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The commodity currency puzzle

Abstract:
This paper addresses the purchasing power parity (PPP) puzzle for commodity currencies. A substantial part of the literature on commodity currencies has found that, despite controlling for the effect of commodity prices, PPP does not hold in the long run. We show that once we also control for the effect of the interest rate differential in the real exchange rate relationship, the discrepancies from PPP are fully accounted for. The analysis is applied to the real exchange rate behaviour in Norway, which has a primary commodity (oil) that constitutes the majority of its exports. We show that with the interest rate differential included in the long run real exchange rate relationship, the real oil price plays a minor role. Adjustment to equilibrium (half-lives) is also substantially reduced, taking no more than one year on average. Hence, contrary to earlier findings on commodity currencies, we have effectively removed the PPP puzzle.

Keywords: Exchange rate, commodity currencies, real oil price, purchasing power parity, uncovered interest parity.

JEL classification: C32, F31.

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1. Introduction

Ever since Meese and Rogoff (1983) reported that a comprehensive range of exchange rate models were unable to outperform a random walk, the role of economic fundamentals in explaining exchange rate behaviour has been scrutinized. Economic theory typically predicts that the behaviour of the real exchange rate should be closely related to the behaviour of deviations from purchasing power parity (PPP). However, there seems to be a widespread agreement that substantial deviations from PPP have occurred since the abandonment of the Bretton Woods fixed exchange rate system in 1971. In particular, time series studies for this period have shown that the real exchange rate is not only very volatile in the short run; the speed of convergence to PPP in the long run is extremely slow, (see e.g. Rogoff, 1996; Froot and Rogoff, 1995, for a survey).[1]

Studies spanning longer periods of time (say a century or more), have traditionally found more evidence in favour of PPP than the studies of the post Bretton Woods period. However, many recent studies have found persistent deviations from PPP also in the long run (see e.g. Serletis and Zimonopoulos, 1997; Cuddington and Liang, 2000; Engel, 2000; Rogoff et al., 2001).

The failure to find support for PPP should encourage researchers to construct exchange rate models that investigate the role of economic fundamentals as sources of deviations from PPP. Long run deviation from PPP suggests the influence of real shocks with large permanent effects, (see Rogoff, 1996). The fact that many empirical studies do not reject the hypothesis of a unit root in the real exchange rate, also supports the argument that the variation in the real exchange rate is attributed to permanent shocks.[2] However, up to now it has been difficult to successfully identify real shocks that can explain these permanent deviations from PPP.

One line of research has been to analyse the effect of different real shocks on the terms of trade and thereby the real exchange rate directly. For countries that have a substantial share of commodity exports, the direct effect of commodity price shocks on the terms of trade has been investigated, without much luck. In particular, Chen and Rogoff (2003) show that for three major commodity exporting countries; Australia, Canada and New Zealand, there is still a PPP puzzle, despite controlling for a significant co-movement between world commodity prices and real exchange rates.

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[1] The rejections are less clear-cut using panel data, see e.g. Frankel and Rose (1996) among many others. However, see O'Connell (1998) and Chortareas and Driver (2001) for critical assessments of these panel data studies. See also the recent study by Holmes (2001), who using a new panel data unit root test, finds clear evidence against PPP.

[2] This finding is also consistent with e.g. Mark (1995), who forecasts the nominal exchange rate at long horizons by predicting that it returns to a target level, which, however, is not the PPP value. Further, Mark and Choi (1997) find that models that allow the long-run real exchange rate to vary over time have a better out of sample forecasting properties than models in which long-run PPP holds.

PPP has its roots in goods markets models. Hence, by analysing the effects of the real shocks on the real exchange rate directly, one effectively assumes that all deviations from PPP will work through the goods market. However, an increase in the commodity price that improves the terms of trade and force the real exchange rate away from PPP has to be captured through the movements in interest rates, since they reflect expectations of future purchasing power. Hence, massive movements in capital flows in response to interest rate differentials can keep the exchange rate away from PPP for long periods. A solution to the PPP puzzle for commodity currencies described above could therefore be to investigate a central parity condition for the exchange rate in capital market models, namely that of uncovered interest parity (UIP).[3]

Empirical evidence has generally led to a strong rejection also of the UIP condition in the Post Bretton Woods period (see e.g. McCallum, 1994; Lewis, 1995; and Engel, 1996, for recent surveys). However, as pointed out by Johansen and Juselius (1992) and MacDonald and Marsh (1997), by combining the PPP and the UIP conditions, one could more effectively capture the interactions between the nominal exchange rate, the price differential and the interest rate differential. However, whether the combination of these parity conditions will be sufficient to account for all real shocks, or whether commodity price shocks will have to be investigated separately as a source of deviations from PPP, is a separate question that will be investigated below.

This paper clarifies and calculates the concept of the long run (equilibrium) real exchange rate in a commodity exporting country, by examining different hypothesis related to persistent real exchange rate variation. In particular, we will test whether the direct effects of commodity price changes as well as the interest rate differential are possible explanations of PPP deviations. The analysis is applied to Norway. Since Norway is a dominant oil exporting country, the real oil price is the relevant commodity price to include in the analysis. Oil price shocks are of particular interest for at least two reasons. First, they have historically been very volatile, thereby causing persistent deviations from PPP. In addition they are also thought to affect the terms of trade, see Backus and Crucini (2000).

Previous studies of the determination of the real exchange rate in Norway, have generally rejected the notion of simple PPP using conventional (time series or panel data) unit root tests, see e.g. Serletis and Zimonopoulus (1997), Papell (1997) and Chortareas and Driver (2001)[4] or by testing for PPP in multivariate studies, see e.g. Jore et al. (1998), Alexius (2001) and Bjørnland and Hungnes

[3] A test of UIP, refers to a test of the interest rate differential as an optimal predictor of the rate of depreciation, providing the conditions of rational expectations and risk neutrality are satisfied.

[4] Papell (1997), analysing a series of countries including Norway, cannot reject the unit root hypothesis in the real exchange rate for Norway at the 5% level. However, using a panel of 20 countries, he finds more evidence of PPP. However, again see footnote 1, for a reference to some critical assessments of these panel data studies.

(2003). An exception is Akram (2000, 2002), who, using a multivariate cointegrating framework, find strong evidence of PPP for Norway. [5]

Empirical tests for the UIP condition, also finds little evidence supporting this parity condition for Norway (see e.g. Holden and Vikøren, 1994; Jore et al., 1998; Nessen, 1997; and Flood and Rose, 2001).

The rest of this paper is organised as follows. In Section 2 we discuss the hypothesis of PPP and possible sources of deviations from PPP. Section 3 identifies an econometric model used to estimate the long run real exchange rate behaviour for Norway. In Section 4 we present the empirical results. Section 5 investigates the speed of adjustment to long run equilibrium by calculating half-lives while Section 6 performs some sensitivity analysis. Section 7 summarises and concludes.

