Tax Cut	0.28*	0.93	2.05*	3.41*	2.59*	3.57* (13)
Bal.-Bud. Spending Increase	0.37*	-0.66*	-2.79*	-4.60*	-4.83*	0.37 (3)
BP (2002)						
Def.-Fin. Spending Increase	0.96*	0.57	0.79	1.17*	0.85*	1.21* (14)
Def.-Fin. Tax Cut	-0.87*	1.79*	1.92*	1.11*	0.33	1.97* (7)
Bal.-Bud. Spending Increase	0.09	-1.22*	-1.13	0.06	0.52	0.71 (17)

Table 1:

	1 qrt	4 qrts	8 qrts	12 qrts	20 qrts	Maximum
MU (2009)						
Def.-Fin. Spending Increase	0.65*	0.27	-0.74*	-1.19*	-2.24*	0.65* (1)
Def.-Fin. Tax Cut	0.28*	0.93	2.05*	3.41*	2.59*	3.57* (13)
Bal.-Bud. Spending Increase	0.37*	-0.66*	-2.79*	-4.60*	-4.83*	0.37 (3)
BP identification scheme applied to MU (2009) VAR						
Def.-Fin. Spending Increase	1.33*	1.40*	0.50	0.72*	-0.10	1.60* (3)
Def.-Fin. Tax Cut	0.02	0.40	1.09*	1.58*	0.60	1.62* (13)
Bal.-Bud. Spending Increase	1.31*	1.01*	-0.60	-0.87	-0.64	1.46 (3)

Table 2:

Distribution for a_1, Sign Restriction Approach			
Restrictions	Pure		Penalty Function
	Median	68% C.S.	
Impact	5.14	(1.61; 16.42)	5.13
Up to 4 Qtrs.	3.62	(1.21; 21.71)	4.01
Up to 8 Qtrs.	3.59	(1.22; 21.46)	4.03

Table 3: Distributions of the output elasticity of tax revenue implied by sign restrictions.

Method	Distribution of a_1		$p\left(TM_0 \vert \hat{\Sigma}_{OLS}\right)$	
	Median	68% C.S.	Median	68% C.S.
Blanchard&Perotti	2.16	-	0.00	-
Sign Restrictions	3.62	(1.21, 21.71)	0.41	(-0.46; 0.63)
Sign Restriction + P.F.	4.01	-	0.48	-

Table 4: Distributions of the output elasticity of tax revenue implied by different methods. Posterior distributions of the impact tax multiplier conditional on $\hat{\Sigma}_{OLS}$.

Distribution for b_1, Sign Restriction Approach			
Restrictions	Pure		Penalty Function
	Median	68% C.S.	
Impact	0.25	(-2.26; 2.60)	0.24
Up to 4 Qtrs.	0.16	(-2.46; 2.21)	0.07
Up to 8 Qtrs.	-0.08	(-2.30; 1.46)	0.03

Table 5: Distributions of the output elasticity of government spending implied by sign restrictions.

Method	Distribution of b_1		$p\left(GM_0 \vert \hat{\Sigma}_{OLS}\right)$	
	Median	68% C.S.	Median	68% C.S.
Blanchard&Perotti	0.00	-	0.57	-
Sign Restrictions	0.16	(-2.43,2.21)	0.15	(-1.44; 1.45)
Sign Restriction + P.F.	0.07	-	0.43	-

Table 6: Distributions of the output elasticity of government spending implied by different methods. Posterior distributions of the impact spending multiplier conditional on $\hat{\Sigma}_{OLS}$.

Estimated Distribution of a_1		
Method	Aggregator based on **Net** Tax Revenue	
	Median	90% C.S.
OECD data	2.10	(2.00; 2.20)
TAXSIM data	1.73	(1.67; 1.78)
OECD + Cobb Douglas	2.51	(2.47; 2.54)
TAXSIM + Cobb Douglas	1.92	(1.88; 1.92)
	Aggregator based on Tax Revenue	
OECD data	1.21	(1.16; 1.26)
TAXSIM data	1.00	(0.97; 1.02)
OECD + Cobb Douglas	1.43	(1.41; 1.45)
TAXSIM + Cobb Douglas	1.09	(1.07; 1.11)

Table 7:

	1 qrt	4 qrts	8 qrts	12 qrts	20 qrts	Maximum
a_1 from OECD + Net Taxes Aggregator						
Def.-Fin. Spending Increase	0.60*	0.27	0.70*	0.84*	0.60*	1.12* (10)
Def.-Fin. Tax Cut	0.02	-0.05	0.20	0.73*	1.11*	1.11* (20)
Bal.-Bud. Spending Increase	0.58*	0.31	0.48*	0.12	-0.52*	0.60* (3)
a_1 from TAXSIM + Taxes Aggregator						
Def.-Fin. Spending Increase	0.54*	0.34*	0.75*	0.88*	0.65*	0.94* (10)
Def.-Fin. Tax Cut	-0.27*	-0.47*	-0.09	0.36*	0.73*	0.73* (20)
Bal.-Bud. Spending Increase	0.80*	0.82*	0.86*	0.52*	-0.06	1.06* (3)

