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Estimating the natural rates in a simple New Keynesian framework

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Abstract The time-varying natural rate of interest and output and the implied medium-term inflation target for the US economy are estimated over the period 1983–2005. The estimation is conducted within the New Keynesian framework using Bayesian and Kalman-filter estimation techniques. With the model-consistent estimate of the output gap, we get a small weight on the backward-looking component of the New Keynesian Phillips curve—similar to what is obtained in studies which use labor share of income as a driver for inflation (e.g., Galí, Eur Econ Rev 45(7):1237–1270, 2001; Eur Econ Rev 47(4):759–760, 2003). The turning points of the business cycle are nevertheless broadly consistent with those of CBO/NBER. We find considerable variation in the natural rate of interest while the inflation target has been close to 2% over the last decade.

Keywords Natural rate of interest · Natural rate of output · New Keynesian model · Inflation target

JEL Classification C51 · E32 · E37 · E52

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1 Introduction

The New Keynesian theory, as developed by Goodfriend and King (1997), Rotemberg and Woodford (1997), McCallum and Nelson (1999), and others, and with policy implications extensively explored in Clarida et al. (1999) and Woodford (2003), has become the leading framework for the analysis of monetary policy. This theory honors the proposition that monetary policy affects only nominal variables in the long run and that the steady-state inflation rate can be governed by monetary policy. Moreover, it assumes that the central bank implements its policy through the setting of the short-term interest rate. Monetary policy influences decisions about real magnitudes due to prices not being fully free to adjust to shocks (price rigidities). The overriding objective of monetary policy is to alleviate the effects of these rigidities while keeping inflation expectations close to a target rate of inflation.

An important point of reference for the policymaker is how the economy would have developed had prices been without rigidities and instead fully flexible. We refer to the rate of interest and the level of output in such an equilibrium as *the natural rates of interest rates* and *the natural level of output* (see Woodford 2003). Consistent with this view, the strategy of monetary policy is often formulated in terms of deviations from these natural rates, that is, in terms of the interest rate gap and the output gap, respectively. The well-known Taylor rule (Taylor 1993) provides an illustration. Under the Taylor rule, the central bank raises the interest rate *relative* to the natural rate of interest if either inflation deviates from the inflation target and/or output deviates from the natural level of output. For these reasons, the natural rates are important indicators for the setting of the policy instrument and the characterization of a neutral monetary policy stance.

The main objective of this article is to present a simple framework in which to derive the natural rates within a New Keynesian model setting. The model is small, yet incorporates the main ingredients of the New Keynesian framework, making it a useful device of analyzing how changes in the natural rates affect the economy and monetary policy. Despite the simple nature of the model, we derive plausible time-varying estimates of the natural rates and the corresponding interest rate and output gaps using Bayesian estimation and Kalman filtering techniques on the US data. Previous studies on the topic include the seminal article by Laubach and Williams (2003) who use the Kalman filter to estimate the (unobserved) natural rate of interest and the output gap. The model is a standard growth model, implying that the natural real interest rate varies over time in response to shifts in preferences and trend growth rate of output.[1] Their models, however, specify the natural rates within a reduced form system devoid of forward looking elements.[2] In this regard, our article is more related to the recent unobserved components study of Basistha and Nelson (2007) who acknowledge that inflation may be dependent on expected future inflation. They derive the output gap assuming that inflation depends on (survey measures of) expected future inflation as well as past inflation rates and the output gap. We extend on their

[1] The idea builds on the articles by Watson et al. (1997) and Gordon (1998), among others, which estimate the natural rate of unemployment (NAIRU) using the Kalman filter.

[2] See also Garnier and Wilhelmsen (2005) for an application to the Euro Area.

contributions by deriving the estimates in a way that is consistent with New Keynesian theory restrictions and furthermore allowing inflation expectations to be rational and theory consistent. Our approach is nevertheless spare relative to a full DSGE approach, in that we neither impose technology restrictions nor model the market for production factors. The supply side of the economy is governed by exogenous processes. This approach allows us to have a relatively simple model while allegedly being less sensitive to possible controversial assumptions required to model, e.g., marginal costs explicitly.

Another facet of the contribution of this article is the allowance of the possibility of a time-varying inflation target. The US inflation history is difficult to reconcile with a constant inflation target. In this regard, our approach is similar to that of Ireland (2007). While we, however, assume that the inflation target reflects a preference of the monetary policymaker and is unrelated to the state of the economy, Ireland assumes that the inflation target is dependent on some of the shocks to private sector behavior. Our conclusions regarding the evolution of the inflation target are nevertheless similar.

A third novelty of our approach is that it does not require detrending of the data prior to analysis (using for instance the HP-filter) or making output stationary by deflating by a trending variable (for instance, by assuming that total factor productivity follows a trend stationary process), as has been common practice in many recent DSGE analyses, including Edge et al. (2007), Juillard et al. (2005), Andrés et al. (2005), and Smets and Wouters (2003, 2007) who also estimate the natural rates.[3]

An important empirical finding in this article is that inflation is primarily a forward-looking process. By allowing inflation to have both forward-looking and backward-looking components, using a hybrid New Keynesian Phillips curve, data prefer a forward-looking specification. Although this is the common conclusion in studies which use labor's share of income as a proxy for marginal costs (see Galí et al. 2001, 2003), it is not common finding when the output gap is the driving process. Interestingly, after accounting for the time-varying inflation target and natural rate of interest, a model-consistent estimate of the output gap gives rise to a Phillips curve specification similar to that of labor's share of income. We interpret this in favor of using the output gap as a valid representation of the inflation driver. This suggests that the approach of studying monetary policy within the a simple model framework with inflation, output gap, and the interest rate, as advocated in Woodford (2003), is also empirically useful.

The remainder of this article is organized as follows. In Sect. 2, we present the New Keynesian framework. Section 3 presents the estimation framework and results. In Sect. 4, we provide some concluding remarks.

2 A simple New Keynesian framework

The New Keynesian framework assumes that firms operate in monopolistic competitive markets and production is constrained by aggregate demand. Prices are assumed

[3] A recent exception is Juillard et al. (2006). They allow for a more general stochastic process where there could be both temporary changes in the growth rate of total factor productivity as well as autocorrelated deviations from steady state.

to be sticky and consequently do not move instantaneously to movements in marginal costs. Owing to the price stickiness, the central bank affects aggregate demand through its influence on real interest rates. By lowering real interest rates, the central bank induces higher aggregate demand, marginal costs, and prices than would otherwise materialize. As noted above, the natural rate of interest rate can be regarded as the neutral stance of monetary policy—the real interest rate that produces zero output gap and stable inflation.

In estimating the natural rates, we build on the economic structure provided by the New Keynesian framework. The basic model is extended with external habit formation in consumption (Fuhrer 2000) and a hybrid New Keynesian Phillips curve that allows for both forward-looking and backward-looking elements. This set up is rationalized by the Calvo (1983) framework with some of the firms setting prices in accordance with an indexation scheme (Christiano et al. 2005) or in accordance with some rule-of-thumb (Galí and Gertler 1999). Our approach remains, nevertheless, conservative regarding the extent of the economic structure regarding production technology and the structure of the labor market imposed in estimation. This reduces the approach's rigor at the gain of not being tied up to a particular description of production technology which may bias the result if incorrect. Specifically, we allow the natural rate of output to follow exogenous autoregressive (AR) processes and in this regard, the article draws on the literature on structural time-series estimation, see e.g., Harvey (1989).