2. Fundamentals and long run exchange rates

A natural starting point for discussing the relationship between exchange rates and fundamentals is the concept of PPP. Assuming no costs in international trade, then domestic prices would equal foreign prices multiplied by the exchange rate. The expression for PPP can then be written (in log-form) as

(1) $$v_t = p_t - p_t^*,$$

where p_t is the log of the domestic price, p_t^* is the log of the foreign price, and v_t is the log of the nominal exchange rate. However, since trade is costly, PPP will not hold continuously. It is therefore informative to define the real exchange rate as

(2) $$r_t = v_t - p_t + p_t^*,$$

where r_t is the real exchange rate. If PPP holds, this implies that the real exchange rate is stationary and fluctuates around a fixed value (its mean) in the short run. In a univariate framework, PPP can be tested by simply testing for whether the real exchange rate is stationary or not. Alternatively, PPP can be cast in a multivariate framework by applying cointegration methods. Note that since we will use price indices in the estimation, we can only test relative PPP. However, for the macroeconomic topic of this paper, this is the relevant hypothesis to test.

5 Note, however, that Akram (2000) is close to rejecting the hypothesis of PPP when testing each cointegrating vector separately (keeping the others unrestricted). However, continuing the analysis with a conditional VAR model, he can clearly accept the hypothesis of PPP.

The massive empirical testing of PPP has generally cast doubt on long run PPP, either by rejecting the hypothesis that PPP follows a stationary process, or by suggesting that the real exchange rate adjusts too slowly back to a long run equilibrium rate to be consistent with traditional PPP (the half time is normally found to be 3-4 years, see e.g. Rogoff, 1996).[6] Instead, long run deviation from PPP suggests the influence of real factors with large permanent effects, like commodity prices, productivity differentials, fiscal policy, and other relevant variables, again see Rogoff (1996) for a survey. These factors will work through the current account, and thereby push the real exchange rate away from PPP.

However, as several authors have emphasised, (see e.g. MacDonald and Marsh, 1997; Juselius and MacDonald, 2000), the balance of payment constraint implies that any imbalances in the current account has to be financed through the capital account. Shocks that force the real exchange rate away from PPP has to be captured through the movements in interest rates, since they reflect expectations of future purchasing power. Hence, massive movements in capital flows in response to interest rate differentials can keep the exchange rate away from purchasing power for long periods. The PPP condition in the goods market will therefore be strongly related to the central parity condition in the capital market, namely that of UIP.

This can be formalised in the following way. According to the UIP condition, the interest rate differential will be an optimal predictor of the rate of depreciation, providing the conditions of rational expectations and risk neutrality are satisfied. The expected gain, g^e, from investing money in Norway is given as the deviation from UIP, i.e.

(3) $$g_t^e = i_t - i_t^* - \Delta v_{t+1}^e ,$$

where Δv_{t+1}^e is the expected depreciation rate from period t to t+1, i_t is the domestic interest rate and i_t^* is the foreign interest rate.

Assume that in the long run, the current account (*CA*) depends on the deviation from PPP whereas the capital account (*KA*) depends on the deviation from UIP, that is, deviations from the nominal interest differentials adjusted for expected exchange rate changes (see also MacDonald and Marsh, 1997). The balance of payment then implies that

[6] In a recent study, Murray and Papell (2002) also find the half life of deviations from PPP for each of 20 countries (including Norway) to lie between 3-5 years. However, their confidence intervals are much larger than previously reported, implying in fact that univariate methods provide virtually no information regarding the size of half life.

(4) $$CA_t + KA_t = \gamma\left(v_t + p_t^* - p_t\right) + \lambda\left(i_t - i_t^* - \Delta v_{t+1}^e\right) = 0,$$

where γ captures the elasticity of net exports with respect to competitiveness and λ represents the mobility of international capital (where $\lambda \to \infty$ so that $g_t^e = 0$, if capital is perfectly mobile). Assuming that in equilibrium, $\Delta v_{t+1}^e = 0$, (4) can be solved for the exchange rate to yield a long run equilibrium relationship

(5) $$v_t = p_t - p_t^* - \upsilon\left(i_t - i_t^*\right),$$

where $\upsilon = \lambda/\gamma$. However, as argued above, the assumption that the effect of real shocks (that force the real exchange rate away from PPP), has to be captured through the movements in interest rates in the long run, may be a too strong assumption for countries that has a sizable share of their export sector dominated by commodity exports. For an oil producing country like Norway, the oil price may also have important effects on the real exchange rate that may lead to deviations from PPP not captured by the interest rates in the long run. More specifically, a higher real oil price will increase natural wealth and raise demand. The additional demand can only be met if the relative prices change in favour of foreign goods, so that the currency experiences a real appreciation (see e.g. Corden and Neary, 1982). This may squeeze the tradable sector. To account for the possibility that the Norwegian krone may be a "petrocurrency", which appreciates when the oil price is high and depreciates with a low oil price, we therefore include the real oil price as an additional variable in the model. With this in mind, the equilibrium exchange rate can finally be written as

(6) $$v_t = p_t - p_t^* - \upsilon\left(i_t - i_t^*\right) - \delta op_t,$$

where op_t is the (log of the) real oil price, and a test for $\delta > 0$ is a test whether the Norwegian krone is a "petrocurrency". Equation (6) states that the nominal exchange rate is a function of the price level differential, the interest rate differential and the real oil price. Another way to interpret (6) is that the non-stationarity of the real exchange rate (v_t-p_t+p_t*) can be removed by the non-stationarity of the interest rate differential (i_t-i_t*) and the non-stationarity of the oil price (op_t).

3. The econometric framework

Until early 1990s, tests of PPP were conducted within a single equation framework, either by modelling a univariate process of the real exchange rate or by establishing a dynamic equation between the nominal exchange rate and prices. The former approach implicitly assumes that the variables have common roots, see e.g. Kremers et al. (1992). The latter approach will only yield the best unbiased estimates if the chosen endogenous variable (normally the exchange rate) is the only variable that adjusts back to the long run equilibrium exchange rate level. If, on the other hand, more than one of the variables adjust to the cointegration vector, one will get a more precise estimate of the coefficients in this vector by modelling the variables jointly in a system.

Here we model the whole system jointly within a full information maximum likelihood (FIML) framework, see Johansen (1988). We first define the stochastic vector process as $z_t = \left(v_t, p_t, p_t^*, i_t, i_t^*, op_t\right)'$, where v, p, p^*, i, i^* and op are defined as above. This process can be reparameterised as a vector equilibrium correction model (VEqCM)

$$\Delta z_t = \mu + \Gamma_1 \Delta z_{t-1} + \Gamma_2 \Delta z_{t-2} + \ldots + \Gamma_{k-1} \Delta z_{t-k+1} + \Pi z_{t-1} + \gamma t + \Psi D_t + u_t, \tag{7}$$

where $u_t \sim NID(0, \Sigma)$. μ is a vector of constants, t is a linear trend and D_t is a vector of other deterministic variables, comprising a vector of centred seasonal dummies, S, and a vector of general country specific impulse dummies, D_{CS}, i.e. $D_t = \left(S_t', D_{CS,t}'\right)'$. The null hypothesis of r cointegrating vectors can then be formulated as

$$H_0 : \Pi = \alpha\beta', \tag{8}$$

where α and β are $6 \times r$ matrices of rank r, (r<6), $\beta' z_t$ comprises r cointegrated I(0) relations, and α contains the loading parameters. In the empirical analysis, we will restrict the trend to lie in the cointegrating space, whereas the vectors of constants and seasonal dummies are left unrestricted.[7]