Table 8: 3-equation VAR

	1 qrt	4 qrts	8 qrts	12 qrts	20 qrts	Maximum
a_1 from OECD + Net Taxes						
Def.-Fin. Spending Increase	0.92*	0.76*	1.06*	1.28*	1.09*	1.33* (10)
Def.-Fin. Tax Cut	0.04	0.34*	0.80	1.20*	1.52*	1.52* (20)
Bal.-Bud. Spending Increase	0.87*	0.44	0.28*	0.07	-0.45	0.87* (1)
a_1 from TAXSIM + Taxes						
Def.-Fin. Spending Increase	0.81*	0.71*	1.03*	1.24*	1.03*	1.30* (11)
Def.-Fin. Tax Cut	-0.24*	-0.17	0.35*	0.75*	1.04*	1.04* (20)
Bal.-Bud. Spending Increase	1.07*	0.86*	0.69*	0.52	0.02	1.22* (3)

Table 9: 7-equation VAR

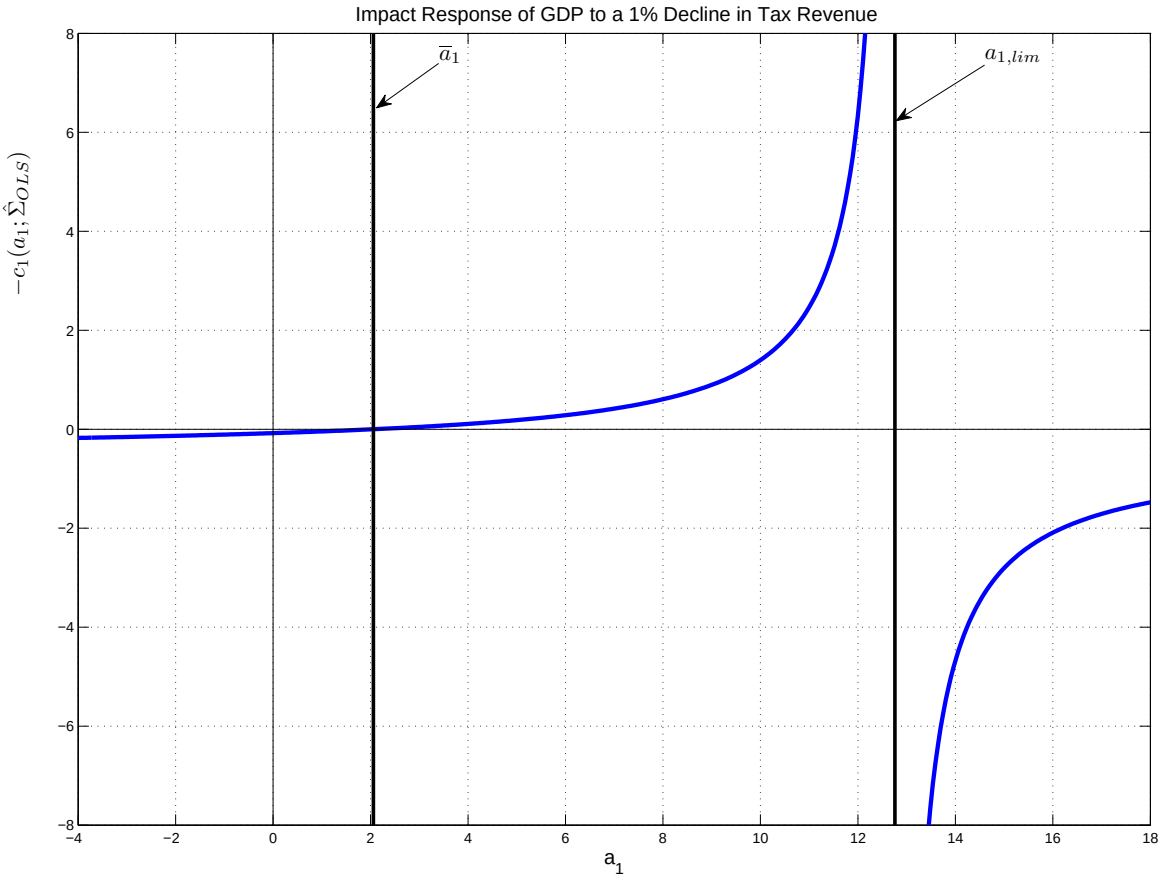
Impact Response of GDP to a 1% Decline in Tax Revenue
$\overline{a}_1$
$a_{1,lim}$
$-c_1(a_1;\hat{\Sigma}_{OLS})$
a_1
8
6
4
2
0
−2
−4
−6
−8
−4
−2
0
2
4
6
8
10
12
14
16
18