2.1 Aggregate demand

We assume that the economy consists of a representative household that lives forever and maximizes expected utility given by

$$U = E_t \sum_{i=0}^{\infty} \left(\frac{1}{1+\delta}\right)^i \left[\frac{1}{(1-\sigma)} \left(\frac{C_{t+i} V_{t+i}}{H_{t+i}}\right)^{(1-\sigma)}\right],$$

subject to the intertemporal budget constraint given by

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \left(\frac{W_t}{P_t}\right) N_t + \frac{M_{t-1}}{P_t} + I_{t-1}\frac{B_{t-1}}{P_t} - \frac{T_t}{P_t} + \Pi_t.$$

δ is the discount rate, σ is the intertemporal elasticity of substitution and C is an CES index of consumption goods. V is a consumption preference shock. The consumer is also assumed to have preferences over money and leisure. The decision processes associated with labor supply decisions are not explicitly modeled and implicitly left exogenous in the model. The reason for doing this is partly simplicity, and partly a reflection of our view that the approaches currently available for modeling the labor market decisions are too simplistic. Hence, imposing restrictions from these theories are likely to be biasing our results. The cost of keeping the production technology "exogenous," however, is that we cannot distinguish between particular shocks on the supply side, e.g., productivity versus mark-up shocks.

The consumer can either hold money (M) or bonds (B) as a store of wealth. Money yields utility (not modeled) whereas bonds yield a gross risk-free return of I_t in every period. Consumption preferences are subject to a shock $V_t \equiv (1 - v_t)$ where

$$v_t = \rho_v v_{t-1} + \tilde{v}_t \tag{1}$$

where ρ_v is degree of persistence in the shock and $\tilde{v}_t$ is a white-noise shock. H_t represents external habit persistence. We introduce habit persistence of order 2. The reason for this is that it allows for a higher-order lag structure of the resulting first-order condition. The habit persistence is specified as follows:

$$H_t = C_{t-1}^{\gamma_1} C_{t-2}^{\gamma_2},$$

where γ_1 and γ_2 are habit parameters. This more general setup allows agents to form habits with respect to the changes in as well as the level of consumption.

The first-order condition for the solution to the problem implies the consumption Euler equation

$$\left(\frac{C_t V_t}{C_{t-1}^{\gamma_1} C_{t-2}^{\gamma_2}}\right)^{1-\sigma} \frac{1}{C_t} = \left(\frac{1}{1+\delta}\right) I_t E_t \left(\frac{C_{t+1} V_{t+1}}{C_t^{\gamma_1} C_{t-1}^{\gamma_2}}\right)^{1-\sigma} \frac{1}{C_{t+1}} \frac{P_t}{P_{t+1}}. \tag{2}$$

Taking the logarithm of the Euler equation and using the resource constraint, we have

$$\begin{aligned} y_t = {} & \frac{\sigma}{A} E_t y_{t+1} + \frac{(\gamma_1 - \gamma_2)(\sigma - 1)}{A} y_{t-1} \\ & + \frac{\gamma_2 (\sigma - 1)}{A} y_{t-2} - \frac{1}{A}(i_t - E_t \pi_{t+1} - \delta) + \frac{(\sigma - 1)}{A}(v_t - E_t v_{t+1}), \end{aligned} \tag{3}$$

where $A \equiv \sigma + \gamma_1 (\sigma - 1)$ and π_t is quarterly inflation at an annual rate. A small letter denotes the log of the corresponding capital letter variable.[4]

Note that due to dynamic homogeneity, we can write the aggregate demand schedule (3) as

$$\begin{aligned} \Delta y_t = {} & \frac{\sigma}{\gamma_1 (\sigma - 1)} E_t \Delta y_{t+1} - \frac{\gamma_2}{\gamma_1} \Delta y_{t-1} \\ & - \frac{1}{\gamma_1 (\sigma - 1)} (i_t - E_t \pi_{t+1} - \rho) + \frac{1}{\gamma_1} (v_t - E_t v_{t+1}). \end{aligned} \tag{4}$$

[4] Note that we have for simplicity ignored Jensen's inequality and used first-order Taylor approximations, implying $\ln E(1 + x) = E \ln(1 + x) = Ex$.

2.2 Aggregate supply

Aggregate supply is represented by the hybrid Phillips curve as

$$\pi_t = \mu E_t \pi_{t+1} + (1-\mu) \sum_{j=1}^{4} \alpha_j \pi_{t-j} + \kappa x_t + \varepsilon_t, \tag{5}$$

where $(1-\mu)$ is the weight on the backward-looking component, ε_t is a cost push shock and $x_t \equiv y_t - y_t^n$ is the output gap, defined as the deviation of output from the natural rate of output. As in Rudebusch (2002a,b), we allow for a lag structure on past inflation to match the dynamics of inflation at the quarterly frequency. Furthermore, we impose dynamic homogeneity, i.e., that $\alpha_4 = 1 - \alpha_1 - \alpha_2 - \alpha_3$.[5]

As noted above, we do not endogenize the input of production factors and specify technology, but instead assume that the natural rate of output is given exogenously by the process

$$\Delta y_t^n = v + \omega_t \tag{6}$$

where ν is the unconditional expected growth rate of output and ω_t is an AR(1) shock to the growth rate (natural rate shock)[6]

$$\omega_t = \phi \omega_{t-1} + \varrho_t. \tag{7}$$

The output gap then follows the process

$$x_t = x_{t-1} + \Delta y_t - \Delta y_t^n. \tag{8}$$

2.3 Monetary policy

The monetary authority is setting the interest rate in accordance with a dynamic Taylor rule as

$$i_t = \psi i_{t-1} + (1-\psi) \left(i_t^n + \theta_\pi \left(\bar{\pi}_t - \pi_t^T \right) + \theta_x x_t \right) + u_t, \tag{9}$$

where ψ measures the smoothing in the interest rate setting. i_t^n is the nominal natural interest rate (defined below) and

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[5] Although we do not provide any microfoundations for these lags, we postulate that these lags will follow from the rules-of-thumb framework of pricing of Galí and Gertler (1999) given that rule-of-thumb allows for longer lags.