In the subsequent sections we will use the framework in the following way. We first estimate the number of cointegrating relations in a well-specified VEqCM. Thereafter, we test whether

[7] Hence, $\gamma = \alpha\beta_0$ and we can write the cointegrating relationships as $\beta^{*\prime} z_t^{*\prime} = \beta' z_t + \beta_0 t$, where $\beta^* = (\beta', \beta_0)'$ and $z_t^* = (z_t', t)'$.

the following hypotheses on the cointegration vector (where the asterics, *, indicates that the coefficient is not restricted).[8]

I Test of no trend

$\beta' = (1,*,*,*,*,*,0)$

II Test of significance of the oil price in the long run relationship

$\beta' = (1,*,*,*,*,0,0)$

III Test of stationarity of (relative) PPP

$\beta' = (1,-1,1,0,0,0,0)$

IV Test of stationarity of the interest rate differential (implied by the UIP condition)

$\beta' = (0,0,0,a,-a,0,0)$

V Test of PPP augmented by interest rate differential and oil price

$\beta' = (1,-1,1,a,-a,*,0)$

The first hypothesis considers whether the trend can be omitted from the cointegrating space. If that is the case, we will continue the inference on the cointegrating space without incorporating the trend. We thereafter test for the significance of the oil price, stationarity of PPP and the interest rate differential,[9] and finally a combination of these hypotheses.

4. The long run exchange rate relationship[10]

Since the collapse of the Bretton Woods agreement, Norway has until 2001 pursued different exchange rate policies, where the task of maintaining stable exchange rates against a basket of currencies has been at the core. In 2001, however Norway formalised an inflation targeting regime. The characteristics of the different regimes can be summarised as the following:

- 1972-1977; After the collapse of the Bretton Woods agreement Norway took part in the Western European "Snake", which implied stabilising the different currencies to each other.

[8] Hypotheses II-V assume that we can restrict the trend in the cointegrating space to be zero. If not accepted, the restriction will not be incorporated in the subsequent tests.

[9] Note that although we use nominal interest rates in the cointegration analysis, the test results will remain unchanged if we instead had used real interest rates (ri). To see the latter, note the following: If i and i^* were replaced with $ri=i-\Delta p$ and $ri^*=i^*-\Delta p^*$ respectively in the cointegration part (i.e. if $\beta' z$ were replaced with $\beta' \tilde{z}$, where $\tilde{z} = (v, p, p^*, i - \Delta p, i^* - \Delta p^*, op)$), all estimated parameters would be unchanged, except some in the coefficient matrix Γ_1. Since we do not impose restrictions on Γ_1, this will not influence any of the results or tests.

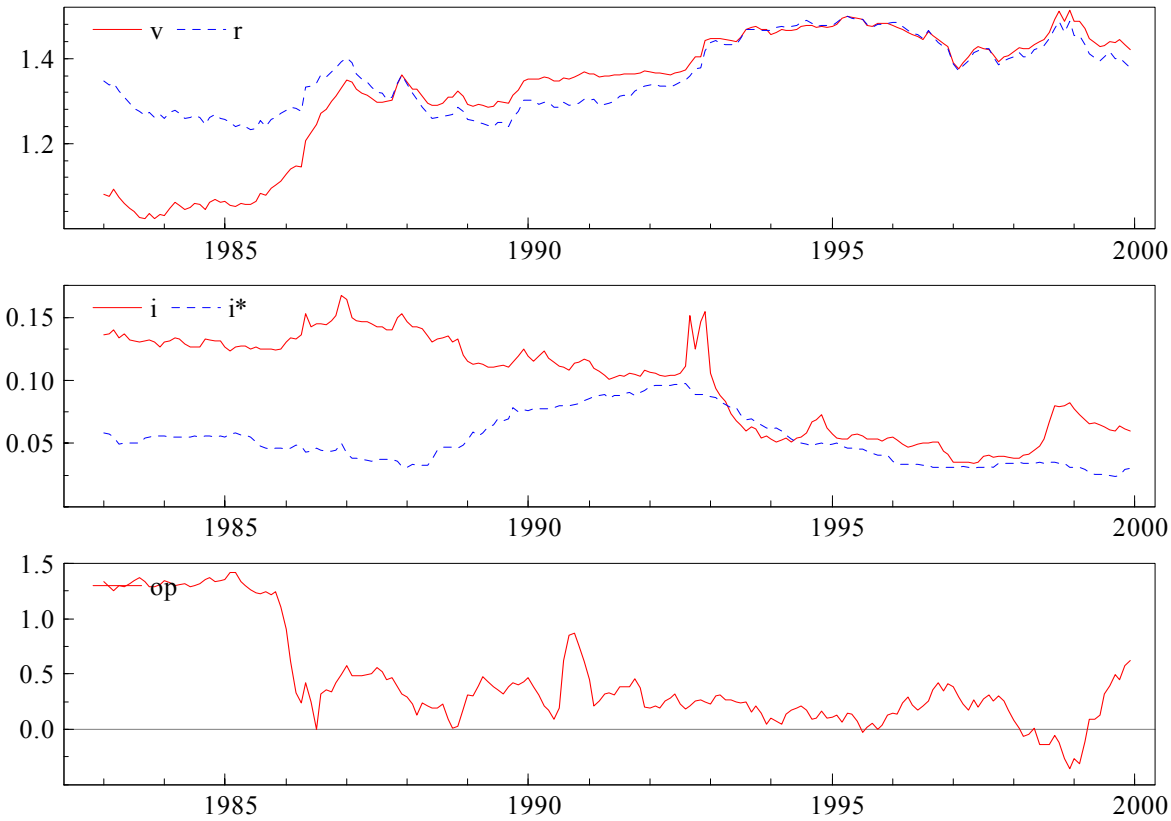
[10] The empirical estimations are conducted using PcGive 9.10 and PcFIML 9.10, see Doornik and Hendry (1996,1997).

- 1978-1990; Norway stabilised the kroner against a basket of currencies, where the weight reflected how much trade Norway had with the different countries. During this period, the Norwegian krone was devaluated a couple of times, most importantly in May 1986 when it was devaluated 12 per cent.
- 1990-1992; Norwegian kroner was stabilised against the European ECU.
- 1993-2001; Due to several speculative attacks, by the end of 1992, the Norwegian government had to let the Norwegian krone float. The floating period was only intended to be temporary. When it turned out to be difficult to return to a normal fixed exchange rate system, Norway formalised the floating regime in May 1994.[11] The Norwegian krone came under appreciation pressure by the end of 1996, and Norges Bank abandoned its attempt to stabilise the krone by intervening in the exchange market in January 1997. From the end of 1997 until August 1998, the Norwegian krone depreciated significantly, leading Norges Bank to increase its interest rates by 4.5 percentage points in that period.
- 2001 to present: Norges Bank adopted an inflation target instead of an exchange rate target. However, this change is not interpreted as a significant change in the monetary policy framework for Norway, since Norges Bank also before 2001 considered low inflation as an aim to stabilise the exchange rate. (Note that this period is not part of the estimation period we apply.)