Figure 1:

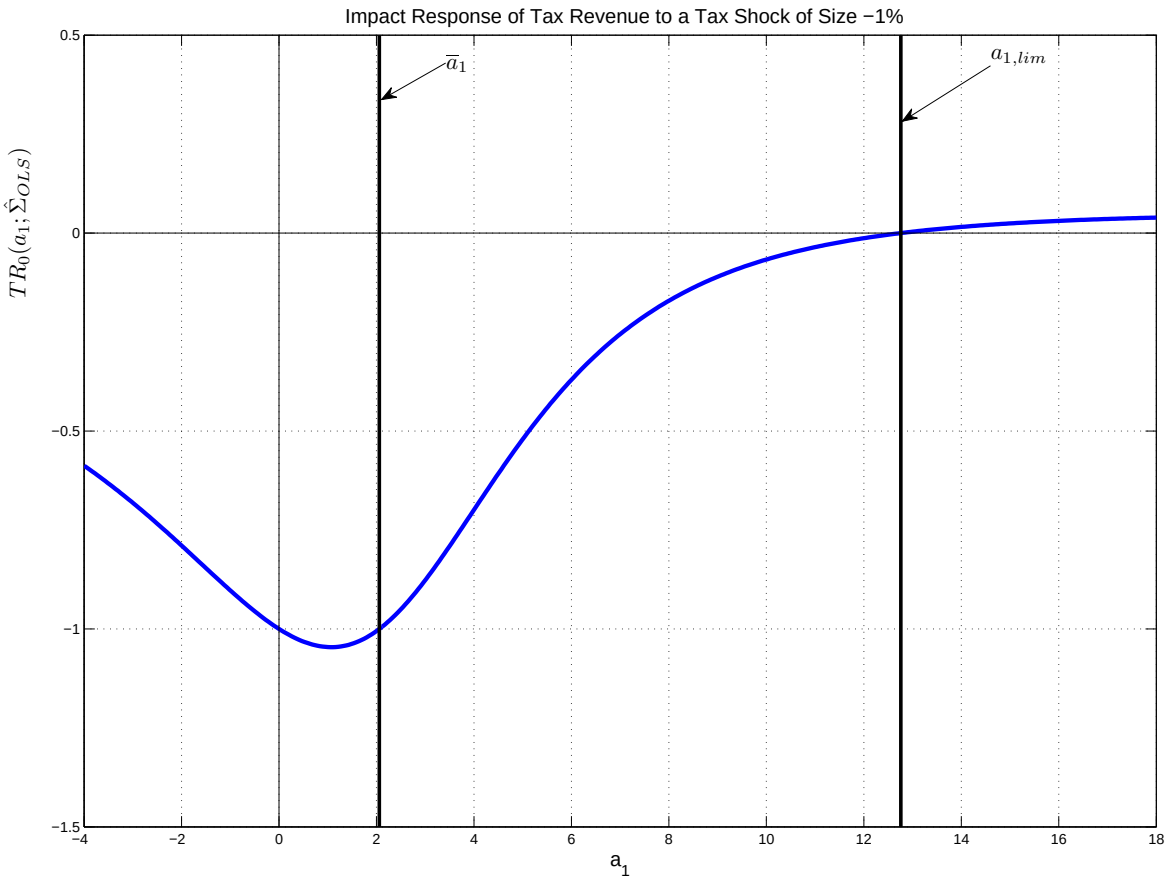
Impact Response of Tax Revenue to a Tax Shock of Size −1%
$\overline{a}_1$
$a_{1,lim}$
$TR_0(a_1;\hat{\Sigma}_{OLS})$
0.5
0
−0.5
−1
−1.5
−4
−2
0
2
4
6
8
10
12
14
16
18
a_1

Figure 2:

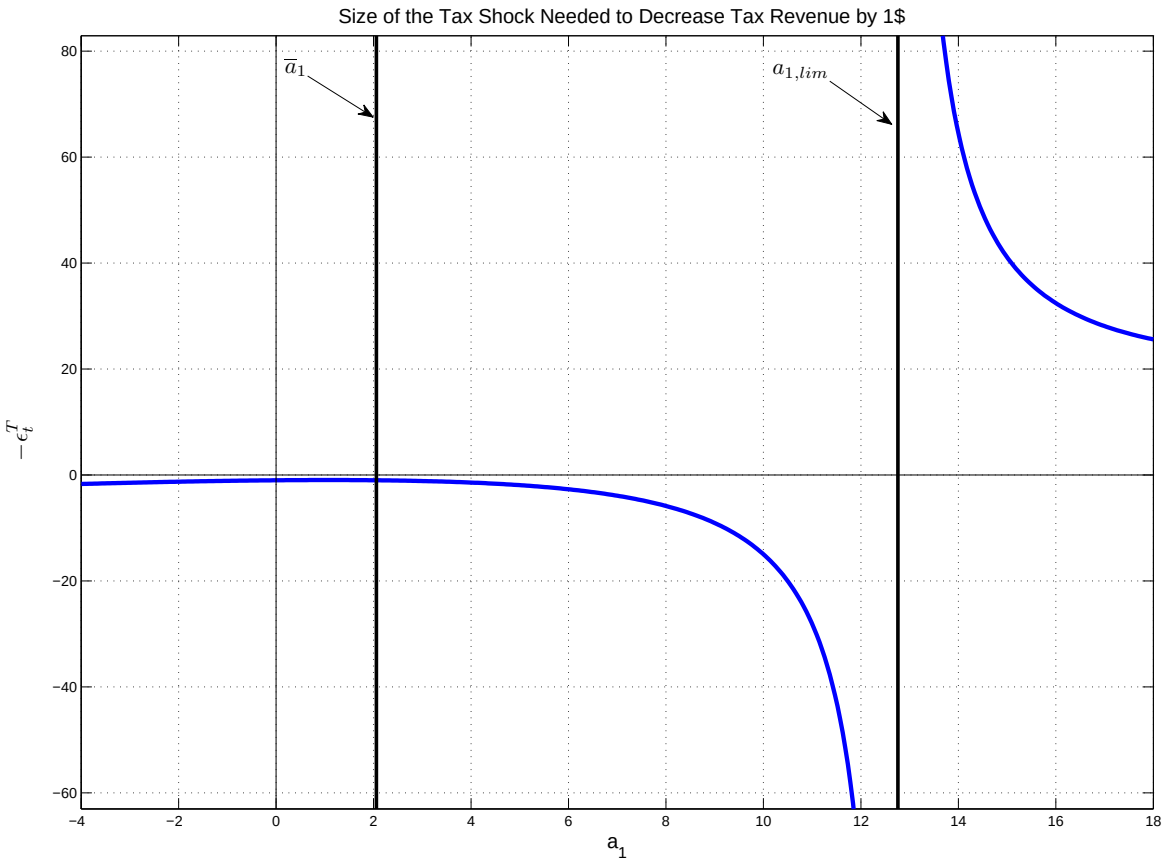
Size of the Tax Shock Needed to Decrease Tax Revenue by 1$
$\overline{a}_1$
$a_{1,lim}$
$-\epsilon_t^T$
a_1
80
60
40
20
0
−20
−40
−60
−4
−2
0
2
4
6
8
10
12
14
16
18

Figure 3:

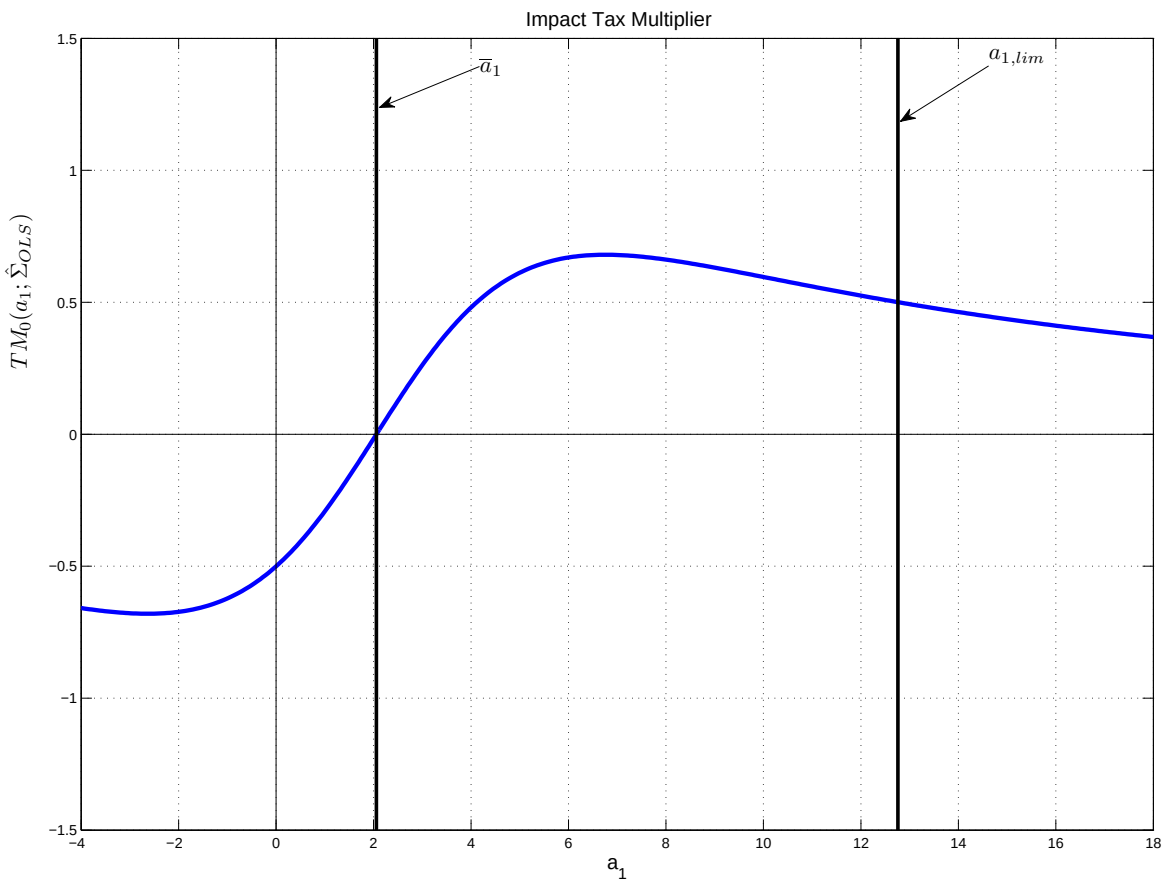
Impact Tax Multiplier
$\overline{a}_1$
$a_{1,lim}$
$TM_0(a_1; \hat{\Sigma}_{OLS})$
a_1
1.5
1
0.5
0
−0.5
−1
−1.5
−4
−2
0
2
4
6
8
10
12
14
16
18