[6] The shock ϱ is best thought of as representing variations in productivity and preferences that influence the marginal rate of substitution between consumption and leisure. Neither sources is modeled explicitly here.

$$\bar{\pi}_t \equiv \frac{1}{4}\sum_{j=0}^{3}\pi_{t-j}$$

is the four-quarter inflation at an annual rate. We assume the FED has an implicit intermediate-run target for inflation that can deviate from the long-run (steady state) inflation target. This could play the role of smoothing inflation dynamics and bring inflation slower back to target once above it, avoiding large output changes. It evolves according to

$$\pi_t^T = (1-\rho_\pi)\,\pi^* + \rho_\pi \pi_{t-1}^T + \xi_t, \tag{10}$$

where π^* is the steady-state inflation rate (or long-run inflation target), and ξ_t is an AR(1) shock to the inflation target, in accordance with

$$\xi_t = \rho_\varkappa \xi_{t-1} + \varkappa_t. \tag{11}$$

2.4 The natural rate of interest

The process for the natural nominal rate of interest can be found by replacing output and the interest rate in Eq. 3 with the natural rates and then solving for the interest rate, i.e.,

$$\begin{aligned} y_t^n &= \frac{\sigma}{A}E_t y_{t+1}^n + \frac{(\gamma_1-\gamma_2)(\sigma-1)}{A}y_{t-1}^n \\ &\quad + \frac{\gamma_2(\sigma-1)}{A}y_{t-2}^n - \frac{1}{A}\left(i_t^n - E_t\pi_{t+1} - \delta\right) + \frac{(\sigma-1)}{A}\left(v_t - E_t v_{t+1}\right), \end{aligned} \tag{12}$$

or

$$\begin{aligned} \Delta y_t^n &= \frac{\sigma}{\gamma_1(\sigma-1)}E_t\Delta y_{t+1}^n - \frac{\gamma_2}{\gamma_1}\Delta y_{t-1}^n \\ &\quad - \frac{1}{\gamma_1(\sigma-1)}\left(i_t^n - E_t\pi_{t+1} - \delta\right) + \frac{1}{\gamma_1}\left(v_t - E_t v_{t+1}\right). \end{aligned} \tag{13}$$

and isolating for the natural interest rate

$$\begin{aligned} i_t^n &= \delta + E_t\pi_{t+1} + \sigma E_t\Delta y_{t+1}^n - \gamma_1(\sigma-1)\,\Delta y_t^n - \gamma_2(\sigma-1)\Delta y_{t-1}^n \\ &\quad + (\sigma-1)\left(v_t - E_t v_{t+1}\right). \end{aligned} \tag{14}$$

The natural real interest rate is then found from the Fisher equation as

$$r_t^n \equiv i_t^n - E_t\pi_{t+1}. \tag{15}$$

The output gap process can be expressed as a function of the natural interest rate by subtracting Eq. 12 from Eq. 3 which gives

$$x_t = \frac{\sigma}{A} E_t x_{t+1} + \frac{(\gamma_1 - \gamma_2)(\sigma - 1)}{A} x_{t-1} + \frac{\gamma_2 (\sigma - 1)}{A} x_{t-2} - \frac{1}{A}\left(i_t - i_t^n\right) \quad (16)$$

where the natural rate of interest is given in Eq. 14 above.

3 Estimation

We estimate the parameters of the model comprising of Eqs. 1, 4, 5, 6, 7, 8, 9, 10, and 11 using Bayesian methods and the Kalman filter. The focus of the analysis will be on the estimation of the natural real rate of interest and the output gap. The use of Bayesian methods to estimate DSGE models has increased over recent years, in a variety of contexts, see An and Schorfheide (2006) for a recent evaluation. The focus is on methods that are built around a likelihood function, typically derived from a DSGE model (see, e.g., Adolfson et al. 2007). With sensible priors, Bayesian techniques offer a major advantage over other system estimators such as maximum likelihood, which in small samples can often allow key parameters to wander off in nonsensical directions.

3.1 Data

We estimate the model laid out in the previous section using the US quarterly time series for three variables: real output, inflation, and interest rates. The sample period is 1983q1–2005q4. The period covers the last part of the Volcker period and the major part of the Greenspan period. The choice of periods follows from the assumption that these two Chairmen shares approximately the same dislike for inflation. The monetary policy regime is, therefore, roughly constant over the sample period. We use the quarterly average daily readings of the US 3-month deposit rates as the relevant nominal interest rate. For real output and inflation, we use real GDP and the CPI, all items, for the entire USA. GDP and CPI are seasonally adjusted by their original source (OECD). We treat inflation, output growth, and the nominal interest rate as stationary, and express them in deviations from their sample mean. Note that all the changes are measured at an annual rate.

3.2 Parameter estimation

As is well known from Bayes's rule, the posterior distribution of the parameters is proportional to the product of the prior distribution of the parameters and the likelihood function of the data. This prior distribution describes the available information prior to observing the data used in the estimation. The observed data are then used to update the prior, via Bayes theorem, to the posterior distribution of the model's parameters.

Table 1 Estimation results for the US economy

Coefficients	Prior mean	Prior SD	Distr.	Support	Post. mean	5%	90%
Phillips curve							
μ	0.50	0.20	Beta	[0, 1]	0.626	0.314	0.908
α_1	0.25	0.10	Norm	None	0.353	0.200	0.506
α_2	0.25	0.10	Norm	None	0.240	0.100	0.366
α_3	0.25	0.10	Norm	None	0.227	0.094	0.369
α_4	0.25	n/a	n/a	n/a	0.180	n/a	n/a
κ	0.20	0.15	Gamm	$[0, \infty]$	0.089	0.005	0.163
IS curve							
δ	0.04	0.02	Gamm	$[0, \infty]$	0.016	0.006	0.027
σ	2.00	0.50	Beta	[1.05, 5]	2.047	1.625	2.448
γ_1	0.50	0.20	Beta	[0, 1]	0.537	0.332	0.727
γ_2	0.40	0.20	Beta	[0, 1]	0.599	0.396	0.870
ρ_υ	0.85	0.10	Beta	[0, 1]	0.945	0.916	0.980
Natural rate process							
ϕ	0.850	0.10	Beta	[0, 1]	0.788	0.678	0.909
υ	0.030	0.005	Gamm	$[0, \infty]$	0.029	0.024	0.035
Monetary policy							
ρ_{pi}	0.800	0.10	Beta	[0, 1]	0.853	0.751	0.950
ρ_χ	0.800	0.10	Beta	[0, 1]	0.795	0.662	0.939
θ_π	0.500	0.10	Beta	[0.1, 1.5]	0.578	0.420	0.720
θ_x	0.500	0.10	Beta	[0.1, 1.5]	0.449	0.284	0.570
ψ	0.700	0.10	Beta	[0, 1]	0.828	0.793	0.872
Standard deviations of shocks							
$\sigma_\varkappa$	0.002	Inf	Invg	$[0, \infty]$	0.0024	0.0008	0.0045
σ_ε	0.001	Inf	Invg	$[0, \infty]$	0.0110	0.0091	0.0127
σ_u	0.001	Inf	Invg	$[0, \infty]$	0.0024	0.0020	0.0028
$\sigma_{\tilde{\upsilon}_t}$	0.001	Inf	Invg	$[0, \infty]$	0.1983	0.1181	0.3045
σ_ϱ	0.001	Inf	Invg	$[0, \infty]$	0.0096	0.0074	0.0119

In order to implement the Bayesian estimation method, we need to be able to evaluate numerically the prior and the likelihood function. Then, we use the Metropolis-Hastings algorithm to obtain random draws from the posterior distribution, from which we obtain the relevant moments of the posterior distribution of the parameters.