Due to the various exchange rate systems pursued throughout the sample, we model the exchange rate in Norway against the bilateral German DM. We do this partly because Germany is an important trading partner, and partly because many other of Norway's trading partners indirectly have stabilised their currencies to the German Mark (due to European Monetary System and the Maastricht treaty). However, in the end, we will repeat the procedure by testing for whether the results hold for a basket of trading partners, where the weights of the different countries in the basket are allowed to vary over the years.

[11] Note that, although the exchange rate could now float, Norges Bank should still aim to stabilise the Norwegian krone. However, there were no target zones for the Norwegian krone, and the exchange rate instruction did not explicitly state which currencies the krone should be stabilised against (though it was generally assumed to be the ECU).

Figure 1: Bilateral exchange rate and the interest rates in Norway and Germany[1]



1) Nominal exchange rate (*v*), real exchange rate (*r*), Norwegian interest rate (*i*), German interest rate (*i**) and real oil price (*op*).

Figure 1, upper frame, plots the monthly German DM nominal and real exchange rate, whereas *Figure 1, lower frame* plots the 3-month interest rate in Norway and Germany. The figure shows that the two exchange rate indices follow each other closely; the correlation coefficient is as high as 0.8. This clearly indicates that the nominal and the real exchange rate are not independent of each other in the short run. However, whether they are independent in the long run (as implied by the PPP theory) or whether the theory needs to be modified in a way to explicitly take account of real factors that influence the equilibrium exchange rate, is a question which has to be answered by the econometric analysis below.

Figure 1 also shows that the real exchange rate does not fluctuate around a fixed value, but instead wanders rather wildly. This suggests that the real exchange rate has a non-stationary stochastic trend. Hence, one cannot use the PPP measure to find a base period in which the real exchange rate is believed to be in equilibrium. Testing for non-stationary using the Augmented Dickey Fuller (ADF) unit-root test confirms our suspicion that the real exchange rate in Norway is

characterised by a stochastic trend. In particular, we cannot reject the hypothesis that the real exchange rate is non-stationary (see Table A-1 in Appendix A).

The lower frame of Figure 1 shows that there has been a substantial interest rate differential between Norway and Germany, in particular from 1985 until the early 1990's, and again from 1998 and to 2000. However, to what extent the interest rate differential can explain the non-stationarity of the real exchange rate or whether changes in the oil price are the main driving force, is being discussed more carefully in the econometric analysis below.

The variables used in the econometric analysis are; the nominal exchange rate in Norway measured against German DM (v), home (p) and foreign (p^*) prices measured by CPI, home (i) and foreign (i^*) interest rates and the real oil price (op), (see Appendix A for a further description of data and their sources). We use monthly data, and the estimation period is from 1983M1 to 1999M12. The start date for estimation is set to exclude the turbulence in the international interest rate markets in the early 1980s, which would necessitate a series of intervention dummies (see the discussion in MacDonald and Marsh, 1999). Some extreme observations are nevertheless remaining in the system, mainly due to changes in the exchange rate regimes not accounted for by the model, but also due to two periods of extreme oil price fluctuations; 1986 and 1990-1991. These observations are best represented with dummies (see Appendix A for the list of dummy variables).

In the model specified above, we assumed that all the variables are integrated of first order, I(1). ADF test results show that for neither of the variables can we reject the hypothesis of I(1) in favour of the (trend) stationary alternative. However, we can, for all variables, reject the hypothesis that they are integrated of order two, I(2) (see Tables A-1, A-2).

To decide on the lag length in the VAR model, a battery of lag reduction tests were applied; suggesting a lag order of two[12] Estimating a VAR-model with two lags, a trend and a set of dummies (including centred seasonal dummies), there is no evidence of serial correlation (see Table A-3 in Appendix A for diagnostic tests). The normality hypothesis is rejected, essentially due to excess kurtosis. However, since the properties of the cointegration estimators are more sensitive to deviations from normality due to skewness than to excess kurtosis, we do not find the rejections of normality serious (see Juselius and MacDonald, 2000). Table A-3 also reveals some problems with ARCH for the exchange rate and heteroscedasticity for both the exchange rate and the oil price. Again cointegration test are not very sensitive to these violations, so we ignore them in the following analysis.

[12] A lag length of two may be regarded as short. However, increasing the lag length marginally to three or four lags, does not really change our results.

The cointegration tests are presented in Table A-4. The table reveals clear evidence of one cointegration vector. In the second half of Table A-4, different tests on α and β are conducted under the restriction of there being one cointegration vector. Assuming the rank to be one, we start by testing on β, that is, we test the different (cointegrating) hypotheses reported above. First, the hypothesis of no trend (hypothesis I) can (just) be accepted, so we restrict it to be zero in the remaining analysis. The zero restriction on the oil price (hypotheses II) is rejected, using a significance level of 5 per cent. Further, we can reject the hypothesis of pure PPP (hypothesis IIIa) and a stationary interest rate differential (based on pure UIP) (hypothesis IVa). However, neither of these two hypotheses can be rejected when the rest of parameters on the cointegrating vector are left unrestricted (hypotheses IIIb,c and IVb), implying that PPP and UIP should be combined with the oil price.[13] This is confirmed when testing hypothesis V, where we identify a cointegration vector with PPP augmented with the interest rate differential and the real oil price. The coefficients of this vector have the expected signs. In particular, it implies a negative relationship between the (real) value of Norwegian kroner measured against German DM and the real oil price in the long run, so that a higher oil price appreciates the real exchange rate. Further if the Norwegian interest rate is high relatively to the German interest rate, the equilibrium real exchange rate must appreciate.

Testing zero restrictions on α implies testing for weak exogeneity. A variable is weakly exogenous with respect to the long run parameters if it is not adjusting to the long run equilibrium. Testing for weak exogeneity shows that we can reject the null hypothesis of weak exogeneity for the exchange rate and Norwegian prices. For the other variables, the null hypothesis of weak exogeneity cannot be rejected when tested individually, although weak exogeneity of German interest rates are close to being rejected. Testing restrictions on α and β jointly (hypothesis V & H), only German prices, the Norwegian interest rate and the oil price turns out to be weakly exogenous. Equation (9) reports the estimated long run exchange rate relationship (standard errors in parenthesis)

$$v = p - p^* - \underset{(0.250)}{1.166}(i - i^*) - \underset{(0.015)}{0.083}\, op \,. \tag{9}$$

Equation (9) implies that although PPP is not by itself a stationary process, the real exchange rate cointegrates with the interest rate differential and oil price, so the residuals from the estimation are stationary. Hence, the long-run interactions between the goods and capital markets cannot be ignored. However, the fact that the oil price also enters explicitly into the cointegrating relationship, may

[13] Note that the hypothesis of augmented PPP is only accepted when we allow for a trend. However, when all the relevant hypotheses are tested jointly, the hypotheses are accepted independent on the inclusion of trend or not.

emphasise that any long run effect that the oil prices may have on the exchange rate, is not captured by the interest rate differential. The coefficient on the oil price suggests that a 10 per cent change in the oil price implies approximately 1 per cent change in the exchange rate. The coefficient on the interest rate differential is close to one, and implies that a one percentage point increase in the interest rate differential will lead to a one per cent appreciation in the exchange rate.