Figure 4:

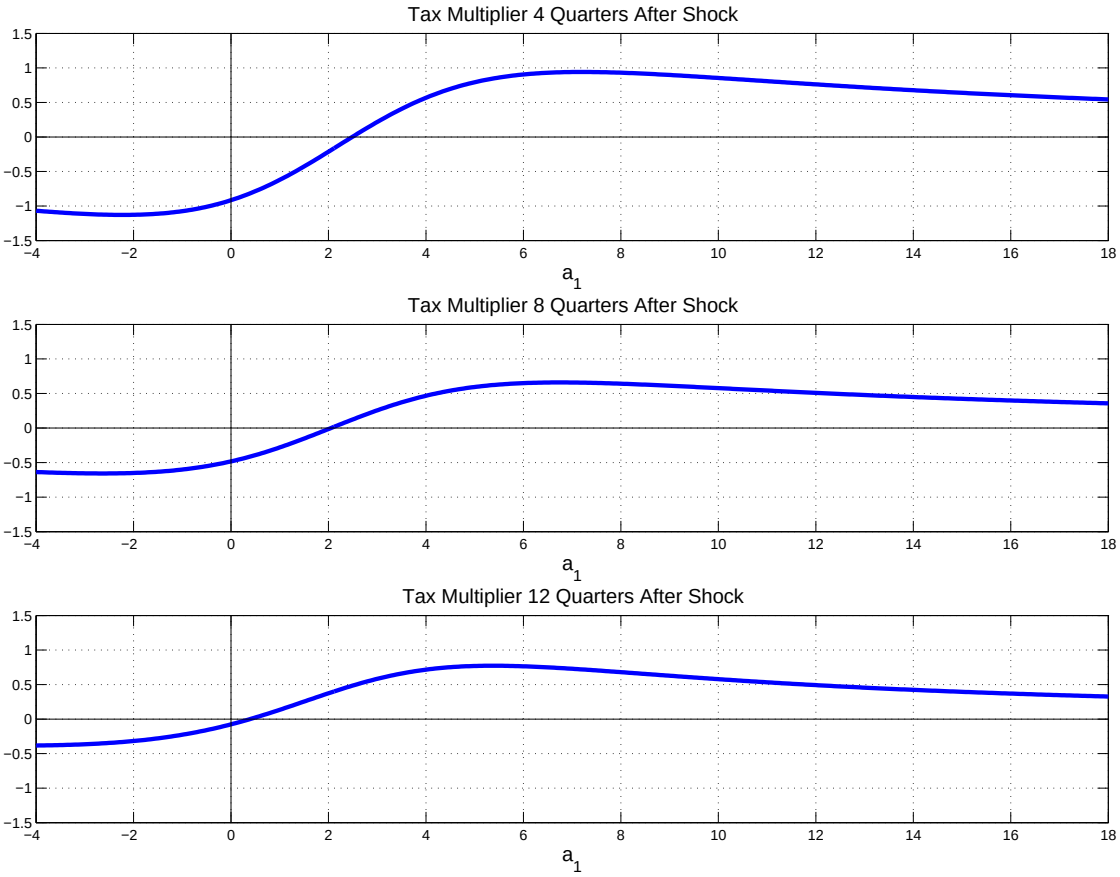
Tax Multiplier 4 Quarters After Shock
1.5
1
0.5
0
−0.5
−1
−1.5
−4
−2
0
2
4
6
8
10
12
14
16
18
a1
Tax Multiplier 8 Quarters After Shock
1.5
1
0.5
0
−0.5
−1
−1.5
−4
−2
0
2
4
6
8
10
12
14
16
18
a1
Tax Multiplier 12 Quarters After Shock
1.5
1
0.5
0
−0.5
−1
−1.5
−4
−2
0
2
4
6
8
10
12
14
16
18
a1