More specifically, the model is estimated in two steps in Dynare-Matlab. In the first step, we compute the posterior mode using 'csminwel,' an optimization routine developed by Christopher Sims. We use the first 3 years of the full sample 1983q1–2005q4 to obtain a prior on the unobserved state, and use the subsample 1986q1–2005q4 for inference. In order to calculate the likelihood function of the observed variables, we apply the Kalman filter. In the second step, we use the mode as a starting point to compute the posterior distribution of the parameters and the marginal likelihood by simulations of the Metropolis-Hasting (MH) algorithm (see for details Schorfheide

2000). The debugging features of Dynare are used to determine whether the optimization routines have found the optimum and whether enough draws have been executed for the posterior distributions to be accurate. Having estimated the parameters, they can then be used to construct the natural rates of interest rates and output.

3.3 Prior and posterior distributions

The Bayesian estimation technique allows us to use prior information from previous micro- and macro-based studies in a formal way. Table 1 summarizes the assumptions for the prior distribution of the estimated parameters and structural shocks. In the first three columns, the list of structural coefficients with their associated prior mean, standard deviations and distribution are shown. On the basis of standard conventions, we use Beta distributions for parameters that fall between zero and one, (inverted) gamma (invg) distributions for parameters that need to be constrained to be greater than zero, and normal (norm) distributions in other cases. For some of the parameters, the distribution is constrained further, as indicated in column four ('support').

The next three columns indicate the posterior mean and the associated 90% uncertainty interval. Starting with the Phillips curve, we provided a prior for $\mu = 0.50$ which puts equal weight on the forward-looking and backward-looking components with a large standard deviation providing a rather diffuse prior. This choice is rationalized by the fact that the literature has suggested estimates in the whole zero-unity interval. We wanted data to determine this coefficient without pushing it in either direction. In the estimation, $\alpha_1, \alpha_2, \alpha_3$, and α_4 were restricted to sum to one (with α_4 determined by this identity). However, since we do not have a strong prior on their magnitudes, we give them the same weight with the standard deviation set to 0.1. κ was estimated at 0.089 which is not far from the estimate of 0.13 obtained by Rudebusch (2002a,b) who used CBO estimate of the output gap.

We find that the Phillips curve is primarily forward looking. It has nevertheless a non-negligible weight on the backward-looking component with $(1 - \mu)$ just below 0.4. This is consistent with the estimates of the New Keynesian Phillips curve found when using labor's share of income as the proxy for marginal costs[7] as opposed to using detrended output. We believe that this result is due to allowing for simultaneous estimation of the Phillips curve parameter and the natural rates, which are both closely connected. The use of detrended output in some studies disregards this important simultaneity. Our results are consistent with the estimation results in Galí et al. (2003, 2005) using a full information, system estimation. We find this result interesting because it suggests that the output gap may be a valid representation of the inflation driving process. Hence, modeling the measures of marginal costs may not be essential to capture a broad representation of the monetary policy transmission mechanism. The results support that monetary policy can be studied within a simple two-equation model framework which explains the development of inflation and the output gap conditional on the policy instrument (as suggested by Clarida et al. 1999 and Woodford 2003).

[7] See, e.g., Galí et al. (2001, 2003, 2005) and Sbordone (2002, 2005).

Table 2 Model fit: standard deviations

Standard deviations	Interest rate	Inflation	Output growth
Theoretical moments	0.0260	0.0144	0.0219
Actual sample moments	0.0269	0.0209	0.0206

Regarding the expectational IS curve, we find that our prior on the intertemporal elasticity of substitution $\sigma = 2$ is well within the range of the estimates in the literature. The posterior has increased somewhat from the prior, although not significantly so (posterior mean equals 2.05). Moreover, the preference shocks display a high degree of persistence, with a coefficient of $\rho_v = 0.95$. In addition, the habit parameters γ_1 and γ_2 are restricted to lie between zero and one, with the prior for γ_1 being the largest, assuming more habit from the immediate past. However, we choose a large standard deviation that provides us with a fairly diffuse prior. The second-order habit persistence is well accounted for in data, as both γ_1 and γ_2 turn out to be above the priors. Finally, the prior for the annual discount rate δ is set to 0.04, reflecting a quarterly discount factor of 0.99. Rather surprisingly, we find that data push the annual discount rate from the prior of 4 to 1.6%.

The prior for the equilibrium natural output growth rate is set equal to the (annual) growth rate in the model (3%), with the posterior mean estimated to $\upsilon = 0.029$. As our data set is small, it is unlikely that we would get any other value than the equilibrium value suggested by the data. As an alternative, we could, therefore, have calibrated this value at 0.029.

The data seem to support a dynamic Taylor rule specification of monetary policy reasonably well. The monetary policy shock (σ_u) has standard deviation of 0.024. Moreover, the weights on inflation and output gap are deviating only marginally from the priors and what Taylor (1993) suggested as likely coefficients (0.5). There is a pronounced gradual adjustment of the interest rate with $\psi = 0.83$. Finally, we calibrate the steady-state inflation rate π^* to be equal to steady state inflation. The results seems to indicate fairly persistent movements in the medium-run inflation target ($\rho_\pi = 0.85$), with also rather persistent shocks to this process ($\rho_\chi = 0.80$). The latter suggest that movements in the medium-run inflation target is done gradually over time.

Finally, we note that the fit of the model seems to be reasonably good in terms of matching moments. In the first row of Table 2, we show the sample moments (standard deviations) from the smoothed posterior predictive distribution of the three observable variables: interest rates, inflation, and output growth. The second row show the same sample moments, but calculated from the actual US data. The estimated model seems to fit reasonably well and is able to explain the larger part of the salient features of the data as the actual sample moments does not lie too far from the posterior predictive distribution.

3.4 Error variance decomposition and impulse responses

Table 3 shows the decomposition of the unconditional variance. Some interesting observations can be made from the table. We first note that the main drivers of infla-

Table 3 Error variance decomposition

Variables and shocks	Inf.-tar. ($\varkappa$)	Cost-push (ε)	Mon. pol. (u)	Preference ($\tilde{\upsilon}$)	Nat. rate (ϱ)
r^n	0.00	0.00	0.00	91.52	8.48
x	25.14	21.75	6.51	44.04	2.56
π	31.58	48.25	1.46	17.56	1.14
i	10.79	3.04	0.85	83.91	1.41
i^n	10.81	2.89	0.44	78.08	7.78
π^T	100.00	0.00	0.00	0.00	0.00

tion variations are the cost-push and inflation-target shocks. These shocks account for about 80% of the variation in inflation. If the central bank adheres to an inflation-targeting loss specification with the loss function having inflation and output gap variations as the two arguments (see Svensson 1997; Clarida et al. 1999), efficiency in policymaking requires that inflation should be driven *only* by cost-push and inflation-target shocks. The ratio is high and can be taken as an indication of efficiency in policymaking. However, by the same logic, the central bank should fully neutralize the impact of preference shocks on both the output gap and inflation. This does not seem to be the case. Although the Taylor rule has allowed strong responses to the preference shocks as they can explain more than 80% of the variation in the interest rate, preference shocks have still influenced inflation and, in particular, the output gap to a large extent. Hence, the estimated Taylor rule does less well in insulating the economy from this type of shock.

The natural real interest rate is driven mainly by preference shocks that make demand deviate from the natural rate of output. Shocks to the natural rate of output play only a minor role in explaining the variation observed. The estimated model suggests that monetary policy main role is to mitigate the effects of demand shocks on aggregate demand, and to lesser extent accommodate the effect of supply shocks. The error variance decomposition of the interest rate suggests that this is also the case.