How do these results compare to previous studies of the determination of the real exchange rate in Norway? As suggested above, many empirical studies have rejected PPP using conventional unit root tests or by testing for PPP in multivariate studies. An exception is Akram (2000, 2002), who, using a multivariate cointegrating framework, finds strong evidence of PPP for Norway. Although the model by Akram differs from ours with respect to focus and choice of variables, the most notably difference is the sample period. In particular, Akram includes the 1970s in his analysis, but ends the period before we do, in 1997. It is reasonable to argue that in the period we have focused on, the interest rate differential plays a more important role for monetary policy, compared to the fixed exchange rate regime of the 1970s. This point is emphasised further in Naug (2003), who finds the interest rate differential to matter substantially for the period 1999-2003. Bjørnland and Hungnes (2003) also show that in a forecasting perspective, only a model that augments PPP with the interest rate differential in the long run, can beat a random walk.

5. Half-lives; Removal of the PPP puzzle

Having identified the long run relationship between the real exchange rate, the interest rate differential and real oil prices, we need to assess the speed of adjustment around equilibrium. A useful indicator of the speed of adjustment is the so-called half-live indicator. More specifically, assuming a system is hit by a shock, a half-live indicator expresses the time it takes for the process to correct for half of the magnitude of the shock. In a univariate process estimated with one lag, the half-live can be expressed by the adjustment parameter. If for instance the univariate process is expressed as $\Delta x_t = \rho x_{t-1} + u_t$, the half-life is given by $\log(1/2)/\log(1+\rho)$. The corresponding expression for a multivariate process with one lag and one cointegrating vector is equal to $\log(1/2)/\log(1+\beta'\alpha)$. However, our system is estimated using two lags, so applying the formula above would give wrong estimates. In particular, had we ignored the second lag, the formula above would yield an estimate for the half-live of a shock to the cointegrating relationship at 18.5 months. However, in the multivariate case with more than one lag, the half-live indicator will depend on the type of shock hitting the system (i.e. in which error is shocked). Half-lives can therefore instead be found by applying impulse responses of the effect of shocks on the cointegrating vector in the system.

In order to calculate half-lives, the deterministic variables in the system can be ignored. Furthermore, we need to augment the VAR model with the definitional relationship: $\beta' z_t - \beta' \Delta z_t = \beta' z_{t-1}$. This yields

$$(10) \qquad \begin{bmatrix} I_6 & 0_{6\times1} \\ -\beta' & 1 \end{bmatrix} \begin{bmatrix} \Delta z_t \\ \beta' z_t \end{bmatrix} = \begin{bmatrix} \Gamma_1 & \alpha \\ 0_{1\times6} & 1 \end{bmatrix} \begin{bmatrix} \Delta z_{t-1} \\ \beta' z_{t-1} \end{bmatrix} + \begin{bmatrix} u_t \\ 0 \end{bmatrix}.$$

This relationship holds for all *t*. Furthermore (assuming all future errors to be zero) the effect of a shock in period *t* can be expressed as (see Appendix B for further elaborations)

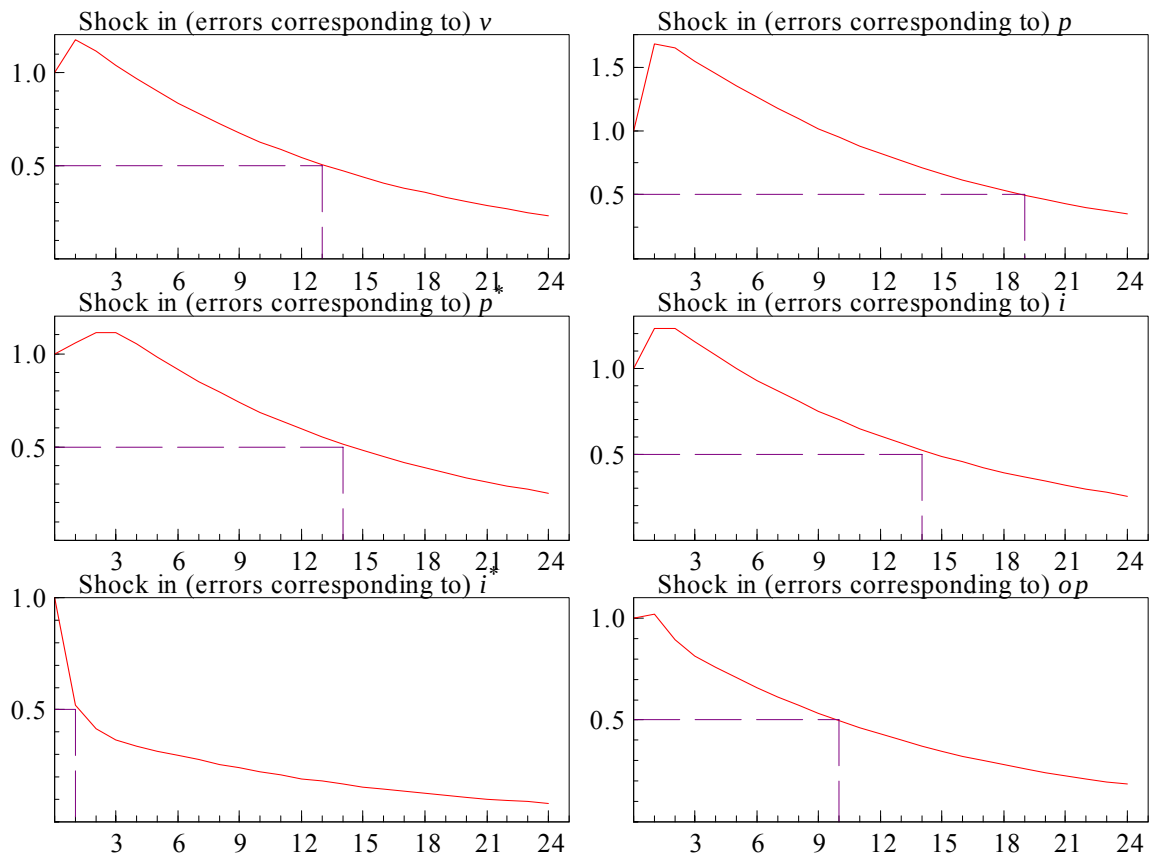
$$(11) \qquad \begin{bmatrix} \Delta z_{t+s} \\ \beta' z_{t+s} \end{bmatrix} = \left(\begin{bmatrix} I_6 & 0_{6\times1} \\ -\beta' & 1 \end{bmatrix}^{-1} \begin{bmatrix} \Gamma_1 & \alpha \\ 0_{1\times6} & 1 \end{bmatrix} \right)^s \begin{bmatrix} I_6 & 0_{6\times1} \\ -\beta' & 1 \end{bmatrix}^{-1} \begin{bmatrix} u_t \\ 0 \end{bmatrix}.$$

In the analysis we focus on the effect from the shocks on the cointegrating relationship $\beta' z_t$, as we are not interested in the effect on the individual variables. All shocks are normalized to be unity in the period they take place (i.e. $\beta' z_t = 1$). The shock in the error corresponding to exchange rate (*v*) can then be given by $u_t = [1,0,0,0,0,0]$, and similarly for the other shocks. Note that as our system is formulated on reduced form, hence we do not give any economic interpretation of these shocks. Figure 2 shows the impulse responses on the cointegrating vector of the different shocks. The graphs are generated with Ox Professional 3.4, see Doornik (2001).

For all shocks except a shock in the error corresponding to the German interest rate, half-lives are between 1 and 1½ year, which is much faster than the speed of convergence to PPP that has been found in the literature by e.g. Rogoff (1996). The half-live for a shock in the German interest rate is only one month. This is mostly due to the rapid adjustment in the Norwegian interest rate. When the German interest rate increases with a percentage point, the Norwegian interest rate increases by 0.45 percentage points the following month.

Figure 2 also shows that all shocks, except the shock corresponding to the German interest rate, overshoots. Cheung and Lai (2000) have argued that with such non-monotonic adjustment, much of the estimation of the half-lives are overstated in the literature. They argue that rather than calculating the half-lives after the initial shock, one should instead calculate the half-live after the responses to shocks have reached their maximum level. If we do this, half-lives are reduced even further, to an average of 9-11 months (except for the shock in the error to the German interest rate, which still has a half-live of 1 month).

Figure 2: Impulse responses measuring half-lives to different shocks



It is interesting to compare these results with those of Akram (2002), who also calculates half-lives from a cointegrating relationship for Norwegian bilateral exchange rates in a uni-variate framework (using the real exchange rate). Akram (2002) finds the half- lives of the bilateral real exchange rate versus German DM to be approximately 11 quarters, which is more than twice the time it takes the exchange rate in our model to adjust to equilibrium. Hence, by including the interest rate differential in the long run cointegrating relationship, we have effectively shown that half-lives are reduced substantially, so that in effect we have removed any purchasing power parity puzzle.

6. Robustness of the results - Trading partners

In this section we perform some alternative specifications of the model, to check the robustness of our reported results. In particular, we replace the bilateral exchange rate with a basket of trading partners, and, eventually using a different model for oil prices.

Trading partners

In the following, the German exchange rate and interest rate are replaced by the equivalent variables from a basket of trading partners, where the weights of each country in the basket are allowed to vary over time (see Bjørnland and Hungnes, 2002, for calculation). Estimating a VAR with two lags, the cointegration tests indicate one cointegration vector at the 1 per cent significance level. Testing restrictions on the parameters on this cointegrating vector, we find that we can reject the long run hypotheses of pure PPP and interest rate differential. In addition, we now also find that the oil price can be excluded from the cointegration vector. Finally, we therefore identify a cointegration vector with PPP augmented with the interest rate differential. Hypotheses of weak exogeneity for the nominal exchange rate, domestic and foreign prices are rejected at a 5 per cent level. The estimated long run exchange rate relation is reported in Equation (12) (standard error in parenthesis)

$$v = p - p^* - \underset{(0.164)}{0.685}(i - i^*) \,. \tag{12}$$

The coefficient on the interest rate differential is significant and still close to one, hence the long-run interactions between the goods and capital markets cannot be ignored. However, oil prices do not enter the cointegrating relationship explicitly, emphasising that any long run effect that the oil prices may have on the exchange rate, is already captured by the interest rate differential. One reason for this could be the fact that the different trading partners respond differently to oil price changes, thereby on average cancelling out any effects. In particular, they may respond asymmetrically to oil price increases and decreases, thereby confounding the true responses (see also Akram, 2004). This is investigated further below.

Oil price and asymmetry

Above we have assumed there are symmetric and linear effects of the oil price on the exchange rate (or between any of the variables). However, Mork et al. (1994) have shown that there are important asymmetries between the effects of oil price increases and decreases on the US economy. More recently, Hooker (1996) has argued that the relationship between the macroeconomy and oil has decreased dramatically in the US since 1973. However, contrary to the results obtained by Mork et al. (1994), Hooker finds that re-specifying the VAR according to asymmetry theories does not restore the oil macroeconomic relationship.

Hamilton (1996), on the other hand, argues that as most of the increases in the price of oil since 1986 have followed immediately after even larger decreases, they are corrections to the previous decline rather than increases from a stable environment. To correctly measure the effect of oil price

increases on the macroeconomy, he suggests that one should compare the price of oil with where it has been over the previous year, rather than with where it was the previous quarter (or month) alone. By constructing what he refers to as the net oil price (the maximum value of the oil price observed during the preceding year), Hamilton (1996) shows that the historical correlation between oil price shocks and the macroeconomy remains.

Below we use a methodology similar to that described in Hamilton (1996) and replace the oil price with a net oil price. We continue using the model where the exchange rate is measured against the trading partners. Assuming one cointegrating vector, the results still suggest that we can identify a long run relationship between the real exchange rate and the interest rate differential. However, there are now some more evidence of a second cointegrating vector, and restricting both in a valid way, we can identify a long run relationship between the exchange rate and the oil price (and a trend), in addition to our originally identified vector (the oil price does not enter significantly there). However, the effect of the oil price is small, but it enters with the right sign and similar in magnitude to that identified for Germany. Overall then, allowing for a non-linear relationship between the exchange rate and the oil price, there seems to be somewhat more evidence that the oil price will affect the nominal exchange rate in the long run (see also Akram, 2004, for a non-linear analysis of the effects of the changes in the oil price).

7. Conclusions

This paper has set out to solve the purchasing power parity puzzle for commodity currencies. A substantial part of the literature on commodity currencies has found that despite controlling for the effect of commodity export prices on the floating real exchange rate, PPP does not hold in the long run. We argue that the main reason for this puzzle is that by focusing on deviations from PPP alone, one ignores the link to the capital account. In particular, an increase in the commodity price that improves the terms of trade and force the real exchange rate away from PPP has to be captured through the movements in interest rates, since they reflect expectations of future purchasing power. Hence, massive movements in capital flows in response to interest rate differentials can keep the exchange rate away from purchasing power parity for long periods. A solution to the PPP puzzle above could therefore be to investigate a central parity condition for the exchange rate in capital market models, namely that of uncovered interest parity, in conjunction with PPP.

The analysis is applied to the real exchange rate behaviour in Norway, which has a primary commodity (oil) that constitutes the majority of its export. We show that despite controlling for the effect of the commodity export price on the real exchange rate, PPP does not hold in the long run. However, when we also allow the interest rate differential to enter the relationship, the real

exchange rate is effectively made stationary. The long run relationship is consistent with a synthesis of PPP and UIP. We show that once the interest rate differential is allowed to matter, the real oil price plays only a minor role in the long run real exchange rate relationship, although the sign of the effect is as expected. In particular, the Norwegian currency can be characterised as a petro-currency, which appreciates when the oil price increases and depreciates when the oil price falls. We further show that adjustment to shocks from the equilibrium relationship is fast, taking no more than one year on average. Hence, contrary to earlier findings on commodity currencies, we have effectively removed the purchasing power parity puzzle.

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Appendix A

The data set

Data sources

All the time series, except those for the oil price and German CPI, are taken from the International Financial Statistics (IFS) from International Monetary Fund (IMF). The nominal oil price is taken from Norges Bank and German CPI is taken from OECD**.** More details on the data can be found in Bjørnland and Hungnes (2002).

The dummies used in the analysis are

$$D_{G,t} = \left(d84m7_t, d84m9_t, d86m5_t, d86_t, d9091_t, d9293_t, Dfloat_t, d97m1_t, dger91_t, dger93_t\right)'$$

- *d84m7*, *d84m9* and *d86m5* are impulse dummies that are unity in one month and zero otherwise, included to take account of different changes in the Norwegian exchange rate regime. In July 1984, the basket of currencies that Norway was holding the krone fixed against changed, from being calculated as an arimetic average to a geometric average (involving a devaluation of 2 per cent). In September 1984 it was decided temporarily to hold the exchange rate index 2 per cent higher than the centre of the exchange band (involving a devaluation of 2 per cent). In May 1986 the value of the Norwegian krone was depreciated with about 10 per cent, due to the fall in the oil price and increased labour cost in production.
- *d86* and *d9091* capture extreme fluctuations in the oil price. *d86* equals 1 from December 1985 to April 1986, and picks up the extreme decrease in the oil price in this period. *d9091* equals 1 in August 1990 and -1 in January and February 1991 and captures the fluctuations in the oil price due to the Golf war.
- *d9293* equals 1 in September and November 1992, -1 in October 1992 and January and February 1993, and zero otherwise. This dummy controls for the speculations against the ERM system in the second half of 1992 (see Figure 2).
- *Dfloat* is a step dummy that equals 1 from December 1992, zero otherwise. It picks up the change to a floating exchange rate regime.
- *d97m1* controls for the appreciation pressure against the Norwegian krone in January 1997. This dummy equals unity in January 1997 and zero otherwise.
- *dger91* equals 1 in July and October 1991, and controls for fluctuations in the German CPI and interest rate. (Probably repercussions of the reunion in 1990.)
- *dger93* equals 1 in January 1993, and zero otherwise, and is probably picking up after-effects of the speculations against ERM in the second half of 1992.
- When modelling the exchange rate against Norway's trading partners, the dummy *dtp92* is included (instead of *dger91* and *dger93*). This dummy equals 1 in August and September 1992, -1 in October and November 1992 and zero otherwise. This dummy is included to control for speculations against the ERM system.

Unit root test

Table A-1: Unit root tests

	Norway	Trading Partners	Germany
Without trend. Critical values: 5%=2.88, 1%=3.46.			
v		1.84	1.71
r		2.39	1.27
i	1.07	0.23	0.55
*i-i**		1.87	1.30
With trend. Critical vales: 5%=3.43, 1%=4.01			
p	1.93	0.06	0.89
op	2.22		

Table A-2: Unit root tests

	Norway	Trading Partners	Germany
Without trend. Critical values: 5%=2.88, 1%=3.46.			
Δv		8.29**	7.26**
Δr		8.93**	7.80**
Δi	10.4**	9.66**	8.53**
Δ(i-i)*		10.6**	9.97**
Δp	5.82**	5.59**	7.99**
Δop	9.04**		

(**) indicates rejection at the 1 per cent significance level.

Table A-3: Diagnostic tests[a,b)]

		v	*p*	*p**	*i*	*i**	*op*
AR 1-7	F(7,162)	1.31 [0.25]	1.68 [0.12]	1.70 [0.11]	0.94 [0.48]	1.34 [0.23]	0.53 [0.81]
Norm	X2(2)	22.1 [0.00]**	34.0 [0.00]**	18.1 [0.00]**	30.7 [0.00]**	24.8 [0.00]**	36.4 [0.00]**
Skewness		0.23	0.01	0.44	0.75	0.23	0.32
Exc. kurt.		1.78	2.28	1.72	3.02	1.93	1.93
ARCH 7	F(7,155)	1.67 [0.12]	0.27 [0.96]	1.46 [0.19]	0.72 [0.66]	2.37 [0.02]*	1.67 [0.12]
Het	F(26,142)	2.11 [0.00]**	1.17 [0.28]	1.11 [0.34]	1.40 [0.11]	1.02 [0.44]	2.39 [0.00]**
System:	AR 1-7 F(252,734)=1.36 [0.00]**		Norm X2(12)=162.5 [0.00]**,			Het F(546,2085)=1.121 [0.04]*	

a) p = 2, b) AR 1-7 is Harvey's (1981) test of residual autocorrelation up to order 7; NORM is the normality test described in Doornik and Hansen (1994); ARCH is the Engle (1982) test for autoregressive conditional heteroscedasticity up to order 7 in the residuals; and Het is a test for residual heteroscedasticity due to White (1980). (*) denotes rejection at the 5 per cent significance level while (**) indicates rejection at the 1 per cent level.

Table A-4: Cointegrating tests

Cointegrating rank tests

H0:rank=r	eigenvalue λ_i	λ-max[a)]	λ-max adj.	95%	λ-trace[b)]	λ-trace adj.	95%
r=0	0.308	75.11**	70.69**	44.0	158.1**	148.8**	114.9
r≤1	0.171	38.23*	35.98	37.5	83.02	78.14	87.3
r≤2	0.133	29.04	27.33	31.5	44.79	42.15	63.0
r≤3	0.045	9.30	8.75	25.5	15.75	14.82	42.4
r≤4	0.022	4.55	4.28	19.0	6.49	6.07	25.3
r≤5	0.009	1.90	1.79	12.3	1.90	1.79	12.3

Unrestricted α and β

	v	*p*	*p**	*i*	*i**	*op*	Trend[c)]
α	-0.046	0.019	-0.005	0.004	-0.008	0.117	---
β	1	-1.341	0.540	1.069	-1.032	0.072	2.326

Cointegrating vectors

Number	Interpretation	Restrictions, β	Estimated β	LR prob
I	No trend	(1, *,*,*, *,*,0)	(1,-1.049,1.498,1.648,-1.944,0.090,0)	3.39 [0.07]
II	No oil/trend	(1, *,*,*, *,0,0)	(1,-1.509,2.124,2.316,-2.655,0 ,0)	7.74 [0.02]*
IIIa	Pure PPP	(1,-1,1,0,0,0,0)	(1,-1 ,1 ,0 , 0 ,0 ,0)	54.6 [0.00]**
IIIb	Augm. PPP, no trend	(1,-1,1,*, *,*,0)	(1,-1 ,1 ,1.154,-1,685,0,074,0)	8.85 [0.03]*
IIIc	Augmented PPP	(1,-1,1,*, *,*,*)	(1,-1. ,1 ,1.315,-1.596,0.105,0.717)	2.48 [0.29]
IVa	Interest rate differential	(0, 0,0,1,-1,0,0)	(0, 0 , 0 ,0 , 1 ,-1 ,0)	28.6 [0.00]**
IVb	Augm. UIP, no trend	(1, *,*,a,-a,*,0)	(1,-1.064,1.534,1.833,-1.833,0.091,0)	3.55 [0.17]
V	PPP, UIP, no trend	(1,-1,1,a,-a,*,0)	(1,-1 ,1 ,1.435,-1.435,0.079,0)	9.33 [0.05]