Figure 5:

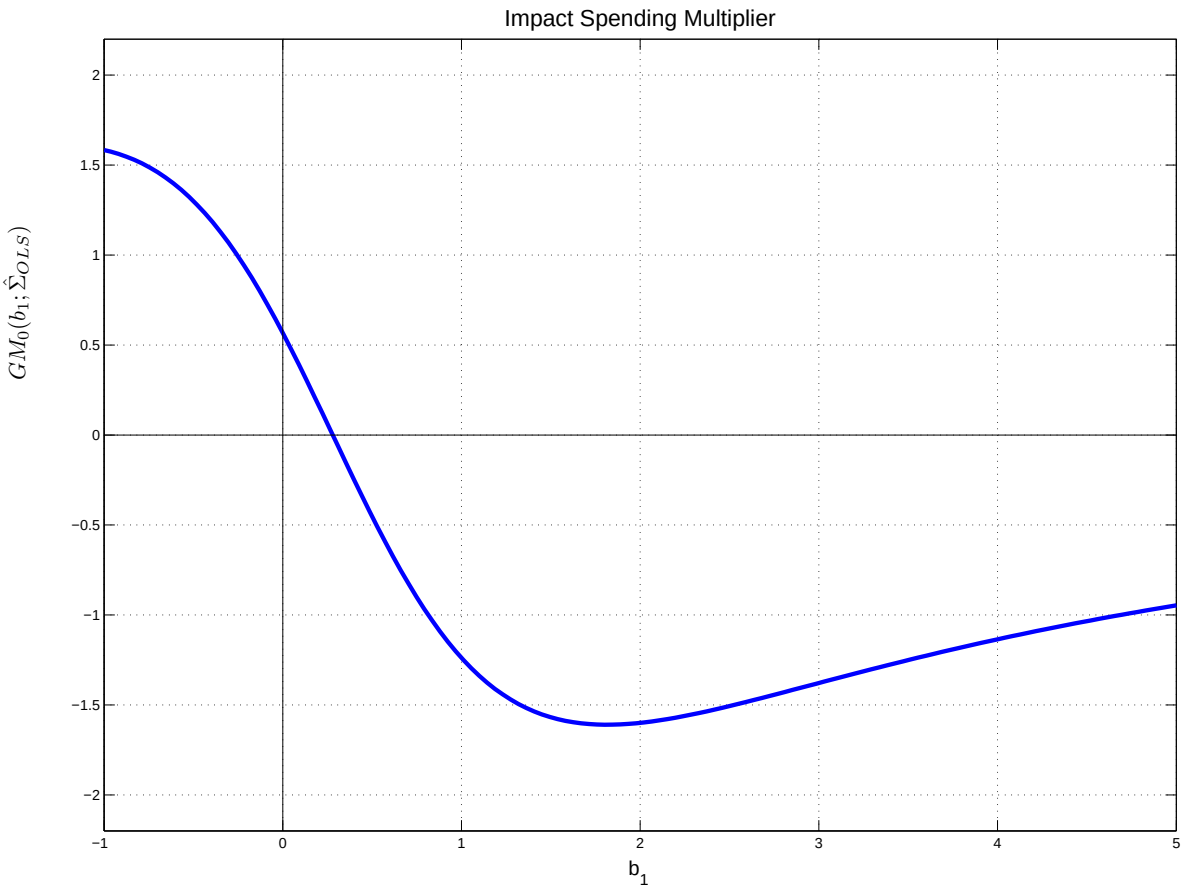
Impact Spending Multiplier
$GM_0(b_1; \hat{\Sigma}_{OLS})$
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−1
0
1
2
3
4
5
b_1

Figure 6:

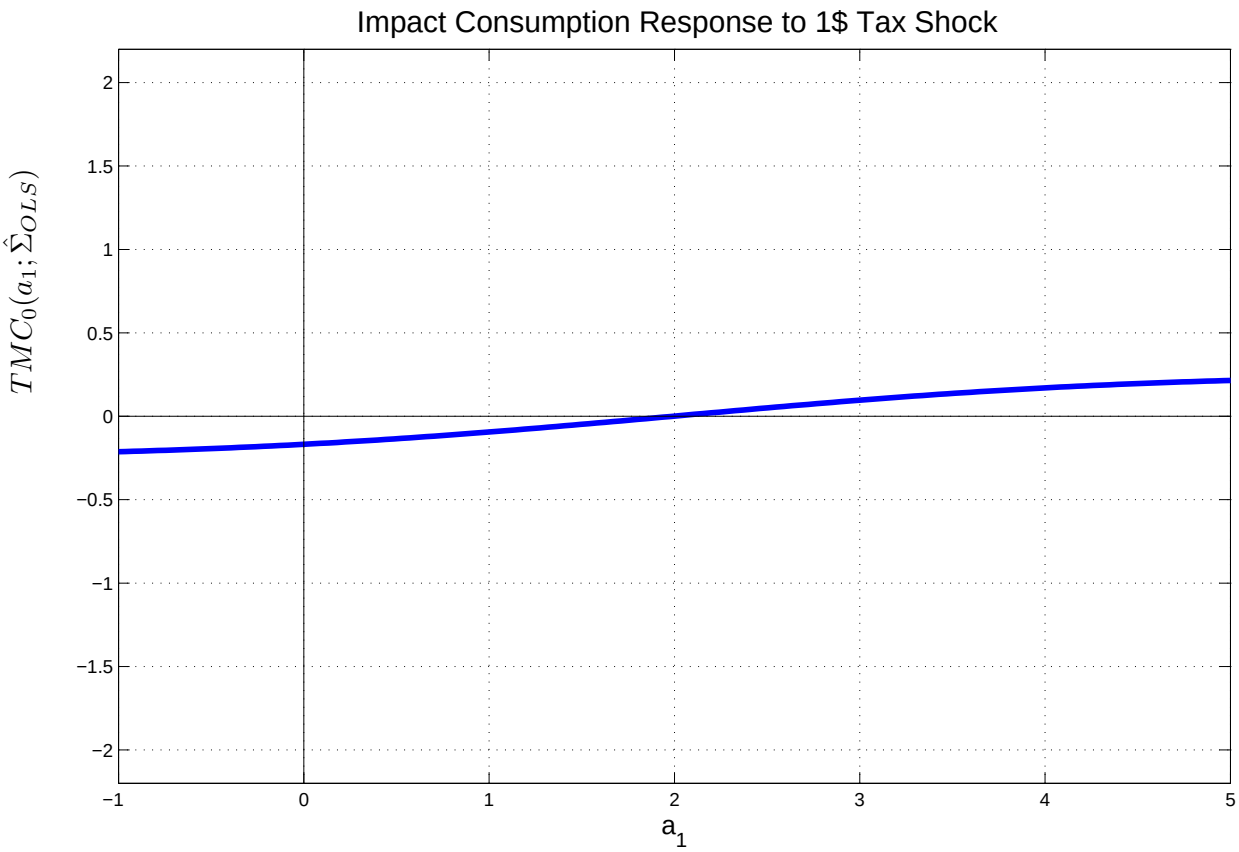
Impact Consumption Response to 1$ Tax Shock
$TMC_0(a_1; \hat{\Sigma}_{OLS})$
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−1
0
1
2
3
4
5
a_1

Figure 7:

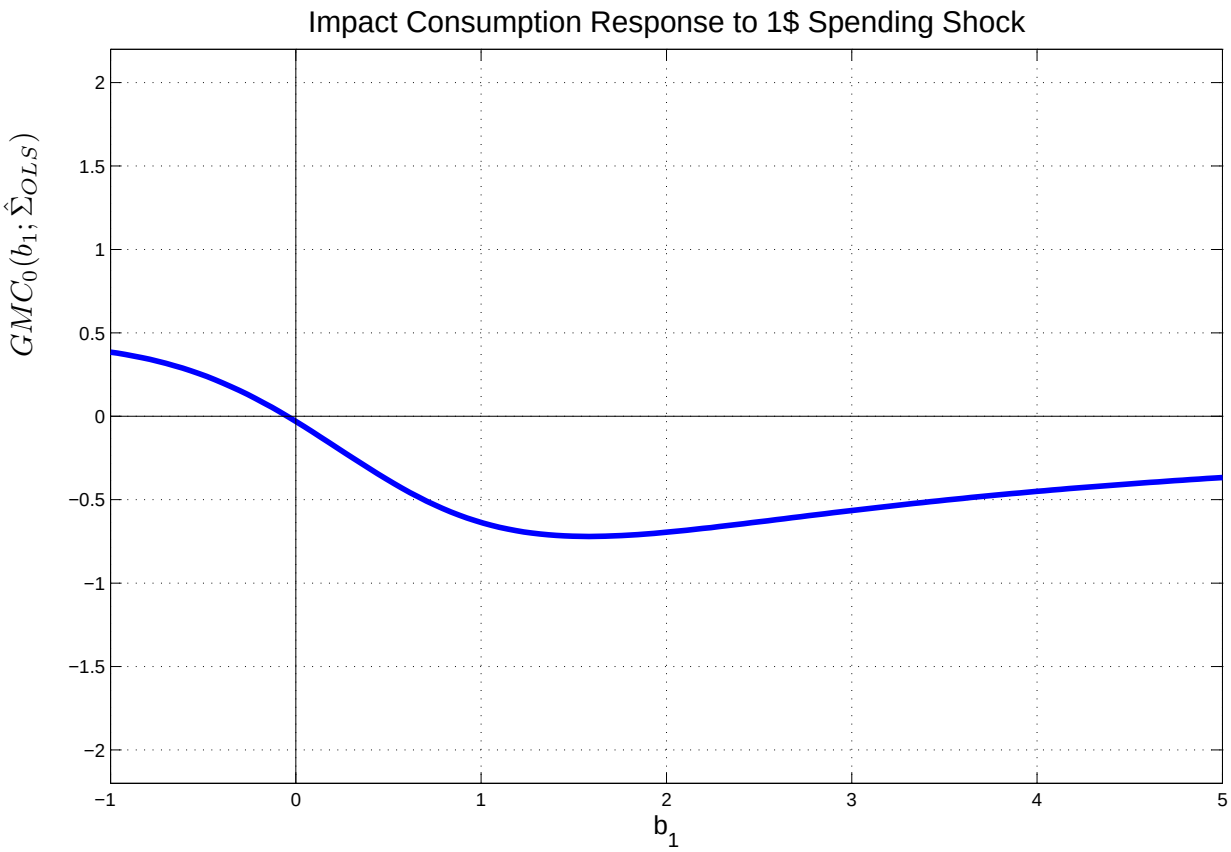
Impact Consumption Response to 1$ Spending Shock
$GMC_0(b_1; \hat{\Sigma}_{OLS})$
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
−1
0
1
2
3
4
5
b_1