The impulse response functions are shown in the appendix. None of these responses deviates from what we understand as conventional thinking, although the responses to some of the shocks seem to be rather fast (preference shocks in particular). The impulses from the monetary policy shock correspond well with results generated from VARs: For a positive shock to the interest rate, the output gap falls on impact and inflation reacts with a lag. The short-term interest rate falls relatively quickly and enters a period in which the policymaker corrects for the shock.

A shock to the medium-run inflation target raises inflation expectations and the current inflation rate on impact due to the expectations channel. The nominal interest rate increases, but the real interest rate falls and creates a temporary increase in the output gap which again increases inflation. Inflation peaks after five quarters and is then brought slowly back to the steady-state rate of inflation over a 5–7 years period. Hence, the medium-term is relatively long, approximately equal to the average business cycle. This gives some indication of the medium-term inflation target being used as an instrument to smooth output as a result of pursuing a constant inflation target over the business cycle.

A preference shock that raises aggregate demand increases the natural real interest rate as a higher interest rate is needed to keep output at the natural rate. The higher natural interest rate together with increased output and inflation gaps, raise the nominal interest rate. After an initial increase in output and inflation, both gaps fall below the long-run equilibrium levels after four to five quarters due to the contractionary monetary policy response. A cost-push shock has no influence on the natural real interest rate, but raises inflation and lowers output in an ordinary fashion. A shock to the growth rate of the natural rate of output raises the natural real interest rate. As people expect income to increase permanently in the future, aggregate demand increases more than the natural rate of output, and hence, the natural real interest rate increases. Monetary policy reacts in a contractionary way, and the output gap is negative after having been positive on the time of impact of the shock. Inflation is consistently below the long-run equilibrium after the shock.

3.5 The estimated variables

The two-sided Kalman-filtered (denoted as "smoothed" in the remainder of the article) output gap, the medium-run inflation target, and the nominal and real natural interest rates are shown with 95% uncertainty intervals in Fig. 1. Furthermore, Fig. 2 shows the smoothed natural rate of output and the natural real interest rate plotted with actual output and the real interest rates, respectively, as well as the real interest rate gap $(r - r^n)$ and the estimated inflation gap $(\bar{\pi}_t - \pi^T)$.

The output gap estimates suggest two recessions over the sample period: the first one with a trough in 1991 and the other with a trough in 2001/2002. The recessions are of approximately the same order of magnitude, suggesting a deviation of output from the natural rate of output of approximately 5%. The recessions correspond to periods with large positive interest-rate gaps (see Fig. 2). Further, as will be discussed in more detail below, the dates for the turning points and the length of the business cycles do not seem inconsistent with NBER/CBO estimates.

The dynamics of the output gap is affected by the forward-looking Phillips curve. Inflationary pressures can be seen not only as a result of the current output gap (and the cost-push shock), but also as a result of expected future output gaps and cost-push shocks. This will change the dynamics relative to other measures of the output gap, as explained under the comparisons with other estimates of the output gap.

The sample average CPI inflation over the period is 3.3%. The estimated medium-run inflation target suggests that the mild run-up of inflation in the late 1980s, due to a positive output gap, was partly accommodated by an increase in the inflation target over the period, see Fig. 1. The reduction in the rate of inflation of the first part of the 1990s, accompanied by the recession in the same period, can partly be explained by a reduction in the inflation target. From 1994 to the end of the sample, the medium-run inflation target is estimated to be around 2% with an uncertainty band of about ±1 p.p. For most of the period, the inflation target is significantly above zero. The inflation gap (see Fig. 2) suggests that for the major part of the 1990s and the period after 2002,

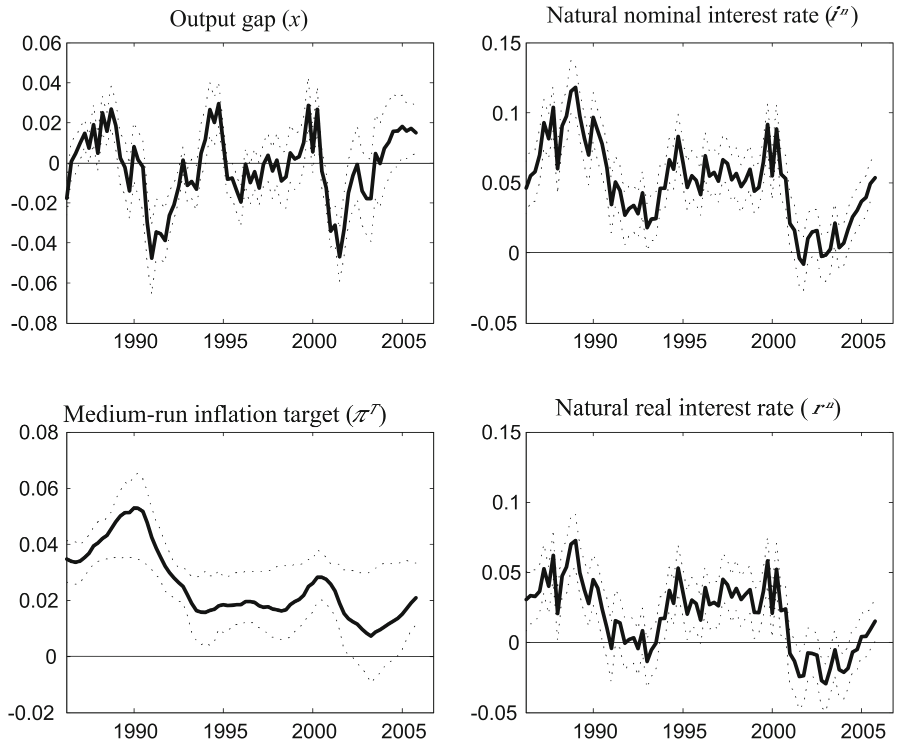

Fig. 1 Inflation target, output gap, and natural interest rates. The figures show the estimated two-sided Kalman filtered (smoothed) variables over the sample period

inflation has in general been above the medium-term inflation target, and, therefore, has exerted an upward pressure on interest rates.

The estimate of the natural real interest rate shows considerable variation over the period—varying between −3 and 6%. The variation in the natural real interest rate is in periods greater than the equivalent real interest rate. This is also found in the DSGE study of Edge et al. (2007), but not by Laubach and Williams (2003) where the natural interest rates appear as smoothed interest rates.[8] Here, the natural rate follows instead from the stochastic processes governing the preference shocks and shocks to the natural rate of output (see Eqs. 14 and 15). As noted above, the high degree of prevalence of preference shocks contribute importantly to the volatility of the natural interest rate.[9] These processes are unaffected by the potential smoothing of interest

[8] Using a similar model to Laubach and Williams (2003), Garnier and Wilhelmsen (2005) also find the volatility of the natural rate of interest having decreased over time.

[9] The zero-bound on nominal interest rates has been disregarded in the estimation of the model. The estimate of the natural nominal interest rate becomes negative (but not significantly so) during short periods of 2002–2004. We suspect, however, that a method taking account of this constraint would not produce any significant changes since the time periods and size of the negative interest rate are rather small.

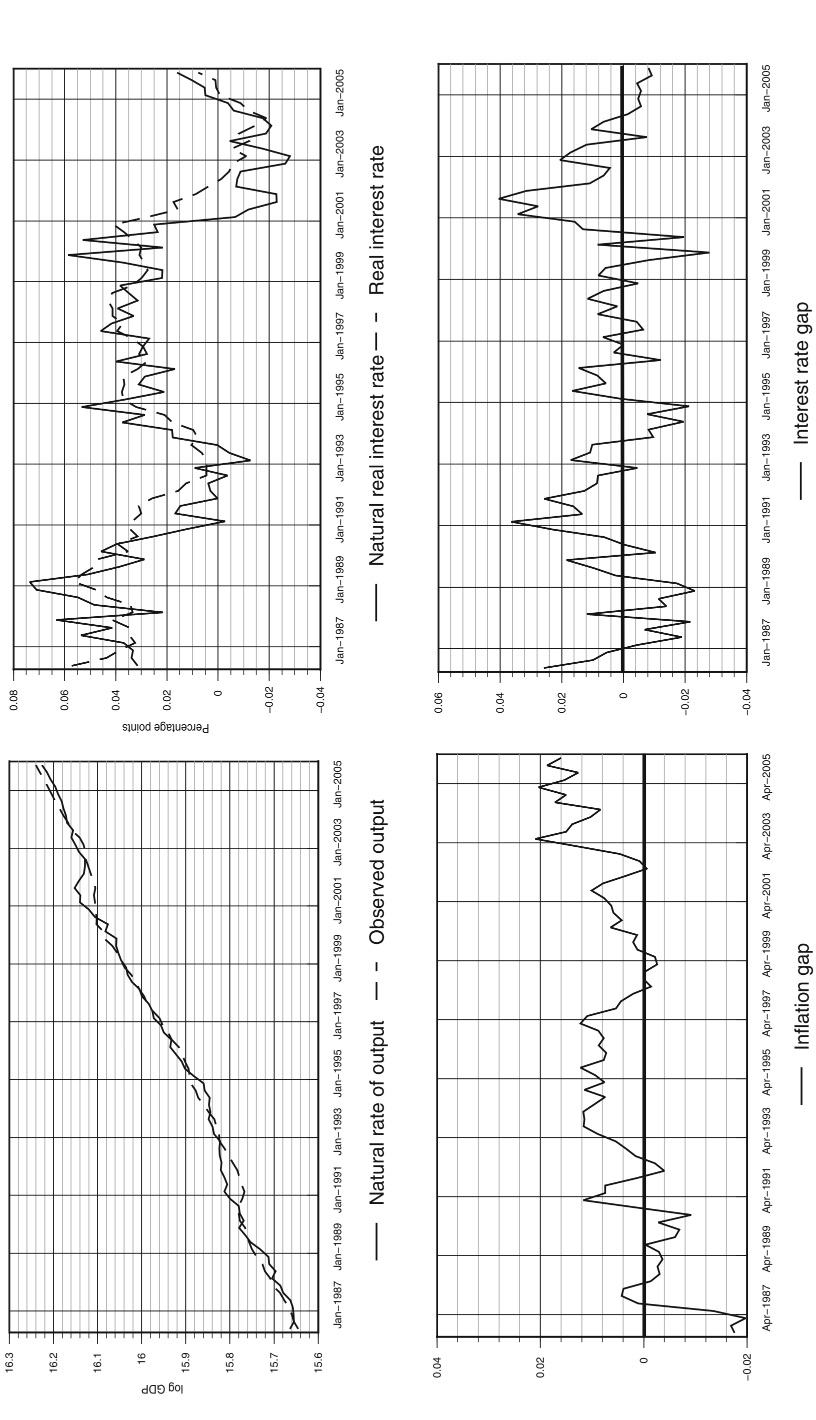

Fig. 2 The natural rate of output, interest-rate, and inflation gaps. From the *top-left panel* and moving clockwise: the natural rate of output and the real rate with actual output and the observed real interest rate, respectively, the inflation gap (inflation minus the inflation target) and the real interest rate gap (real interest rate minus the natural real interest rate)

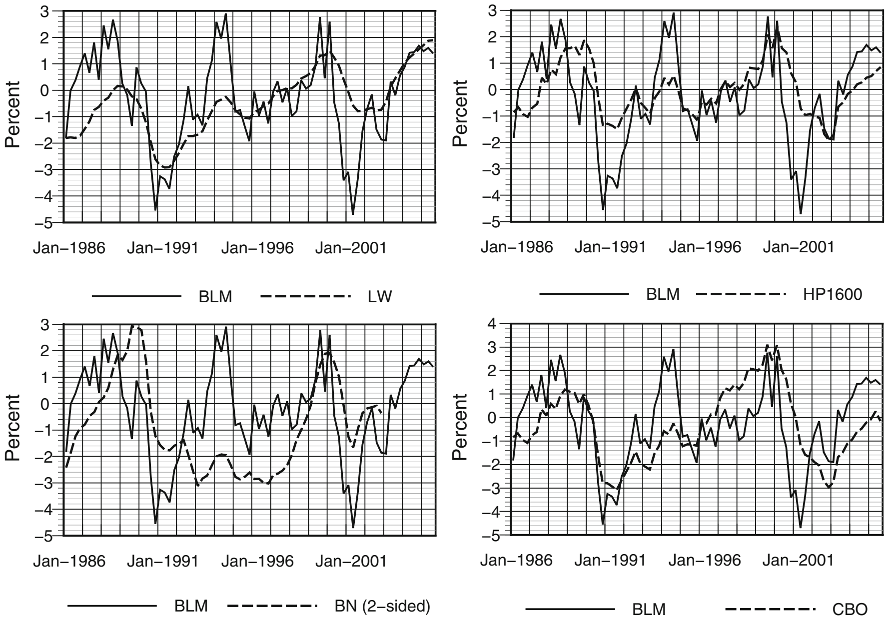

Fig. 3 Alternative estimates of the output gap. *BLM* our measure of the output gap, *LW* an updated version of Laubach and Williams (2003), *BN* the two-sided output gap estimate of Basistha and Nelson (2007), *CBO* the Congressional Budget Office estimate, *HP* the Hodrick Prescott's filtered output gap. See the main text for more details

rates done by the central bank in the sticky-price equilibrium.[10] Moreover, the mode of the natural rate of interest is in the range 3–4% which does not seem unreasonable for the average real interest rate. The average natural interest rate is remarkably stable over the period 1994–2000 where the variation is in the region ±1 p.p. This is a result also found by Edge et al. (2007). The recession of the first half of 2000s imply negative real interest rates for this period, suggesting a rather expansionary monetary policy that would have been needed to keep aggregate demand equal to the natural rate of output.

It has been relatively common to estimate monetary policy reaction functions conditional on the natural rate of interest being equal to a constant plus the inflation rate. The relatively large variation in the natural interest rate suggests that the estimates could be severely biased if the central bank is not taking account of the time-varying nature of the natural rate of interest when setting interest rate. In particular, the high degree of persistence in the natural rate in then likely to bias the coefficient on the past interest rate upwards. Moreover, failing to take account of the interdependence between the output gap and natural interest rates (estimates) may also bias the estimates.

[10] By the same logic, there is nothing that ensures that the evolvement of the natural rate of output is smoother than output itself. Woodford (2001, p. 234) notes "In theory, a wide variety of real shocks should affect the growth rate of potential output[…] [T]here is no reason to assume that all of these factors follow smooth trends. As a result, the output-gap measure that is relevant for welfare may be quite different from simple detrended output."

Table 4 Correlation and standard deviations

Estimate	BLM	LW	HP1600	BN (2-sided)	CBO
Crosscorrelations and standard deviations					
BLM	1.74	0.55	0.58	0.27	0.51
LW		1.17	0.66	0.45	0.69
HP1600			0.98	0.69	0.87
BN (2-sided)				1.69	0.47
CBO					1.60
Autocorrelations					
	0.76	0.97	0.89	0.95	0.95

The standard deviations are shown on the diagonal of the matrix, while correlations are shown off diagonal. See note on Fig. 3 for explanations on the gaps

Table 5 Concordance

Estimate	BLM	LW	HP1600	BN (2-sided)	CBO
BLM	1.00	0.68	0.76	0.69	0.62
LW		1.00	0.76	0.79	0.74
HP1600			1.00	0.79	0.85
BN (2-sided)				1.00	0.79
CBO					1.00

Concordance the proportion time the gaps move in the same direction. See note on Fig. 3 for explanations on the gaps

With high volatility in the natural rate of interest, a “neutral” monetary policy stance requires considerable changes in the interest rate. If the policymaker nevertheless regards the natural rate of interest as a constant, policy is likely to induce inefficient movements in inflation and output.

Some readers may object to the arguments by claiming that interest rates should be smoothed over time and, for this reason, the variability in the natural rate should largely be ignored. We claim that such an argument mixes up two things. Interest rate smoothing can be welfare-enhancing (see Woodford 1999) in its own right due to its impact on private sector expectations. But optimal smoothing of interest rate does not imply the removal of some arguments over which the smoothing should be done. While the interest rate may be more volatile if responding to the natural interest rate, the benefits of interest rate smoothing can still be extracted.

3.6 Alternative output gap series

We now return to the output gap in more detail, to compare our measure to some alternative measures of the gap previously found in the literature. Figure 3 compares our measure of the output gap (BLM henceforth) to (i) the output gap derived from an updated version (2006) of Laubach and Williams (2003) (LW henceforth),[11] (ii) the two-sided output gap estimate of Basistha and Nelson (2007) (BN henceforth),[12] (iii) the Congressional Budget Office (CBO) estimate of potential output as well as (iv) the Hodrick Prescott's filtered output gap, with the smoothing parameter set to 1600 (HP henceforth).[13] Tables 4 and 5 finally show, respectively, the correlation and the concordance (i.e., the time proportion that the cycles of two series spend in the same phase, see McDermott and Scott 2000)[14] between the different estimates.

Our output gap series is picking the major NBER recession periods (of 1991 and 2001) efficiently. The gap is also broadly consistent with that of the other gaps, although there are notable differences. Differences are hardly surprising given that our estimate is consistent with a rational expectation's forward-looking Phillips curve, whereas the others are not. Our estimate has the highest volatility and the smallest persistence of the series. Our Phillips curve allows for longer lags, and this implies that, for a given value of κ, the output gap needs to move more to have the same effect on inflation. The high degree of inflation persistence in the Phillips curve also implies that the needed persistence in the output gap is lower to explain the observed persistence in the inflation. Inflation is also more responsive to *persistent* changes in the output gap due to the large coefficient on future expected inflation in the Phillips curve. In order for the model to match inflation dynamics and volatility, the output gap then needs to be somewhat less persistent compared to a situation with a smaller forward-looking term in the Phillips curve. The deviations from the other series are likely to be attributable to the differences needed for the output gap to better reflect underlying marginal costs, as discussed above.

The differences show up in the measures of correlation. Table 4 indicates that there is modest degree of co-movement, with correlation coefficients varying around 0.5. The lowest correlation is found between our estimate (BLM) and that of BN.[15] This is partly explained by the early 1990s, when all the output gaps except the BN output gap increase, with our measure suggesting a pronounced peak in 1994. Our estimate of the natural rate of interest rose sharply over the period 1993–1995 and the interest rate gap became negative (ref. Table 2). An expansionary monetary policy contributed to the output gap peak. The measures of concordance in the output gap, stated in Table 5, are slightly larger than the correlation coefficient for the alternative estimates. This implies that the estimates differ more in their sizes than their phases, that is, the different methods tend to pick the same phase for their respective output gap estimate. This is important information for Central Banks when comparing different gaps.

[11] We thank John Williams for providing us with the updated simulation results.

[12] Their output gap series was downloaded from http://www.be.wvu.edu/divecon/econ/basistha/gap.htm.

[13] The Hodrick Prescott method is a univariate statistical method designed to extract the low frequency component of a time series. Lambda penalizes the variation in the trend, and is determined a priori. A smoothing parameter of 1600 is commonly used in many international studies.

[14] The measure of concordance is useful when the focus of the analysis is on the sign of the gap and not necessarily its magnitude.

[15] In fact, the BN gap displays low correlation with all the other gaps as well.

4 Concluding remarks

This article provides estimates of the natural real interest rate, the output gap and the implicit inflation target for the US economy. The inflation target since 1994 has been remarkably stable around 2%. The natural real interest rate has, however, been varying a lot. The assumption often made in the monetary policy literature that the natural real interest rate is exogenous or even constant, might be very misleading and biasing the results. For the conduct of monetary policy, acknowledging the variation in the real interest rate and conducting policy in accordance with it, seems to be important.

By estimating the hybrid New Keynesian Phillips curve with a model-consistent estimate of the output gap, we find that the structure of the curve is very similar to that found by estimating the Phillips curve with the labor share of income. Our results are, therefore, a contribution to the debate of whether it is the output gap or the labor share of income, which provides the best representation for the inflation driving process. If the output gap is a good representation of the inflation driving process, our results support the idea that a simple two-variable system in inflation and the output gap (see, Clarida et al. 1999; Woodford 2003) is a good representation of the monetary policy transmission mechanism.

Appendix

Extra figures: Impulse response functions

See Figs. 4, 5, 6, 7, and 8.

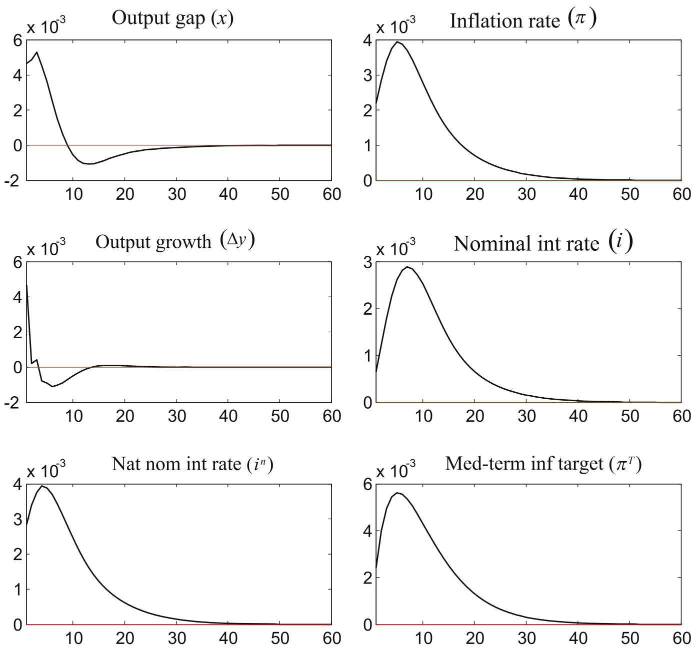

Fig. 4 Monetary policy shock to the medium-term inflation target, $\varkappa_t$. The impulse response function due to a shock to the medium-term inflation target

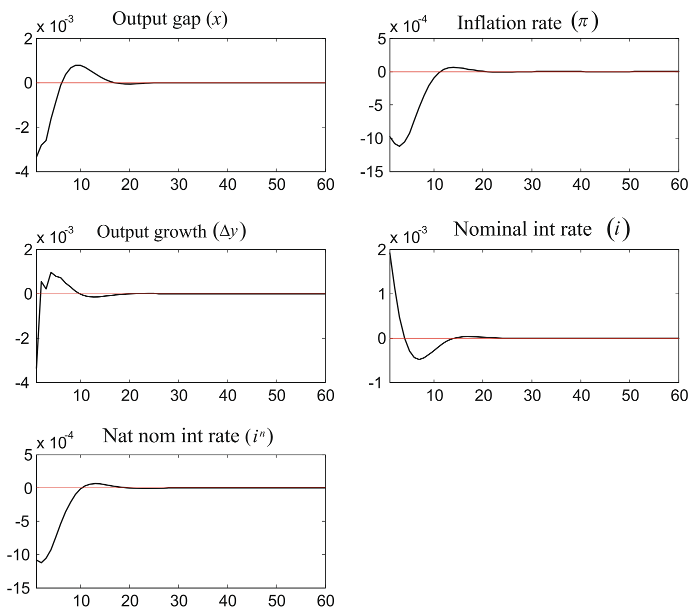

Fig. 5 Monetary policy shock to short-term interest rate, u_t. The impulse response functions due to a shock to the short-term interest rate

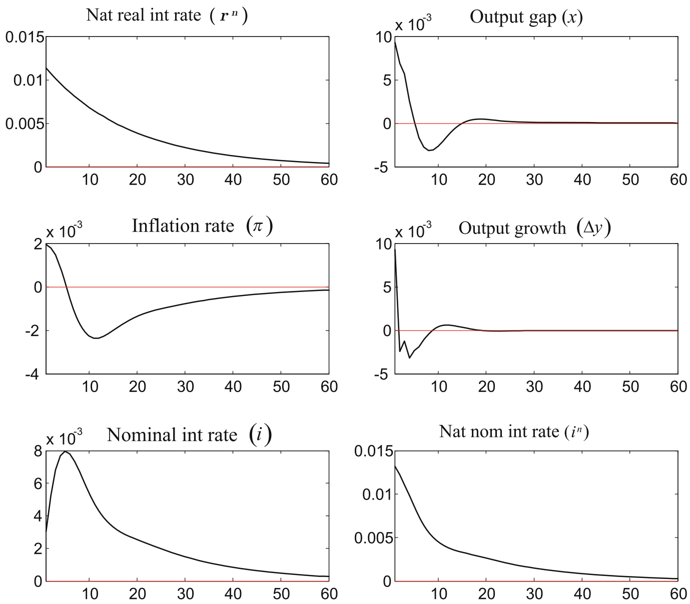

Fig. 6 Preference shock, $\tilde{v}_t$. The impulse response functions due to a preference shock

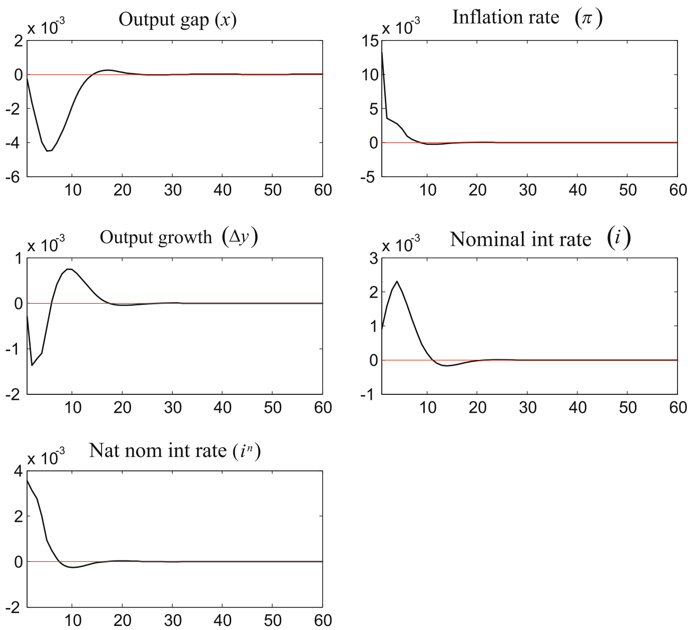

Fig. 7 Cost-push shock, ε_t. The impulse response functions due to a cost-push shock

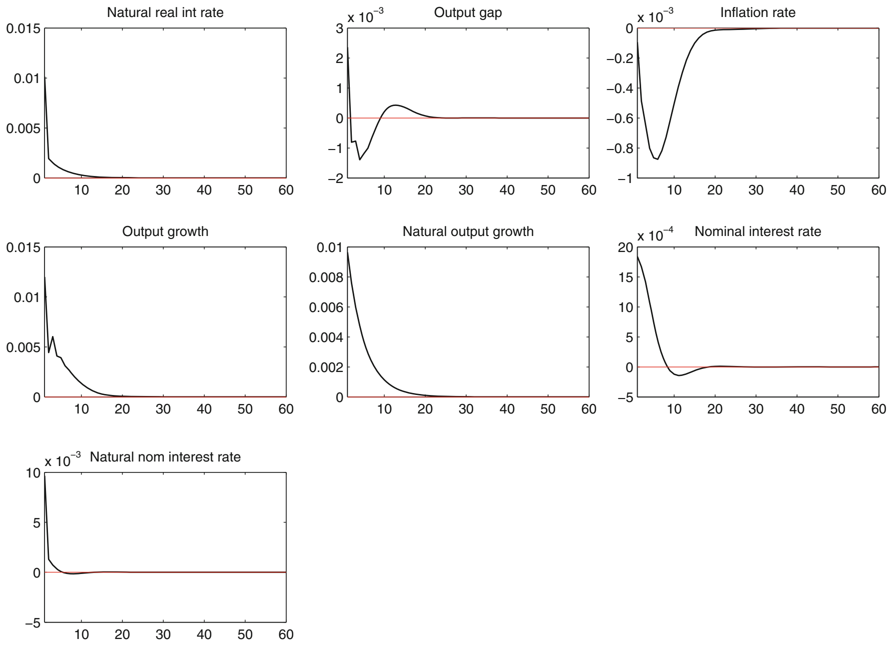

Fig. 8 Natural rate shock, ω_t. The impulse response functions due to the natural rate of output

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