Weak exogenity

Number	Variable	Restrictions, α	LR prob
A	*v*	(0,*,*,*,*,*)	6.08 [0.01]*
B	*p*	(*,0,*,*,*,*)	21.0 [0.00]**
C	*p**	(*,*,0,*,*,*)	2.94 [0.09]
D	*i*	(*,*,*,0,*,*)	0.41 [0.52]
E	*i**	(*,*,*,*,0,*)	3.75 [0.05]
F	*op*	(*,*,*,*,*,0)	0.89 [0.34]
G=D+F	*i* and *op*	(*,*,*,0,*,0)	1.39 [0.50]
H=C+D+F	*p**, *i* and *op*	(*,*,0,0,*,0)	5.14 [0.16]
J=D+E+F	*i*, *i** and *op*	(*,*,*,0,0,0)	5.38 [0.15]
K=C+D+E+F	*p**, *i*, *i** and *op*	(*,*,0,0,0,0)	13.3 [0.01]*

Joint tests of α and β

Comb.	Estimated α	Estimated β	LR prob
V & H	(-0.032,0.019, 0 ,0,-0.012,0)	(1,-1 ,1 ,1.166,-1.166,0.083,0)	11.6 [0.11]
V & J	(-0.021,0.014,-0.005,0, 0 ,0)	(1,-1 ,1 ,2.469,-2.469,0.073,0)	19.0 [0.01]**

a) The λ-max test the null hypothesis of r cointegrating vectors against the alternative r+1, r=0,1,2,..,n-1, where n is the number of variables in the model, see Johansen (1988). In the adjusted version of the test, the number of observations T is replaced by T-nk, where k is the number of lags, see Reimers (1992). b) The λ-trace test for r=0,1,2,..,n-1. Also here we report the adjusted version (see Reimers, 1992). c) The estimated coefficient for the trend is multiplied by 1000. Critical values are taken from Osterwald-Lenum (1992), and (*) denotes rejection at the 5 per cent significance level while (**) indicates rejection at the 1 per cent level.

Appendix B

Calculating half-lives

In order to calculate half-lives the deterministic variables in the system can be ignored. Without the deterministic variables, the system can be written as

$$\text{(B.1)} \qquad \Delta z_t = [\Gamma_1, \alpha] \begin{bmatrix} \Delta z_{t-1} \\ \beta' z_{t-1} \end{bmatrix} + u_t ,$$

where

$$\alpha' = [-0.032 \quad 0.019 \quad 0 \quad 0 \quad -0.012 \quad 0]$$
$$\beta' = [1 \quad -1 \quad 1 \quad 1.166 \quad -1.166 \quad 0.083]$$
$$\Gamma_1 = \begin{bmatrix} 0.239 & 0.216 & 0.134 & 0.415 & 0.078 & -0.000 \\ 0.046 & 0.114 & -0.088 & -0.027 & 0.178 & 0.007 \\ -0.016 & -0.259 & 0.108 & -0.049 & 0.002 & 0.005 \\ 0.022 & -0.011 & 0.212 & 0.012 & 0.453 & -0.007 \\ -0.023 & 0.065 & -0.026 & -0.035 & 0.059 & 0.001 \\ -0.208 & -5.663 & -6.170 & -1.593 & 1.827 & 0.194 \end{bmatrix}$$

To effectively use (B.1) for calculating half-lives, we need to augment (B.1) with the definitional relationship: $\beta' z_t - \beta' \Delta z_t = \beta' z_{t-1}$. This yields

$$\text{(B.2)} \qquad \begin{bmatrix} I_6 & 0_{6\times1} \\ -\beta' & 1 \end{bmatrix} \begin{bmatrix} \Delta z_t \\ \beta' z_t \end{bmatrix} = \begin{bmatrix} \Gamma_1 & \alpha \\ 0_{1\times6} & 1 \end{bmatrix} \begin{bmatrix} \Delta z_{t-1} \\ \beta' z_{t-1} \end{bmatrix} + \begin{bmatrix} u_t \\ 0 \end{bmatrix},$$

or

$$\text{(B.3)} \qquad \begin{bmatrix} \Delta z_t \\ \beta' z_t \end{bmatrix} = \begin{bmatrix} I_6 & 0_{6\times1} \\ -\beta' & 1 \end{bmatrix}^{-1} \begin{bmatrix} \Gamma_1 & \alpha \\ 0_{1\times6} & 1 \end{bmatrix} \begin{bmatrix} \Delta z_{t-1} \\ \beta' z_{t-1} \end{bmatrix} + \begin{bmatrix} I_6 & 0_{6\times1} \\ -\beta' & 1 \end{bmatrix}^{-1} \begin{bmatrix} u_t \\ 0 \end{bmatrix}.$$

Since the relationship should hold for all *t*, we can write

$$\text{(B.4)}\quad \begin{bmatrix} \Delta z_{t+s} \\ \beta' z_{t+s} \end{bmatrix} = \left(\begin{bmatrix} I_6 & 0_{6\times1} \\ -\beta' & 1 \end{bmatrix}^{-1} \begin{bmatrix} \Gamma_1 & \alpha \\ 0_{1\times6} & 1 \end{bmatrix} \right)^{s+1} \begin{bmatrix} \Delta z_{t-1} \\ \beta' z_{t-1} \end{bmatrix} + \sum_{i=0}^{s} \left(\begin{bmatrix} I_6 & 0_{6\times1} \\ -\beta' & 1 \end{bmatrix}^{-1} \begin{bmatrix} \Gamma_1 & \alpha \\ 0_{1\times6} & 1 \end{bmatrix} \right)^{s-i} \begin{bmatrix} I_6 & 0_{6\times1} \\ -\beta' & 1 \end{bmatrix}^{-1} \begin{bmatrix} u_{t+i} \\ 0 \end{bmatrix}$$

Furthermore, assuming that the system is in equilibrium in period *t*-1 (this is not a necessary assumption; it only simplifies the presentation because the left hand side of (B.4) then can be interpreted as the effect of the shock) and that all future errors are zero ($u_{t+i} = 0, \forall i \geq 1$), the effect on the variables in the system in period *t*+*s* of a shock in period *t* can be written as

$$\text{(B.5)}\quad \begin{bmatrix} \Delta z_{t+s} \\ \beta' z_{t+s} \end{bmatrix} = \left(\begin{bmatrix} I_6 & 0_{6\times1} \\ -\beta' & 1 \end{bmatrix}^{-1} \begin{bmatrix} \Gamma_1 & \alpha \\ 0_{1\times6} & 1 \end{bmatrix} \right)^{s} \begin{bmatrix} I_6 & 0_{6\times1} \\ -\beta' & 1 \end{bmatrix}^{-1} \begin{bmatrix} u_t \\ 0 \end{bmatrix}.$$

We are not interested in the effect on the whole system of the shock, but only in the effect on the cointegrating relationship. Therefore, we pre-multiply with the selection vector [0,0,0,0,0,0,1]. The effect on the cointegrating relationship of a shock in the errors *u*, i.e. after *s* periods (here; months), is hence given by the expression

$$\text{(B.6)}\quad \beta' z_{t+s} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} I_6 & 0_{6\times1} \\ -\beta' & 1 \end{bmatrix}^{-1} \begin{bmatrix} \Gamma_1 & \alpha \\ 0_{1\times6} & 1 \end{bmatrix} \right)^{s} \begin{bmatrix} I_6 & 0_{6\times1} \\ -\beta' & 1 \end{bmatrix}^{-1} \begin{bmatrix} u_t \\ 0 \end{bmatrix}.$$