Figure 8:

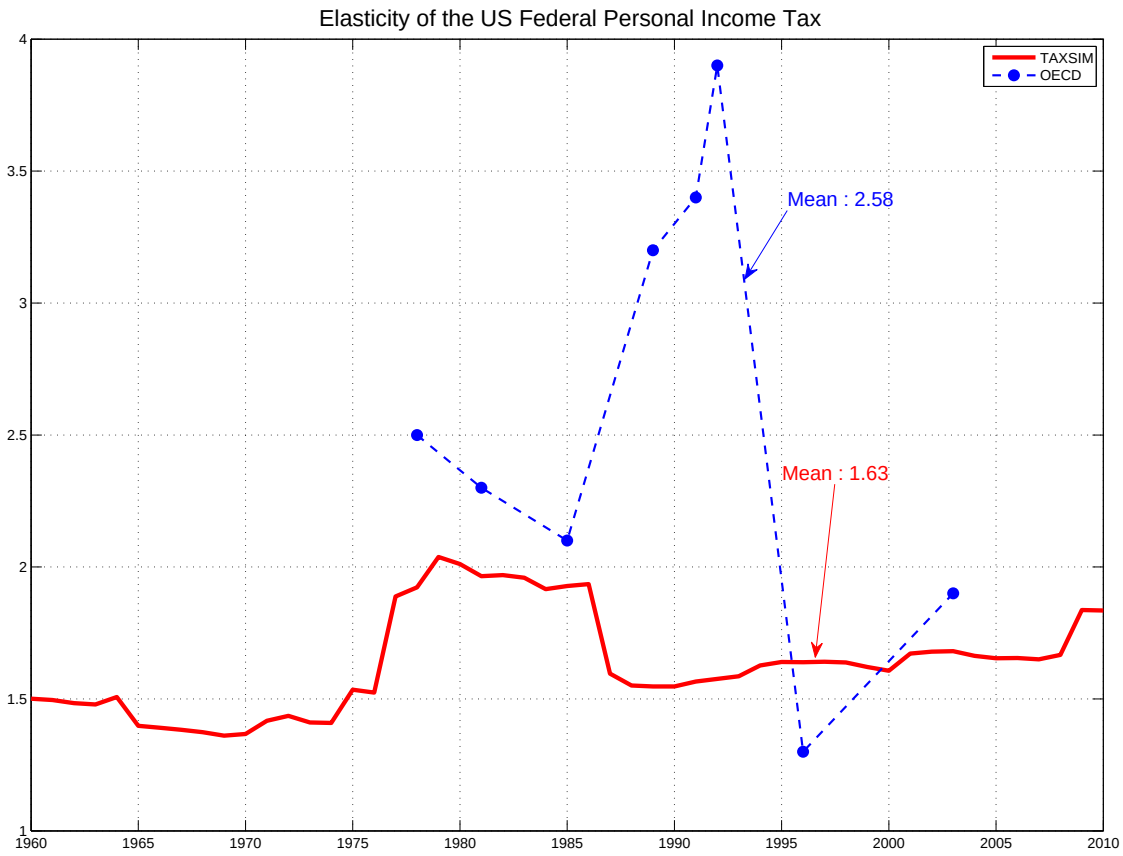
Elasticity of the US Federal Personal Income Tax
TAXSIM
OECD
Mean : 2.58
Mean : 1.63
4
3.5
3
2.5
2
1.5
1
1960
1965
1970
1975
1980
1985
1990
1995
2000
2005
2010

Figure 9: